AIRCRAFT LAYOUT
and
DETAIL DESIGN

BY

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WITH A FOREWORD BY

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AIRCRAFT LAYOUT AND DETAIL DESIGN

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Dedicated to

MY MOTHER AND FATHER,

who are responsible for any
success I may achieve

PREFACE

This text covers the three main subjects with which every good aircraft layout draftsman must be familiar:

1. Descriptive geometry
2. Detail design
3. Fitting analysis

Since, in order to get started on his layout, the draftsman must have a working knowledge of certain parts of descriptive geometry, only those portions that have common application to aircraft structures are included.

After the geometry of his problem has been solved, the draftsman must know good detail design practices. There has been no attempt made to stress shop methods or procedure of manufacture, as the basic idea of this book is to teach good aircraft draftsmen certain facts and principles so that they will be able to create a design that can be made in the shop with a minimum of effort and expense. There is much related technical information that enables the draftsman to understand better why he does certain things, but that is beyond the scope of this text. For example, under Sand and Permanent Mold Castings, the draftsman is taught to do and not to do certain things when he designs a casting. If he follows these instructions, he will design a part that can be made in the shop with a minimum of expense, although he may know very little about pattern shop and foundry practice. Much of this related technical information may be obtained from the references for outside reading that appear at the end of many of the chapters.

He should understand enough stress analysis to make a preliminary strength check on his own design. No attempt is made to determine loads or required margins of safety, as he is expected to obtain that information from the stress department. Sufficient stress analysis is taken up in this book to enable the draftsman to design intelligently parts having the required strength with a minimum of weight; parts that will not have to

be redesigned when the drawings are submitted to the stress department for an official strength check.

This book is designed for men who understand aircraft detail drafting and who want to prepare themselves for higher levels of aircraft layout and design. Hence no time is spent on the principles of orthographic drawing, as it is assumed all those using this text are good aircraft detailers.

Since this material was prepared for advanced students in aircraft schools and colleges, and for detailers in the engineering department of aircraft plants, constant reference is made to various specialty groups in these engineering departments, such as stress group, weight group, etc. A small aircraft engineering department may only have individuals as specialists in these various lines, in which case these specialists should be consulted.

Reference is also made to various manuals such as "Drafting Room Manual" and "Design Manual," since most engineering departments have collections of various data and design sheets bound in loose-leaf form for the guidance of their men. Many schools and colleges have copies of these manuals issued by large aircraft companies, which should be a valuable aid in supplementing this text when used in advanced classes.

Practical aircraft problems are provided—the solution of which should prove valuable to the student as well as an aid to the instructor. No specific information is given as to the form that should be followed for homework, but in general, the descriptive geometry and design problems should be drawn to as large a scale as practical; full size may be used in many cases. All work should be done in pencil on vellum if possible, thus following standard drawing-room practice. In the descriptive geometry problems, the reference lines should be colored red for easy identification, and the student should point to the lines or angles from which the answers were obtained, and note the answer on the layout. It has been found that this greatly facilitates the grading of the papers and simplifies the task of determining errors. The stress analysis problems should be solved by slide rule, since it is important for every layout draftsman or designer to know how to operate one. The solution of any of these problems by logarithms or by longhand should be discouraged owing to the time element involved. The use of the slide rule is not explained as that is considered too elemental for

this text. All examples in this book are slide-rule work; hence only accuracy of a 10-in. rule should be expected.

To complete the problems satisfactorily, the student should have the usual drawing instruments, scales, etc., as well as a drawing board, T square, and have no less than a 10-in. slide rule. A shorter rule is not recommended because of its inaccuracies. An expensive rule is not necessary, since a 10-in. Mannheim is considered by many to be ideal for general stress analysis work. The student should also own a revised edition of *ANC*-5, which may be purchased from the Superintendent of Documents, Washington, D. C. An Army-Navy Standards Book (AN Book) should be available for reference.

The following books, although not absolutely necessary for the solution of the problems, are considered valuable aids:

"Airplane Structures" by Niles and Newell, vol. I, John Wiley & Sons, New York, 1938.

"Machinery's Handbook," 10th ed., Industrial Press, New York.

The following Government publications are valuable for reference and may be obtained from the Superintendent of Documents, Washington, D. C.:

War Department *Technical Manual*, TM1-410, "Airplane Structures," Nov. 30, 1940.

War Department *Technical Manual* TM1-435, "Aircraft Sheet Metal Work," Feb. 10, 1941.

Civil Aeronautics Bulletin 27, "Pilots' Airplane Manual," September, 1940.

When reference is made to these publications, only the numbers are used, such as, TM1-410 or C.A.B. 27.

If difficulty is experienced in obtaining a copy of the following technical note from the National Advisory Committee for Aeronautics, Navy Building, Washington, D.C., a copy may be borrowed from the larger libraries:

N.A.C.A. Technical Note 567, "Tests of N.A.C.A. Airfoils in Variable-Density Wind Tunnel, Series 230," 1936.

If the student wishes to go further into descriptive geometry, either of the two following books, which proved valuable in the preparation of this text, are recommended:

"Applied Descriptive Geometry," by Frank M. Warner, McGraw-Hill Book Company, Inc., New York, 1938.

"Descriptive Geometry," by Smutz and Gingrich, D. Van Nostrand Company, Inc., New York, 1938.

The descriptive geometry portion of this book is based on the lecture "Simplified Methods of Projection" given on March 25, 1935, by Mr. D. J. Bosio, chief draftsman at the Santa Monica plant of the Douglas Aircraft Company, Inc. In this lecture, Mr. Bosio presented descriptive geometry in a simplified form by reducing the operations to a series of mechanical steps which are extremely simple to follow. This material was originally presented during the spring and summer of 1940 to two groups of students composed of aeronautical draftsmen. As a result of valuable suggestions and criticisms made by them, this part has been rewritten. Portions of the text material have been clarified, new examples added, and the subject matter arranged in a more logical manner. Appreciation is expressed here to Mr. Frank N. Fleming, assistant secretary and manager of the Douglas Aircraft Company's Washington, D.C., office, and to Mr. Bosio for their help and advice in the preparation of this part; and to Mr. Gene Vedder of the Douglas El Segundo engineering department for his valuable suggestions. Mr. Foster Gruber of the Douglas production design group also deserves much credit for his help in the preparation of the information on airfoils.

In the preparation of the material on detail design, Mr. Alfred Swanson of the Douglas production design group offered invaluable suggestions and criticisms which are deeply appreciated. Mr. C. J. Hertel, supervisor of production design, Mr. O. A. Wheelon of the production design group, and Mr. L. Best, assistant process engineer of the Douglas Tulsa Plant, deserve much credit for their kind cooperation and assistance.

Mr. David L. Moseley, chief of the Douglas Long Beach stress department, Mr. Alfred Swanson, and Mr. Edward Harpoothian of the Douglas Santa Monica engineering department have rendered valuable aid in the preparation of the material on fitting analysis by their many helpful suggestions and comments.

With few exceptions, the illustrations throughout this text are the fine work of Mr. Jack Dunford of the Douglas El Segundo engineering department, whose superior drawings greatly enhance the value of this material.

The author is deeply indebted to Mr. A. E. Raymond, vice-president and director of engineering, and to Mr. Fred W. Herman, chief engineer of the Long Beach plant, Douglas Aircraft Company, for without their kindly help and encouragement, this book would not have been written.

NEWTON H. ANDERSON.

SANTA MONICA, CALIFORNIA,
September, 1941.

CONTENTS

CHAPTER 4

GENERAL DESIGN CONSIDERATIONS

CHAPTER 5

CONNECTIONS

CHAPTER 6

FABRICATION METHODS

FOREWORD

To supply the topmost rung of the ladder of development of aeronautical drafting skill, where the treatment is so technical that the work is distinctly engineering, the Douglas Aircraft Company's educational supervisors have long felt the need of a new course of study. The desired course would be one with an intensely practical flavor and would closely delineate those situations and treatments that are encountered upon the job in an airplane factory's engineering department. Nonpractical considerations would be excluded so that all emphasis might be placed upon those training items which are of direct pay-roll value. Such a course would give thorough ground in the fundamentals by applying them invariably to the everyday work problems.

Much experience along this line of training has been gained by the Douglas organization in the coaching of new employees through a break-in period. Men with generalized college training only have been helped to design for practical manufacturing considerations. Men experienced at both design and manufacture but outside the aircraft industry have been helped to learn of the peculiar demands of this industry. All this has been carried out without any suitable textbook. In the spring of 1940, the Douglas Company incorporated such a layout and design drafting course among its sponsored public-school classes for extension training outside working hours for those employees who might be ambitious enough to attend voluntarily. Mr. Newton H. Anderson, one of the company's most experienced and highly skilled engineers, with background in both designing and project engineering, was asked to undertake not only the instruction but the full organization of teaching material as well. By fall, several more classes under Mr. Anderson's instruction had to be added to meet the demand. At this time, having seen conclusive evidence of the benefits derived from the training, the company placed Mr. Anderson on full-time teaching status in its

engineering department where, throughout a lengthy period of expansion of the engineering force, he discharged the duty of carrying group after group of new drafting employees through the necessary coaching to fit them for valuable work under aircraft design requirements.

In the spring of 1941 the course was raised to even higher prestige by blending into the new engineering defense training plan of the federally reimbursed colleges.

At this writing Mr. Anderson continues under University of California sponsorship to give to the industry and to industrial education at large the benefit of his thirteen years of aircraft engineering experience as reflected in the fine course of aeronautical engineering-drafting study that comprises this volume.

C. T. Reid,
Director of Education,
Douglas Aircraft Co., Inc.

June, 1941.

AIRCRAFT LAYOUT AND DETAIL DESIGN

CHAPTER 1

REQUIREMENTS FOR A GOOD LAYOUT

Before the three main subjects covered in this text are considered, it would be well to give a little thought to layouts, why and how they are made, and enumerate some of the more important information that should appear on them.

1:1. Why Layouts Are Necessary. The parts for an airplane are made from blueprints, which are usually obtained from drawings on tracing cloth, because a drawing on cloth produces a good print. Although the simple parts might be designed right on the cloth, they would probably be quite dirty and greasy from the draftsman's hands by the time all necessary approvals were secured, and would not make very good blueprints. Therefore, it is customary first to lay a problem out on separate paper and then, after it is completed, have it traced or redrawn by men known as "detailers." The men making these layouts are the layout men.

1:2. Materials Used. When a layout man starts out on a problem, he will have certain general ideas as to the ultimate solution; but invariably, as he progresses, his original ideas change, his supervisor offers suggestions, and he finds himself continually having to change his design. It is common practice to make these layouts on thin semitransparent paper known as "vellum," since it is relatively inexpensive and permits lines that have to be removed to be erased easily. They are always made in pencil, usually a fairly hard one. Many designers prefer a 4H to a 6H pencil for this work as it enables them to make clear sharp lines, an essential feature of a good layout.

1:3. Recommended Sizes. Practically all aircraft layouts are made to definite scales, which are as follows:

One-half size	Full size
One-quarter size	Double size
One-eighth size	Four times size
One-sixteenth size	

In general, the scale is chosen to suit the problem. Since the maximum recommended width of vellum is 36 in., the layout man should plan his work so that he can get his problem on this width of paper. Full size is preferred, but it is obvious that many parts cannot be laid out full size on 36-in. vellum. Hence he may have to use one-half or one-quarter size for his problem. For very small parts, it is often convenient to use twice or even four times size.

1:4. Arrangement of Layout. Before starting on any layout, the draftsman should spend the time necessary to acquaint himself with the problem at hand. This will enable him to arrange his layout in a neat and orderly manner; such a layout leads to clear thinking and thus to a more rapid solution of the problems. A crowded layout, where views overlap each other or where there is insufficient room to make the necessary projections, will prove to be more expensive than one where space seems to be wasted. On the other hand, a poorly planned layout may have the views so widely separated that it is difficult to follow through the projections; this wastes time of all concerned.

1:5. Use of Onionskin. For roughing out the changing ideas, it is generally a good idea to use plenty of onionskin, which is a very thin yet strong tissue paper, useful for sketching various ideas right on top of the main vellum layout. The generous use of onionskin enables the designer to present several ideas with a minimum of effort for the approval of his supervisors.

1:6. Information That Should Appear on a Layout. It is impossible to set forth definite rules to follow as to just what information should appear on a layout, since each aircraft engineering department has its specific rules which should be adhered to. In the absence of more definite information, a good layout contains the following:

All center lines, mold lines, and station lines should be clearly shown and marked, and all basic dimensions to reference lines or planes should be given. Often the layout man believes that he will always remember what a certain base line means, what sta-

tion it is, or what reference plane it is, and neglects to mark that information on his layout. If he has made an error in his layout, it may not be discovered until months later when the shop finds that the part does not fit in the airplane. Then the layout man tries to go back over his work, and the chances are that he has forgotten just what that line represented. Quite often, it is impossible for the man who made the layout to go back over his work; someone else must take the layout and determine where the error is. If complete information is not on the layout, much time will be wasted in determining what the various lines represent.

All surrounding parts should be shown in order to check clearances. It is a common fault of the inexperienced layout man to show these surrounding parts in great detail when in most cases the outline would be sufficient. Needless to say, he has wasted much valuable time in showing these surrounding parts in detail and, although he may have a pretty picture, the value of the layout has not been increased. For example, suppose it is desired to determine the clearance between a fuel strainer and an engine mount tube. In most cases, only two views are required, in each of which only the outline of the strainer would be necessary. Any interference would show without spending hours drawing all the detail of the strainer itself.

Moving parts should be shown in both extreme positions and in any intermediate critical positions that may be necessary. Imagine a bell crank in the aileron control system and its movement as the controls are operated. If the layout man is not alert, he may show it in the normal position only, and find there is ample clearance for the wing structure, overlooking the fact that it moves from this normal position constantly. It is unnecessary to elaborate on the difficulties that may arise due to such carelessness. No set rules can be given as to the amount of clearance necessary around moving parts. Machined parts may have very small clearances; other parts that vary in shape slightly, such as rough castings, should have more clearance. The layout man should consult his supervisor as to just what clearance is deemed necessary for each particular problem.

Sufficient sections and views should be shown so that all parts can be detailed directly from the layout without further calculations or design work. It is the common fault of many layout men to take the attitude, "Let the detailer figure it out." This may

be all right in some cases but, in the majority of cases, time and expense will be saved if the layout man shows enough sections so that the detailer will not have to "figure it out." The layout man is better trained and can "figure it out" more quickly and accurately than the detailer; that is why he is a layout designer. Also, when a layout is turned over to a detailer before it is completed—if he is a capable detailer—he may discover interferences, such as bolts riding radii, which often cause considerable redesign. If that layout man had exercised a little more caution and had done a thorough job, this delay would have been unnecessary.

Of course, there is also the danger of having too many views and sections on a layout. Some men apparently lack a complete understanding of their problem and have views shown for everything. If in doubt whether a certain section or view is necessary, it is advisable to consult your supervisor and possibly save much time. The shading or crosshatching of sections on layouts at times is advantageous; however, much time may be wasted on unnecessary crosshatching. It should be done only where it clarifies the construction, such as several parts that overlap. The use of colored pencils is considered by many designers to be superior to crosshatching; however, care should be exercised not to use indelible pencils, for it is impossible to erase their marks completely from a layout.

1:7. Coordination of Layouts. Layouts by other groups must be checked frequently to avoid interference, to provide supports as needed, and to ensure proper coordination throughout. Assume the problem is to provide a pulley bracket on the rear spar for part of the flap control mechanism. The layout man assigned this task should not only lay in all surrounding structure, but should check other layouts to be sure the electrical group has not put a junction box right where he wanted the pulley bracket. Perhaps it is a large airplane, and the heating and ventilating group have run an air duct right through that region. Without a thorough check of other layouts, it is often difficult, if not impossible, to tell whether or not the space chosen will provide the necessary clearances.

1:8. Adherence to Customer's Requirements. All parts must satisfy the requirements of the customer, whether it is the Army, Navy, or C.A.A. (Civil Aeronautics Authority). The Army has its "Handbook of Instructions for Airplane Designers," the Navy

its SD-24 "General Specifications for the Design and Construction of Airplanes," and the C.A.A. its .04 "Airplane Airworthiness." All layout men should be familiar with these design specifications; yet it is not recommended that a man spend much time looking up customers' requirements without first consulting his supervisor. In many cases the supervisor will know the answer; if he does not, he will probably refer to the specialist in whose field the question lies.

1:9. Practical Considerations. Extreme care should be used by the layout man to see that his design is practical. Wrench clearance must be provided for nuts and bolts, both of which must be accessible for installation. The layout man would do well to pause and ask himself, "Can I install that bolt?" "Is there sufficient room to tighten it?" He should also remember that all rivets must be driven and bucked. which requires generous clearance on both sides.

1:10. Useful Notes. There is much essential information on a layout in the form of notes which should include: the material from which the part is made; special heat-treats; numbers and titles of adjoining layouts; numbers of detail drawings when made; important fits and tolerances; and any other descriptive notes that may be of help to the detailer. The layout man must remember that detailers will usually make the actual production drawings and that, unless this complete information is on the layout, much time will be lost.

If it is necessary to follow a definite sequence of operations to put a complicated assembly together, it is wise to list this sequence right on the layout. For example: At the joint between the wing and fuselage of a pressure cabin transport, there are many parts coming together, and these parts have to be riveted together securely to make the cabin airtight. In order to drive all the rivets in this connection, a definite sequence of operations has to be followed; if this sequence is not followed, the shop will find it impossible to drive some of the rivets. If this sequence is on the layout and if the detailer shows this sequence on the assembly drawing, much time will be saved in the shop.

1:11. Required Dimensions. The question of necessary dimensions on a layout is one that can have no definite answer, for required dimensions will vary with the problem. In general, all master dimensions should appear on layouts, as well as tolerances

when close fits are required; but as layouts are made to scale, the majority of the dimensions may be left to the detailer to scale.

1:12. Strength Calculations. Many layouts represent parts that need a strength check, that is, if the parts being designed carry any appreciable load, a preliminary stress analysis will probably be required; and the principal loads should be indicated by arrows on the layout. These loads are obtained by the designer from the stress group, who also tell him the required margins of safety when using the given load. This information with its source and date should be noted right on the layout. The preliminary stress calculations should also be a part of the layout unless the calculations are extensive. These calculations may be in any convenient place, but it is important that they be on the layout, if possible.

1:13. Final Approval. After the layout has been completed, it is customary to have it approved before actual detailing begins. The supervisor should be consulted as to the necessary approvals, as each engineering department has its own systems.

1:14. Neatness and Accuracy. Accuracy is highly important in layouts and too much care cannot be exercised to draw lines carefully. In general, lines should be within $\frac{1}{64}$ in., which is not difficult after a little practice, especially if a sharp pencil is used. Neatness is also important, since a neat drawing is easier to understand and simplifies the task of the detailer. A smudged and messy layout indicates a careless workman and creates a bad impression on all who see it.

1:15. Timesaving Hints. When a man first starts making layouts, he usually finds it difficult to know just how much to show and tends to make finished detail drawings, instead of layouts. The layout man must remember that he is the creator, and work out his problems in a practical manner; he does not make a finished drawing for that is the duty of the detailer. It is usually a waste of time to add all detail to layouts. Do not spend unnecessary time on lettering since it is customary to trace parts of the layouts for the production drawings. Careful lettering is essential on shop drawings; in most cases it is a waste of time to draw guide lines and to letter carefully on the layout itself. Legibility is far more important on a layout than beautiful lettering, and there is nothing to be gained by underlining notes on a layout, as is customary on a production drawing. Some designers prefer to

write their notes in longhand on the layout, a practice that saves much time and is permissible if the writing is clear and distinct. Some layout men waste much time in showing all detail of castle nuts and cotter pins, which in most cases is unnecessary. It is true that often the outline of the nut is necessary in order to check for clearance, but in many cases, a note such as "AN4 bolt and AC365 nut" would be sufficient. It is usually unnecessary to calculate the length of the bolt such as "AN4-7 bolt"; the detailer can do that. Much time may also be wasted in showing rivets. A circle ○ or a + is usually sufficient to show the location of rivets; the use of a double circle ◎ for rivets is to be discouraged, as it consumes unnecessary time.

1:16. Summary. It was stated earlier in this chapter that these are only general recommendations for layouts in the absence of more specific information. If there is a specialty manual for the group in which the man working, such as a hydraulic manual for the hydraulic group, he should follow that; or the supervisor may, in many cases, realize that some particular layout need not be so complete as outlined in this text. In any case, *the layout should be made to suit the supervisor.*

Problems

1:1. *a.* For a specialty group, where is information to be obtained about the design and layout problems?

b. What are some of the practical considerations the designer must bear in mind?

1:2. Suppose the supervisor desires something to be done contrary to information contained in this chapter, what procedure should be followed?

1:3. When should the instructions given in this text be followed?

1:4. Before actually starting to draw lines for the layout problem, what should be done? Why?

1:5. What is onionskin and why is it advisable to use plenty of it?

1:6. How should the layout man determine whether other equipment or structure is in the region he will be working on?

1:7. Tell whether the following statements are true or false:

a. Center lines and mold lines must be clearly shown and marked and all basic dimensions to reference lines or planes must be given.

b. No surrounding parts should be shown.
c. Never show stress calculations on a layout.
d. Moving parts should be shown in both extreme positions and in any intermediate critical position.
e. Sufficient sections and views should be shown so that the detailer will not have to find the layout man to ask questions.
f. The supervisor should not be bothered about detail requirements of the customer. The layout man should look them up himself.
g. Layouts should invariably be approved by the chief draftsman before detailing can start.
h. All layouts should be made full size, half size, one-fifth, one-tenth, or one-twentieth size.
i. Fancy lettering is important in layouts.
j. All layouts should be made on vellum in pencil.

1:8. *a.* Why are layouts considered necessary?
b. The general notes usually include what items?

1:9. Explain why accuracy and neatness are important.

1:10. Tell whether the following statements are true or false:
a. The views on a layout should overlap wherever possible.
b. The layout man should let the detailer figure out the various sections for himself.
c. The supervisor should be consulted when in doubt as to the necessity of a view or section.
d. All notes on layouts should be underlined.
e. Never show all detail of castle nuts and cotter pins on layouts.
f. All layouts should be completely dimensioned as it is not permissible for the detailer to scale them.
g. Sections are crosshatched only when necessary to clarify the details of construction.
h. If colored pencils are used, they should be indelible pencils.
i. The loads and margins of safety to maintain in using these loads are obtained from the stress group.
j. Rivets should always be indicated on a layout by a double circle ◎.

CHAPTER 2

DESCRIPTIVE GEOMETRY FOR AIRCRAFT LAYOUT

2:1. Determining the True Length of Any Line. As set forth in the Preface, it is assumed that all students using this book are already competent aeronautical draftsmen. The principles of projection illustrated throughout this text are the same as are applied daily in ordinary detail drafting. In other words, descriptive geometry as presented herein is orthographic drawing applied to the solution of more advanced problems.

However, there is one difference: In the complicated problems that follow, it is necessary that there be some consistent place from which to take measurements and some standard method of measuring. The reference line system provides these basic requirements and is therefore followed throughout; this system should be used by the student in all his home assignments.

Although the student may not have been conscious of doing so, he has been using reference lines in making the several views of an object required in detail drafting. These reference lines may have been one edge of the object, its center line, or some similar known line. The multiplicity of lines in some problems of descriptive geometry, however, require that *the reference lines must be separated from the views* to avoid confusion and minimize error. This is the fundamental difference in the method of projection between detail drafting and descriptive geometry.

The lines labeled "reference lines" in this text are called "datum lines" by Profs. Smutz and Gingrich in their "Descriptive Geometry"; Prof. Warner refers to them as "folding lines." Whatever they are called, they serve the same purpose.

It is well known that when two orthographic views are given, a third view may be drawn. For example, if a plan view (looking down or up, usually the former) and a front view (looking aft, that is, facing the pilot) are given, the side view (looking inboard toward the center line of the airplane) may be drawn. This is

one of the fundamental principles of orthographic drawing and will not be explained in this text as it is considered too elementary.

In descriptive geometry, *any two orthographic views may be used,* although there may be times when it would be more convenient to draw a third view, working from this third view and one of the given views. Some problems may be simplified by drawing this third view; for example, a front view of a landing gear may show the brace tubes where one is behind the other, although the plan view would show one attaching to the fuselage some distance behind the other. In this case, it would probably be better to work from the side and plan views instead of from the front and plan views.

Illustration

Given: Two orthographic views of any line in space, such as the front and side views of the line A-B in Fig. 2:1.
To find: The true length of the line A-B.

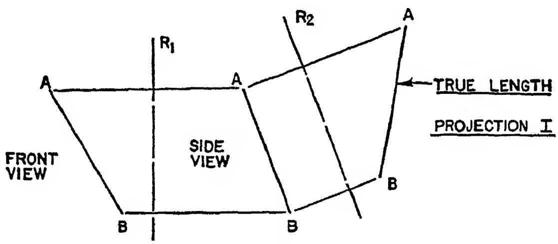


FIG. 2:1.

Discussion: This is a simple descriptive geometry problem, and one with which most students are probably familiar. However, for those who have never studied this subject, it will be carried through in detail.
Solution: To see the true length of a line, that line must be in the plane of the paper.

Operations:

1. At any point between the front and side views, draw a reference line R_1, perpendicular to the construction lines. The first reference line R_1 is always drawn perpendicular to the construction lines between the two given views.

2. At any point to the right of line A-B in the side view, draw another reference line R_2, parallel to line A-B of the side view.
3. Draw construction lines normal to R_2 through points A and B of the side view. Points A and B lie on these lines for projection I.
4. These points are located from R_2 for projection I the same distance as they are from R_1 in the front view.
5. Connecting points A and B in projection I will place line A-B in the plane of the paper. The distance between A and B will then be the true length of line A-B.

Note: One of the most important principles of descriptive geometry should be emphasized at this time. In transferring measurements from one view to another, there must always be an intervening view used for purposes of projection only and not measurement. Hence, in locating A and B in projection I, the dimensions are taken from the front view, the side view serving only to determine the direction in which projection I is made. If the student will keep this basic principle in mind, the more complicated problems will be greatly simplified.

References

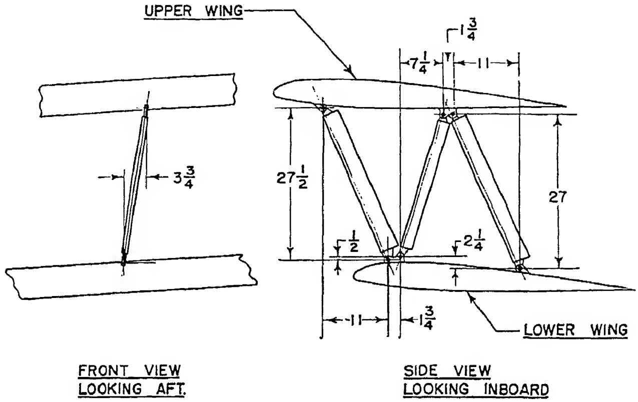
"Applied Descriptive Geometry," by Frank M. Warner, pp. 19–22.
"Descriptive Geometry," by F. A. Smutz and R. F. Gingrich, Art. 52, pp. 45–47.

Problems

2:1:1. *a.* Upon what does the accuracy of the result depend?
b. Is it necessary to have the front and side views given, or will *any* two orthographic views be sufficient?
c. In Fig. 2:1, why is the distance between point B and line R_2 in projection I the same as the distance between point B and line R_1 in the front view?

2:1:2. The accompanying diagram shows the interplane struts of a biplane wing. Determine the true length of each of the struts, assuming that the $3\frac{3}{4}$-in. dimension in the front view is constant for all three struts. The usual fairing around the

ends of these streamline struts has been omitted to avoid complicating the picture.



PROB. FIG. 2:1:2.

2:2. Determining the True Angle between Any Two Intersecting Lines. Many problems are encountered in actual aircraft practice where it will be necessary to determine the true angle between lines. For example, in welded steel tube engine mounts, someone must determine the true lengths of the various tubes and determine the true angle between them to provide suitable gussets and supports. In working out the geometry, it is customary in the majority of cases to use only the center lines, for in many of the views nothing would be gained by having the tubes show. A very simple method of determining the true angle between two lines is a continuation of the method learned in Art. 2:1 for determining the true length of a line, since the true angle is measured in a view where both lines appear in their true lengths.

Illustration

Given: Two orthographic views of two intersecting lines, such as lines *A-B* and *B-C* intersecting at *B* in the plan and front views of Fig. 2:2.

To find: The true angle between lines *A-B* and *B-C*.

Discussion: The method outlined here is admittedly not the shortest one, but it is a simple way to solve this type of problem. It is presented in the belief that it is better to understand a simple method, although it involves an extra projection, than to introduce at this time a shorter way which is more complex and difficult to understand.

Solution: The true angle may be measured only when both lines are in the plane of the paper.

Operations:

1. At any point between the plan and front views, draw a reference line R_1, perpendicular to the construction lines.
2. At any point to the right of the line B-C in the front view, draw another reference line R_2, parallel to B-C.
3. Reference line R_3, parallel to R_2, is inserted merely for convenience in establishing the first projection in a position where it will not interfere with the plan view.

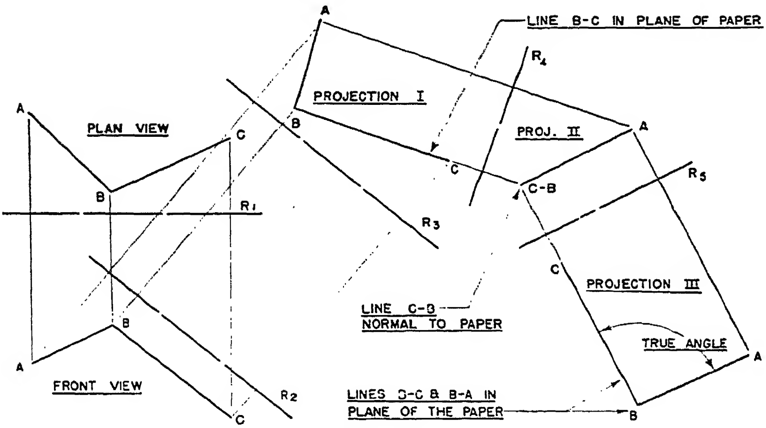


FIG. 2:2.

4. Draw construction lines normal to R_2 and R_3 through points A, B, and C of the front view. Points A, B, and C lie on these lines for projection I.
5. These points are located from R_3 for projection I the same distance as they are from R_1 in the plan view. In projection I the line B-C is in the plane of the paper.
6. At any point to the right of point C in projection I, draw another reference line R_4, normal to the line B-C.
7. Draw construction lines normal to R_4 through the points A, B, and C.
8. These points are located from R_4 for projection II the same distance as they are from R_2 in the front view. In projection II, line C-B is perpendicular to the paper.
9. At any point below the line A-B in projection II, draw another reference line R_5, parallel to A-B.

10. Draw construction lines normal to R_5 through the points A, B, and C.
11. These points are located from R_5 for projection III the same distance as they are from R_4 in projection I.
12. Connect the points A, B, and C in projection III. Lines A-B and B-C are now in the plane of the paper; their true lengths are shown, and the angle between them is the true angle.

Reference

"Descriptive Geometry," by F. A. Smutz and R. F. Gingrich, Art. 66, p. 69.

Problems

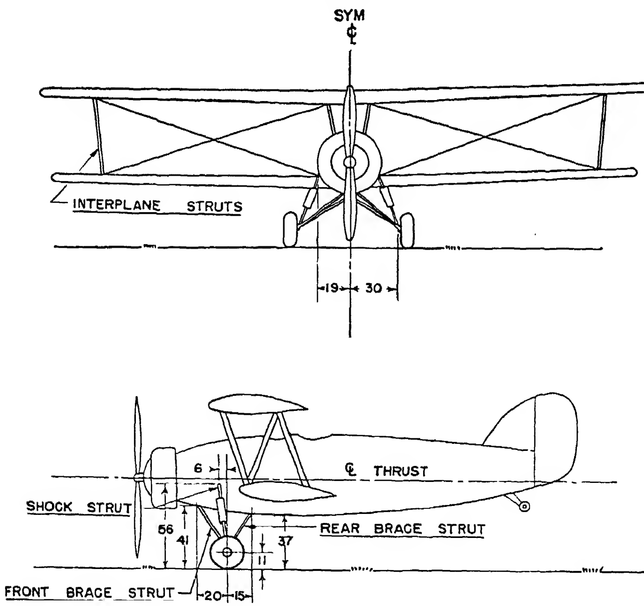
2:2:1. *a.* Was it necessary to get projection I from line B-C in the front view?

b. Could any two orthographic views have been used to determine the true angle between the lines?

c. Was reference line R_3 necessary?

d. Does accuracy have any effect on the results of a layout?

2:2:2. Using the data in the accompanying diagram, determine



PROB. FIG. 2:2:2.

a. The true length of the front brace strut.
b. The true length of the rear brace strut.
c. The true length of the shock strut.
d. The true angle between the front and rear brace struts.
e. The true angle between the front brace strut and the shock strut.
f. The true angle between the rear brace strut and the shock strut.

2:3. Determining the Clearance between Any Two Apparently Intersecting Lines. The layout man must often find the clearance between a cable and a strut, or between a cable and some brace tubes. This problem in terms of descriptive geometry becomes the clearance between lines, since the logical solution would involve the use of the center lines of the cables, tubes, etc. The word "clearance" is used to denote the shortest distance between two lines This shortest distance between two lines is measured perpendicular to both lines.

Illustration

Consider a V brace and a cable, which apparently intersect in both views given, and determine the actual clearance between them.

Given: Two orthographic views of a V brace whose center lines are 2-1-3 in Fig. 2:3 and a cable represented by its center line *A*-*B*.
To find: The actual clearance between the V brace and the cable.
Solution: Obtain a view in which one line is normal to the paper (seen as a point), then measure the shortest distance between the point and the nearest line. In this problem, the line *A*-*B* should be normal to the paper.

Operations:

1. At any point between the plan and the front view, draw a reference line R_1 perpendicular to the construction lines.
2. At any point below the line *A*-*B* in the front view, draw a reference line R_2 parallel to the line *A*-*B* in the front view.

3. By use of projection lines drawn perpendicular to reference line R_2, locate the points A, B, and 1, 2, 3 for projection I. The distances of these points from R_2 in projection I are the same as the respective distances from R_1 in the plan

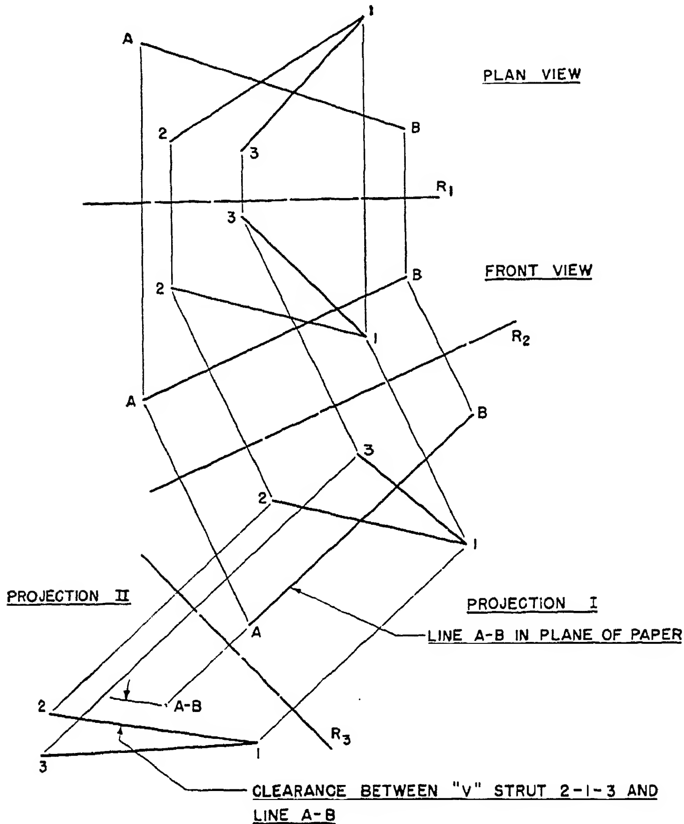


FIG. 2:3.

view. In projection I, the line A-B is in the plane of the paper.

4. At any point to the left of point A in projection I, draw reference line R_3, normal to line A-B.
5. Draw construction lines perpendicular to R_3 through the points A, B, and 1, 2, 3, and locate these points for projection II. The distances from R_3 to the points in projection II are

the same as the distances from R_2 in the front view. In projection II, the line A-B is perpendicular to the paper and appears as a point.

6. The actual clearance between the line A-B and the other two lines may now be determined in projection II by measuring the shortest distance from point A-B to each of the other two lines. In this problem, the V strut leg 2-1 is nearest the cable; hence the minimum clearance is measured between line 2-1 and the point A-B. In actual practice, of course, it would be necessary to subtract from this dimension an amount equal to half the diameters of the tube and cable to determine the net clearance.

This example is necessarily a simple one to illustrate a method, but the same principle used in this article may be applied to any member. Suppose a channel or an H section passes through the V brace instead of the cable; it would still be necessary to obtain a view where the channel or H section is normal to the paper. However, instead of one line representing the center line of the cable, it would be necessary to have two or more lines representing the corners of the section. The required lines would be drawn in the given views and carried through the various projections into the view where these lines appear as points, which establishes the position of the structural section in relation to the V-brace tubes. No matter what sections are used, it is always necessary to have a view where the section is normal to the paper in order to determine the clearance.

Reference

"Applied Descriptive Geometry," by Frank M. Warner, Art. 12.3, pp. 41–44.

Problems

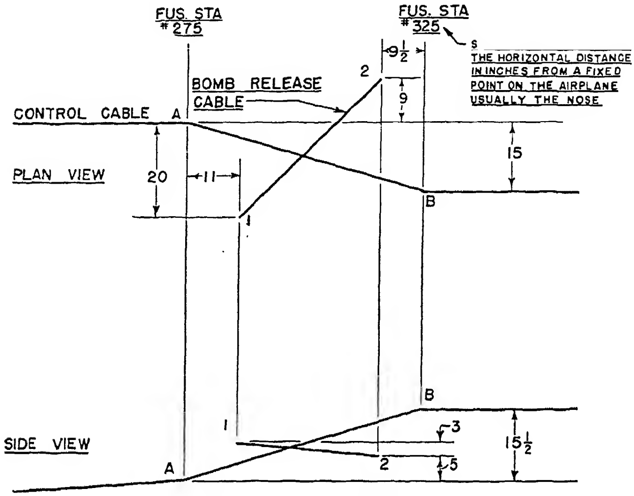
2:3:1. *a.* Why was line A-B chosen as the one to project into the plane of the paper?

b. Could the clearance have been determined by projecting either of the other two lines into the plane of the paper?

c. Could any two orthographic views be used?

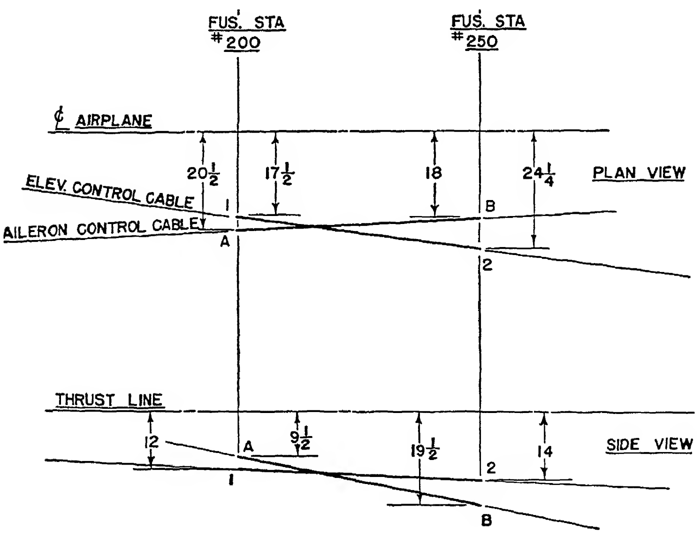
2:3:2. A bomb-release cable passing into the bomb bay of an airplane apparently interferes with a control cable as shown in the

accompanying diagram. Does an actual interference exist? If not, what is the clearance?



PROB. FIG. 2:3:2.

2:3:3. The two accompanying views show an aileron cable and an elevator cable in their desirable positions. If there is any inter-



PROB. FIG. 2:3:3.

ference, the forward end of the aileron cable at A is the only end that can be moved, and it must be moved *outboard only;* no vertical movement is permitted. How far outboard from its present position must A be moved to provide 2-in. clearance between the two cables? Using phantom lines, dash and two dots (— · · — · · —), show the new location of the cable in the plan view.

2:4. Determining the Piercing Point of a Line on a Plane and the True Angle between the Line and the Plane. Thus far in this chapter, only lines have been discussed. The next logical step is the consideration of planes. A plane is defined by Prof. Warner

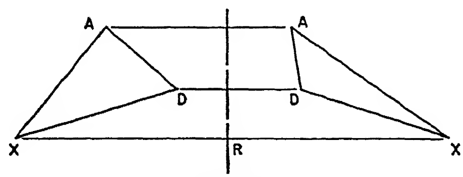


FIG. 2:4.

in his textbook on descriptive geometry as "A surface such that, if any two of its points are connected by a straight line, that straight line always lies wholly on the surface; or every point in that line is on the surface."

It must be understood that a plane extends to infinity; however in solving problems it is necessary to set up definite boundaries on that plane. In this text, usually, the boundaries of a plane will be three points connected by straight lines forming a triangle as shown in Fig. 2:4.

Four points could be used, forming a rectangle; however that introduces an additional point which must be carried through the various projections, making the solution longer. Hence, it is recommended that students use three points to represent a plane wherever possible. Quoting Prof. Warner again, "For the purpose of solving problems, all planes and all straight lines may be considered indefinite in extent, even though they may really have a definite size. A plane or a line, even though it is on a definite object, may be extended beyond the limits of the object if this extension makes the solution easier. The position of the plane or the line in space is not altered by such extension."

Illustration

Given: Two orthographic views of a line *A-B* and a plane 1-2-3 (see Fig. 2:5).

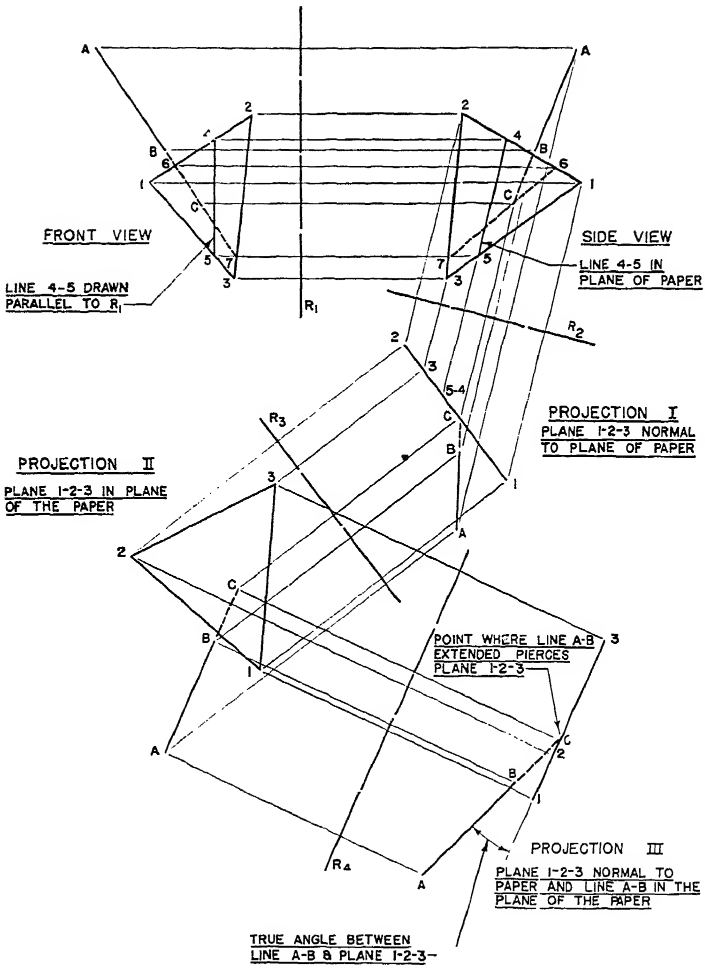


Fig. 2:5.

To find:

1. The point where the line *A-B* extended pierces the plane 1-2-3.

2. The true angle between the line and the plane.

Discussion: If the problem were merely to find the point where the line pierces the plane, only the two given orthographic views would be needed, as is explained in operations 1 to 4 of this article. But when the true angle between them is required, other projections are needed.

Solution:

1. The point where the line pierces the plane is found by the projecting plane method, described fully under operations 1 to 4 inclusive.
2. The true angle between the plane and the line is measured in a view where the plane is normal to the paper and the line is *in* the plane of the paper.

Operations:

1. In the front view of Fig. 2:5, consider the line *A-B* as the edge view of a plane drawn normal to the paper, and extend that plane until it intersects the plane 1-2-3 at points 6 and 7.
2. Project these points 6 and 7 into the side view and draw the line 6-7, which is the intersection of that imaginary plane on the plane 1-2-3. Hence line 6-7 lies both in the imaginary plane and in plane 1-2-3.
3. In the side view, extend the line *A-B* until it intersects the line 6-7 at point *C*. Point *C* is the piercing point of line *A-B* on the plane 1-2-3.

   The reason for this may be explained as follows: lines 6-7 and *A-B* are both in the imaginary plane, hence line *A-B* may be extended until it intersects line 6-7 at point *C*. In operation 2, it was pointed out that line 6-7 is not only on the imaginary plane but also on plane 1-2-3; therefore, point *C* is also in plane 1-2-3. Since point *C* is in plane 1-2-3 and is on an extension of line *A-B*, it must be the point where line *A-B* pierces plane 1-2-3.
4. Project point *C* back onto the front view, thereby locating the piercing point in both given views.

   The first step in determining the true angle between the line and the plane is to obtain a view in which the plane is seen as a line. The layout man frequently needs to obtain

a view of an oblique plane so that the plane will be normal to the paper; hence the student should thoroughly familiarize himself with operations 5 to 7, which illustrate a convenient method of accomplishing this projection.

5. At any point between the front and the side view, draw a reference line R_1, perpendicular to the construction lines.
6. In the front view draw *any* line 4-5 parallel to the reference line R_1 and project 4-5 into the side view. The line 4-5 in the side view is in the plane of the paper. To understand this operation, think of the line 4-5 in the front view as an edge view of a plane drawn perpendicular to the paper. Then line 4-5 becomes the line of intersection between this plane and plane 1-2-3, and lies in *both* planes. In the side view, this plane will lie in the plane of the paper, and any line on that plane will also be in the plane of the paper. Since line 4-5 is on that plane, it must be in the plane of the paper in the side view.
7. At any point below the side view, draw a reference line R_2, normal to the line 4-5. By means of projection lines drawn perpendicular to reference line R_2, locate the points A and B and 1, 2, 3, 4, and 5 for projection I. The distances of these points from R_2 in projection I are the same as the distances of these points from R_1 in the front view. The plane 1-2-3 in projection I is normal to the paper.

   After the given plane is normal to the paper, take a view showing the plane in the plane of the paper, projecting the known line along with it. Finally project it normal to the given line, thus obtaining a view where the line is in the plane of the paper and the plane is normal to the paper.
8. At any point below projection I, draw a reference line R_3 parallel to the line 1-2-3. Draw construction lines perpendicular to R_3 through the points A and B and 1, 2, and 3 and locate these points for projection II. The distances from R_3 in the projection II are the same as the distances from R_2 in the side view. In projection II, the plane 1-2-3 is in the plane of the paper.

Note: The line 4-5 was not carried into projection II, as that line had served its purpose in the side view when it determined the direction for projection I.

9. At any point to the right of projection II, draw a reference line R_4, parallel to the line *A*-*B*. Draw construction lines perpendicular to R_4 through the points *A* and *B* and 1, 2, and 3 and locate these points for projection III. The distances from R_4 in projection III are the same as the distances from R_3 in projection I. In projection III, the plane 1-2-3 is normal to the paper and the line *A*-*B* is in the plane of the paper. In this view, the true angle between the line *A*-*B* and the plane 1-2-3 may be measured.
10. In projection III, extend the line *A*-*B* until it intersects the line 1-2-3 at point *C*. This point *C* is the piercing point of the line *A*-*B* on the plane 1-2-3.
11. Point *C* may now be carried back through projections II and I into the side and front views. If all the work was accurately done, point *C* as determined in operations 1 to 4 will coincide with the location just obtained.

Before attempting to solve the following problems, it is advisable to give a little thought to the simpler aspects of the geometry of an airplane wing.

In the preliminary layout stage, a horizontal plane is passed through the wing at some convenient point and is known as the "reference plane." After the location of this plane is once established, it is never changed; engineering, loft, tooling, and all shop departments use this reference plane as a common datum plane.

The drawings for Prob. 2:4:3 are in what is known as "rigged position." That is, the plan view is drawn looking down on the wing when the thrust line or the fuselage reference line of the airplane is horizontal. Therefore, in the front view (looking aft) the thrust line or fuselage line is normal to the paper. The airplane is then referred to as in "flying position" or "rigged position."

The reference plane is rotated for the dihedral angle, which is 6 deg. in our problem and, in the front view, is seen as a line since in this view it is perpendicular to the paper.

After the reference plane has been established, the wing ribs are usually set in a fore-and-aft position but placed at 90 deg. to the reference plane. Hence, in the front view the plane of a normal rib (a rib placed at 90 deg. to the wing reference plane) is seen as a line at right angles to the wing reference plane.

The wing stations are measured along the wing reference plane, measurements being made perpendicular to the center line of the airplane in the plan view.

The term "skewed rib" is applied to a rib that is bent inboard or outboard along its line of intersection on a spar. In Prob. 2:4:3, the nose rib 63¼ was bent inboard about its intersection on the front spar until the leading edge of the rib coincided with the wing station 43½.

Note that in skewed ribs there is no rotation about the reference plane. In the diagram for Prob. 2:4:3, the line of intersection of the front spar *A-B* is still at 90 deg. to the wing reference plane.

This is in direct contrast to "cant ribs," where the rotation of the rib is about the wing reference plane. This often occurs where it is desirable to have a rib vertical to the ground, such as a landing gear bulkhead. In the engineering department it is usually referred to as a vertical rib; however it is simply a rib, canted about its line of intersection on the wing reference plane until it is vertical to the ground.

It is wise for the student to remember the difference between "cant" and "skewed" ribs, for they are terms often used in the design of airplanes.

After the student understands this explanation of some of the simpler aspects of wing geometry, he is ready to lay out the following problems.

References

"Applied Descriptive Geometry," by Frank M. Warner, Art. 18.3, pp. 47–49.

"Descriptive Geometry," by F. A. Smutz and R. F. Gingrich, Art. 65, pages 67–68; Arts. 72 and 73, pp. 80–82.

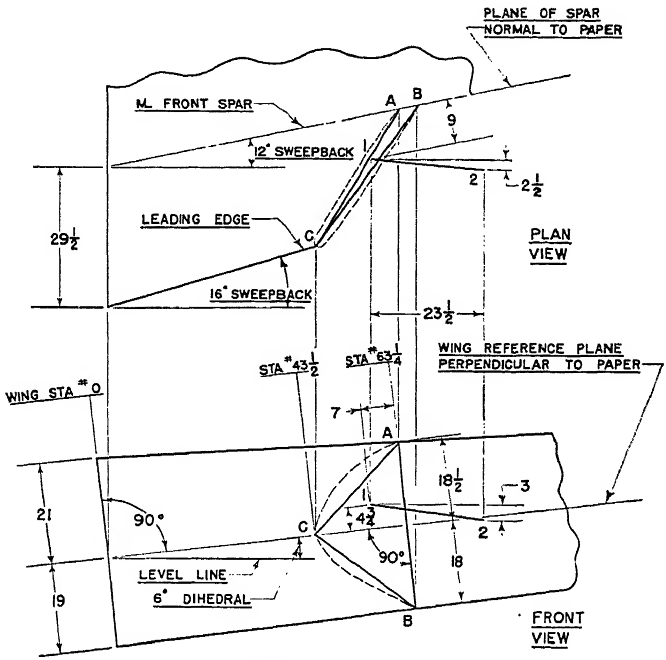
Problems

2:4:1. *a.* In operation 3, Fig. 2:5, why is the intersection of line *A-B* extended, and line 6-7 the piercing point of the line on the plane?

b. Why is line 4-5 in the plane of the paper in the side view?

2:4:2. In the landing gear shown in Prob. 2:2:2, what is the true angle between the plane containing the front and rear brace struts, and the shock strut?

2:4:3. The accompanying diagram shows a portion of a wing. It was necessary to attach a support (line 1-2) to a skewed nose rib of a nacelle for the engine controls. Find the true angle between the plane of the skewed nose rib (*A*-*B*-*C*) and the support.



PROB. FIG. 2:4:3.

2:5. Determining the Intersection of Planes. Frequently complicated problems are encountered involving the intersection of planes. This is especially true in cases where both known planes are oblique planes (neither parallel nor perpendicular to the paper in either of the two given orthographic views) and the defined boundaries of these known planes do not intersect.

In order to prepare for these more advanced problems, it is necessary first to consider simpler cases. This article discusses these various conditions progressively.

2:5:*a*. One oblique plane. One plane normal to the paper. Boundaries intersect.

2:5:*b*. One oblique plane. One plane normal to the paper. Boundaries do not intersect.

2:5:*c*. Two oblique planes. Boundaries intersect.

2:5:*d*. Two oblique planes. Boundaries do not intersect.

Of course, there is more than one method of determining the intersection of planes; however in this article only one will be used. In order that the student may follow this procedure, it is essential that he thoroughly understand the principles used in Art. 2:4 to determine the intersection of a line on a plane.

Illustrations

2:5:a. One oblique plane. One plane normal to the paper. Boundaries intersect.

Given: The plane 1-2-3 perpendicular to the paper in the plan view of Fig. 2:6. The oblique plane *A-B-C* passes through plane 1-2-3.

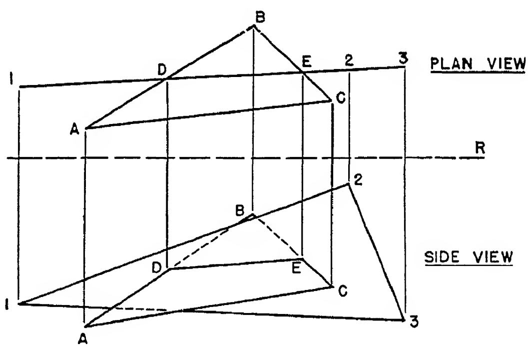


Fig. 2:6.

To find: The line of intersection of these two planes.

Discussion: Here a fundamental fact is used: any point on both planes lies on the line of intersection of these planes.

Solution: In the plan view of Fig. 2:6, line *A-B* intersects plane 1-2-3 at point *D* and line *B-C* intersects plane 1-2-3 at *E*. It is obvious that these points *D* and *E* lie on both planes, hence lie on the required line of intersection. Projecting these points into the side view and drawing the line *D-E* give the line of intersection.

2:5:b. One oblique plane. One plane normal to the paper. Boundaries do not intersect.

Given: In Fig. 2:7, the plane 1-2-3 is normal to the paper in the plan view. The oblique plane *A-B-C* does not intersect plane 1-2-3.

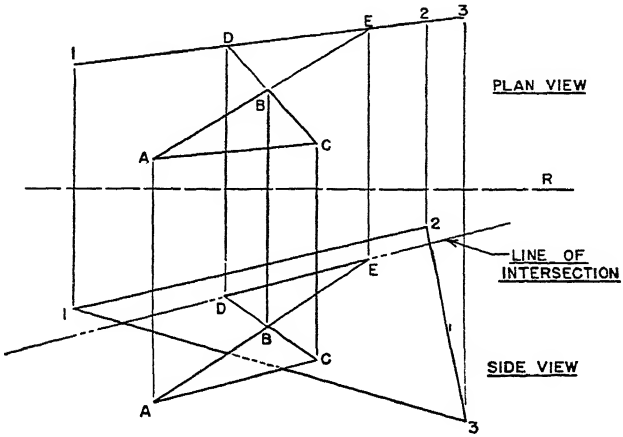


Fig. 2:7.

To find: The line of intersection on plane 1-2-3 when plane *A-B-C* is extended.

Operations:

1. Extend line *A-B* in the plan view until it intersects plane 1-2-3 at point *E*.
2. Extend line *C-B* in the plan view until it intersects plane 1-2-3 at point *D*.
3. Draw projection lines normal to the reference line through points *D* and *E* of the plan view.
4. Extend line *A-B* in the side view until it intersects the projection line drawn through point *E* of the plan view. This locates point *E* in the side view.
5. Extend line *C-B* in the side view until it intersects the projection line drawn through point *D* of the plan view. This locates point *D* in the side view.
6. Connect points *D* and *E* in the side view. The line *D-E* is the intersection of plane *A-B-C* on plane 1-2-3.

2:5:c. Two oblique planes. Boundaries intersect.

Given: Two oblique planes *A-B-C* and 1-2-3 so located that they intersect (see Fig. 2:8).

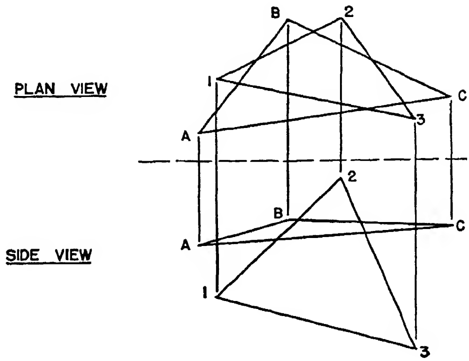


FIG. 2:8.

To find: The line of intersection of these two planes.

Solution: Choose *any* line in one of the planes and find its intersection on the other plane by the method described fully in operations 1 to 4 of Art. 2:4. This point will lie on *both* planes. Then choose another line in one of the planes and find its intersection on the other plane by the same method. This point will also lie on *both* planes. By connecting these two points, the line of intersection of the two planes may be determined. To check his work, the student should choose a third line and likewise find its intersection on the other plane. If the work has been correct, all three points will lie on a straight line.

In the example illustrated in Fig. 2:9, three points of intersection were determined in order to obtain a check on the first two points. It will be noted that Fig. 2:9 is similar to Fig. 2:8, but is enlarged to clarify the construction lines and various points of intersection.

Operations:

1. Draw any line 4-5 in plane 1-2-3 in the side view.
2. Extend this line until it intersects line *A-B* at point *D* and line *A-C* at point *E*.

3. Project points 4, 5, *D*, and *E* into the plan view and draw lines 4-5 and *D-E*.
4. Extend these lines in the plan view until they intersect at *X*. Point *X* lies in *both* planes.
5. Now draw any line 6-7 in plane 1-2-3 in the side view.

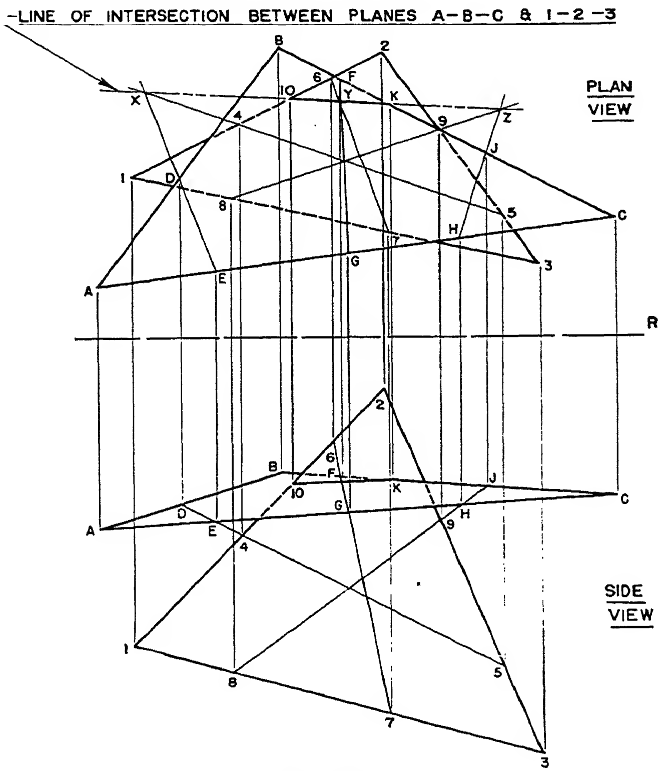


FIG. 2:9.

6. The intersection of this line on line *B-C* is point *F* and on line *A-C* is point *G*.
7. Project these points 6, 7, *F*, and *G* into the plan view and draw the lines 6-7 and *F-G*.
8. The intersection of these lines in the plan view is point *Y*, which lies on both planes.

Two points *X* and *Y*, lying on both planes, have now been determined; a straight line drawn through these points will

give the line of intersection of the two planes. However, it is recommended that a third point be located as a check on the first two, all three points lying on a straight line if the work is correct. The third point may be determined as follows:

9. Draw any line 8-9 in plane 1-2-3 in the side view.
10. Extend this line until it intersects line *B*-*C* at point *J* and line *A*-*C* at point *H*.
11. Project these points 8, 9, *J*, and *H* into the plan view and draw the lines 8-9 and *J*-*H*.

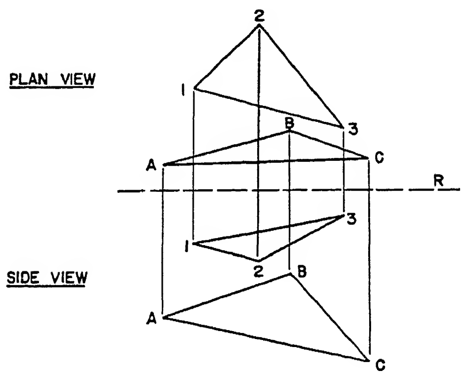


Fig. 2:10.

12. Extend these lines in the plan view until they intersect at *Z*. Point *Z* lies on both planes.
13. Connect the points *X*, *Y*, and *Z*. Since they are on a straight line, the work is proved correct. This line *X*-*Y*-*Z* is the line of intersection of the two planes as seen in the plan view. At this point, a portion of the triangles should be dotted in to indicate that they are hidden, giving a finished appearance to the work as shown in the plan view of Fig. 2:9. To obtain the line of intersection in the side view, proceed as follows:
14. The intersection of lines *X*-*Z* and 1-2 is point 10; point *K* is the intersection of lines *X*-*Z* and *B*-*C*.
15. Project these points 10 and *K* into the side view and draw the line 10-*K*. This is the line of intersection between the planes as seen in the side view.

2:5:d. Two oblique planes. Boundaries do not intersect.

Given: Two oblique planes *A-B-C* and 1-2-3 so located that they do not intersect (see Fig. 2:10).

To find: The line of intersection when one of the planes is extended.

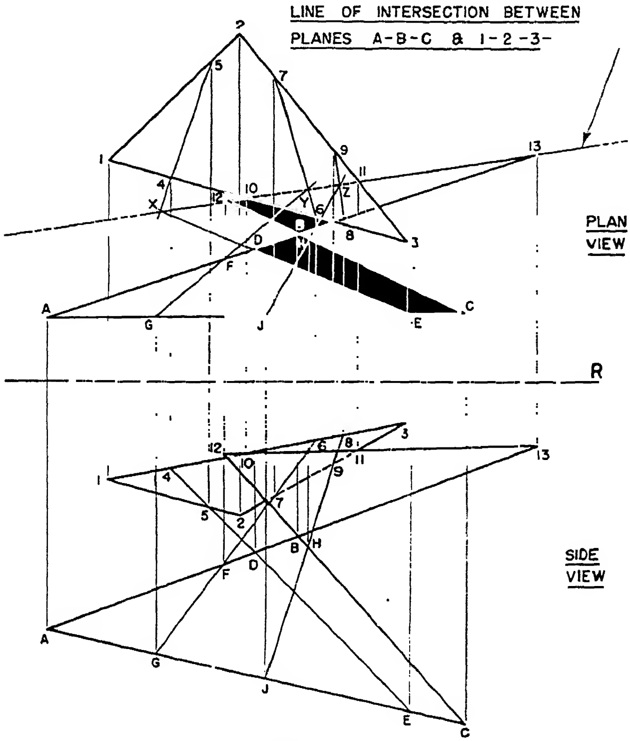


FIG. 2:11.

Solution: The same method is used here as was used in Art. 2:5:*c*. The points of intersection of lines drawn in one plane on the other plane are found. Since these points are on both planes, they determine the entire line of intersection. Figure 2:11 is an enlarged view of the planes shown in Fig. 2:10. Referring to Fig. 2:11, the procedure is as follows:

Operations:

1. Draw the line *D-E* in plane *A-B-C* in the side view.
2. Extend this line until it intersects line 1-3 at point 4 and line 1-2 at point 5.

3. Project these points 4, 5, D, and E into the plan view and draw the lines 4-5 and D-E.
4. Extend these lines in the plan view until they intersect at X, which is a point that lies in *both* planes.
5. Now draw the line F-G in plane A-B-C in the side view.
6. Extend this line until it intersects line 1-3 at point 6 and line 2-3 at point 7.
7. Project these points F, G, 6, and 7 into the plan view and draw the lines F-G and 6-7.
8. Extend line F-G in the plan view until it intersects line 6-7 at Y. Point Y lies on both planes.

   Two points X and Y, which lie on both planes, have now been determined. A third point to check the first two points should next be located; if all the work has been correct, the three points will lie on a straight line.
9. Draw the line H-J in plane A-B-C in the side view.
10. Extend this line until it intersects line 1-3 at point 8 and line 2-3 at point 9.
11. Project these points H, J, 8 and 9 into the plan view and draw the lines H-J and 8-9.
12. Extend line H-J in the plan view until it intersects line 8-9 at point Z, which lies in both planes.
13. Connect the points X, Y, and Z. Since they lie on a straight line, the work is proved correct. This line X-Y-Z is the line of intersection when plane A-B-C is extended until it intersects plane 1-2-3.
14. In the plan view extend the line C-B until it intersects the line X-Z at point 12.
15. Also extend the line A-B until it intersects the extension of line X-Z at point 13.
16. Through points 12 and 13, draw projection lines perpendicular to the reference line into the side view.
17. Extend lines A-B and C-B in the side view until they intersect these projection lines. These points of intersection are 13 and 12, respectively.
18. Connect these points 12 and 13 to obtain the line of intersection of the two planes as seen in the side view.
19. Point 10 is where line X-Z intersects line 1-3 and point 11 is where X-Z intersects line 2-3. If the work is accurately done, these points 10 and 11 in the side view may be projected into the plan view.

If portions of the planes are dotted to show that they are hidden, the work has a finished appearance and is easy to visualize, as seen in Fig. 2:11.

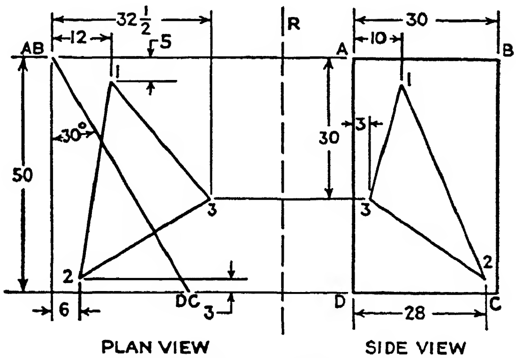
References

"Applied Descriptive Geometry," by Frank M. Warner, Art. 16.3, pp. 45 and 46; Art. 20.3, p. 49; Art. 21.3, pp. 50 and 51.

"Descriptive Geometry," by F. A. Smutz and R. F. Gingrich, Art. 71, pp. 78–80.

Problems

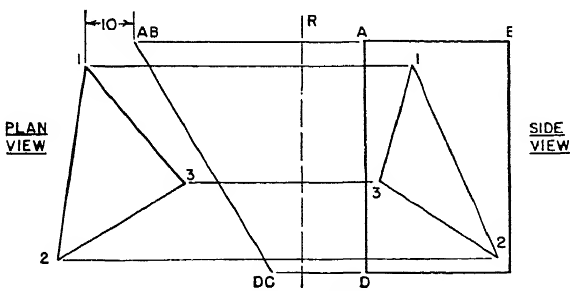
2:5:1. In the accompanying diagram are two orthographic views of two intersecting planes 1-2-3 and *A-B-C-D*. Determine



Prob. Fig. 2:5:1.

the line of intersection of these planes and in the side view locate this line of intersection, by dimensions, with respect to the line *A-D*.

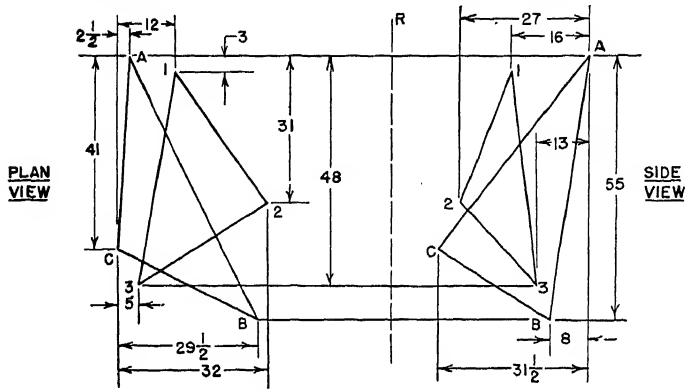
2:5:2. The accompanying diagram shows the same planes of Prob. 2:5:1 separated so that they do not intersect. Extend



Prob. Fig. 2:5:2.

these planes until they intersect and, as in Prob. 2:5:1, locate this line of intersection, by dimensions, with respect to line *A-D* in the side view.

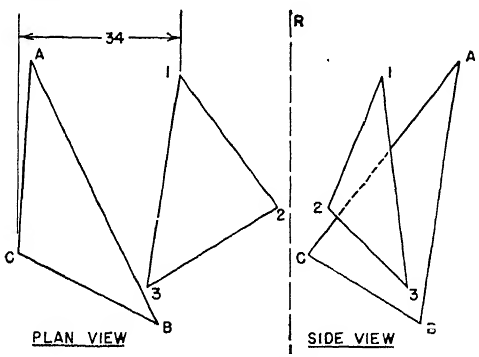
2:5:3. In the accompanying diagram, there are two intersecting triangles, 1-2-3 and *A-B-C*, representing two planes.



PROB. FIG. 2:5:3.

Determine the line of intersection of these two planes and in the side view locate this line, by dimensions, with respect to a line parallel to the reference line through point *A*.

2:5:4. In the accompanying diagram are the same planes of Prob. 2:5:3, separated so that the triangles representing them



PROB. FIG. 2:5:4.

will not intersect. Extend these planes until they intersect and, as in Prob. 2:5:3, locate this line of intersection, by dimensions, with respect to a line drawn parallel to the reference line through point *A* in the side view.

2:6. Determining the True Angle between Any Two Planes. This article will discuss the following two conditions:

2:6:*a*. Defined boundaries of the planes intersect.

2:6:*b*. Defined boundaries of the planes do not intersect.

In order that the student may solve problems that fall under condition 2:6:*b*, it is desirable that he thoroughly understand the preceding work since it may be necessary to determine the line of intersection between planes before the true angle (sometimes referred to as the "dihedral angle") between them is found.

Illustrations

2:6:a. Defined boundaries of the planes intersect.

Given: Two orthographic views of two intersecting planes *A-B-C* and 1-2-3, shown in Fig. 2:12. The line of intersection of these two planes is 2-3.

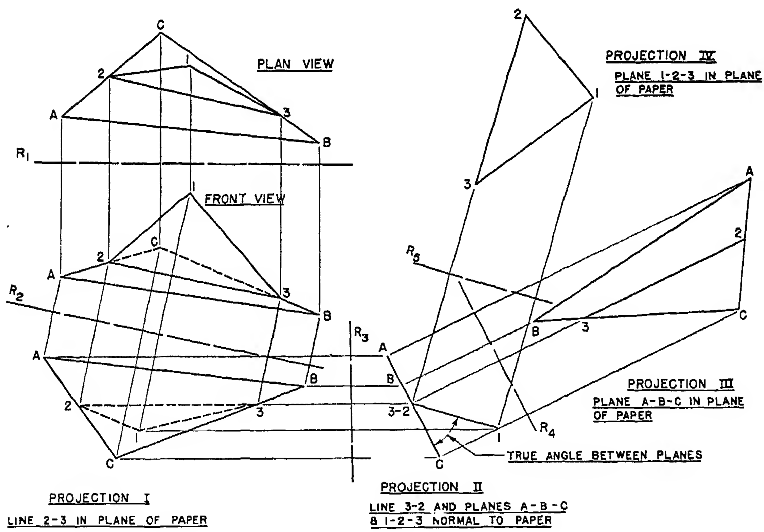


FIG. 2:12.

To find: The true angle between them.

Discussion: To see the true angle between any two planes, it is necessary to draw a view in which the line of intersection between these planes will be perpendicular to the paper, that is, this line of intersection will be seen as a point.

Solution: Take a projection off the front view so that the line 2-3 is placed in the plane of the paper. Then take a view normal to this line of intersection; the line will then appear as a point.

Operations:

1. At any point between the plan and the front view, draw a reference line R_1, perpendicular to the construction lines.
2. At any point below the line *A-B* in the front view, draw a reference line R_2, parallel to the line 2-3 in the front view.
3. By use of projection lines drawn perpendicular to reference line R_2, locate the points *A*, *B*, *C*, and 1, 2, 3 for projection I. The distances of these points from R_2 in projection I are the same as the respective distances from R_1 in the plan view. The line 2-3 in projection I is in the plane of the paper.
4. At any point to the right of point *B* in projection I, draw reference line R_3 normal to line 2-3.
5. Draw construction lines perpendicular to R_3 through the points *A*, *B*, *C*, and 1, 2, 3 and locate these points for projection II. The distances from R_3 in projection II are the same as the distances from R_2 in the front view. In projection II, the line 2-3 is normal to the paper, and the angle between line *A-C* and line 1-2 is the true angle between the planes.
6. If it is desired to obtain plane *A-B-C* in the plane of the paper, as shown in projection III, the reference line R_4 is drawn at any convenient point to the right of projection II parallel to line *A-C* of projection II. The points *A*, *B*, and *C* are located from R_4 the same distances as they are from R_3 in projection I.
7. If it is desired to obtain plane 1-2-3 in the plane of the paper, as shown in projection IV, the reference line R_5 is drawn at any convenient point above projection II parallel to line 1-2 of projection II. The points 1, 2, and 3 are located from R_5 the same distances as they are from R_3 in projection I.

2:6:b. Defined boundaries of the planes do not intersect.

Given: In Fig. 2:13 are two orthographic views of planes *A-B-C* and 1-2-3 so located that they do not intersect.
To find: The true angle between these two planes.

Discussion: These two planes are the same ones shown in Figs. 2:10 and 2:11 and are repeated here for the convenience of the student.

Solution: Determine the line of intersection when one of the planes is extended by the method described in Art. 2:5:*d*. Knowing this line of intersection, proceed as in Art. 2:6:*a*, drawing a view in which the line of intersection is normal to the paper and in this view measuring the true angle between the two planes. In Fig. 2:14, line 10-11 in both plan and side views is the line of intersection on plane 1-2-3 when plane *A-B-C* is extended. As this was determined in Art. 2:5:*d*, the work is not repeated here.

Operations:

1–19. Are for determining the line of intersection of the two planes and are identical to operations 1 to 19, Art. 2:5:*d*.

Operations 20 to 27 are similar to operations 2 to 7, Art. 2:6:*a*. They are repeated here as different points are used for the planes in this illustration.

20. At any point below the line *A-C* of the side view, draw a reference line R_2, parallel to the line 10-11 in the side view.
21. Draw construction lines normal to R_2 from points 1, 2, 3, 10, and 11 and *A*, *B*, and *C* of the side view.
22. These points are located from R_2 in projection I the same distances as they are from R_1 in the plan view. In projection I the line 10-11 is in the plane of the paper.
23. At any point to the right of point *C* in projection I, draw a reference line R_3, normal to line 10-11.
24. Draw construction lines normal to R_3 from the points 1, 2, 3, 10, and 11 and *A*, *B*, and *C* of projection I.
25. These points are located from R_3 in projection II the same distances as they are from R_2 in the side view. In projection II the line 10-11 is perpendicular to the paper, and the planes 1-2-3 and *A-B-C* are also normal to the paper. In this view the true angle between the planes may be scaled.
26. If it is desired to obtain plane 1-2-3 in the plane of the paper, as shown in projection III, a reference line R_4 is

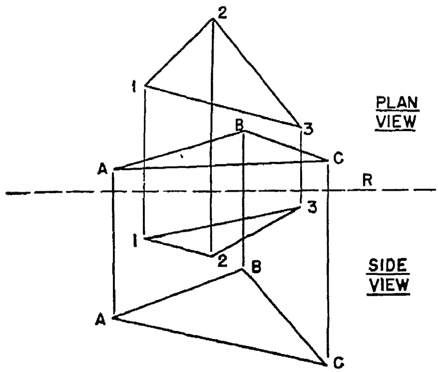


FIG. 2:13.

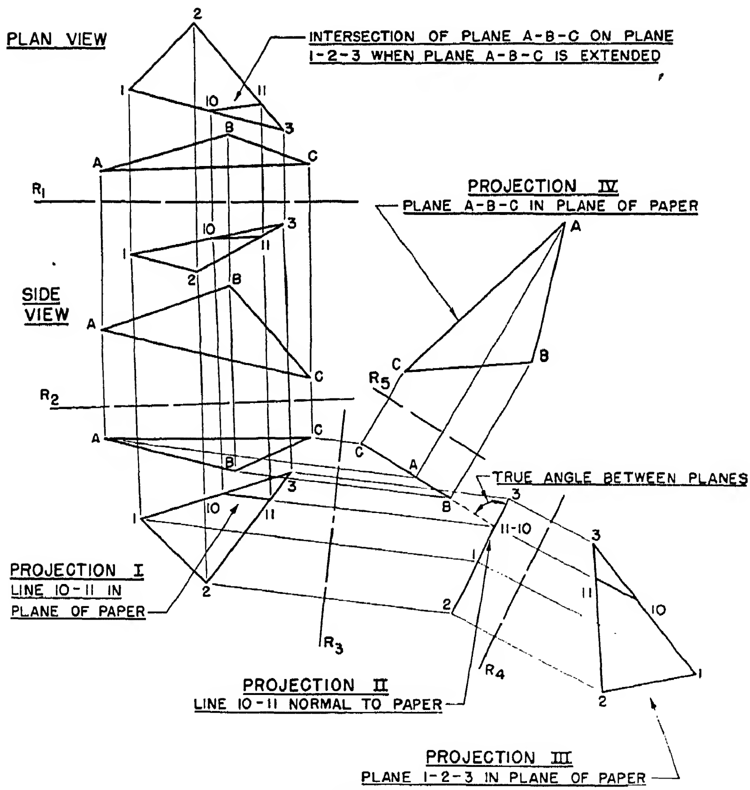


FIG. 2:14.

drawn parallel to line 2-1-3 of projection II. The points 1, 2, and 3 are located from R_4 the same distances as they are from R_3 in projection I.

27. If necessary to have plane *A-B-C* in the plane of the paper, as shown in projection IV, a reference line R_5 is drawn parallel to line *C-A-B* of projection II. The points *A*, *B*, and *C* are located from R_5 the same distances as they are from R_3 in projection I.

Another method of determining the true angle between planes and at the same time obtaining the line of intersection between them, is shown in Fig. 2:14*a*. Although this method has one more projection than the method shown in Fig. 2:14, it is preferred by many as it eliminates the necessity of determining the line of intersection in the two given views.

Given: Two orthographic views of two oblique planes, 1-2-3 and *A-B-C*, the defined boundaries so located that they do not intersect (see Fig. 2:14*a*).

To find: 1. The true angle between these two planes.
2. The line of intersection when one of the planes is extended.

Solution: A view is obtained where one plane is perpendicular to the plane of the paper, using the method of drawing a line parallel to a reference line (see Art. 2:4, operations 5 to 7). Then this method is applied to the other plane, which gives a view where both planes are normal to the paper. In this view, one plane may be extended until it intersects the other, which locates the line of intersection. Also the true angle between the planes may be scaled in this view.

Operations:

1. At any point between the plan and front views, draw a reference line R_1, normal to the construction lines.
2. In the plan view of plane 1-2-3, draw any line 4-5 parallel to reference line R_1.
3. Project the line 4-5 into the front view where it is in the plane of the paper.
4. At any point to the right of the front view, draw the reference line R_2, normal to the line 4-5 of the front view.

5. Draw construction lines perpendicular to reference line R_2, through all the points of the two planes A, B, C, and 1, 2, 3, 4, 5; and on these lines locate the points for projec-

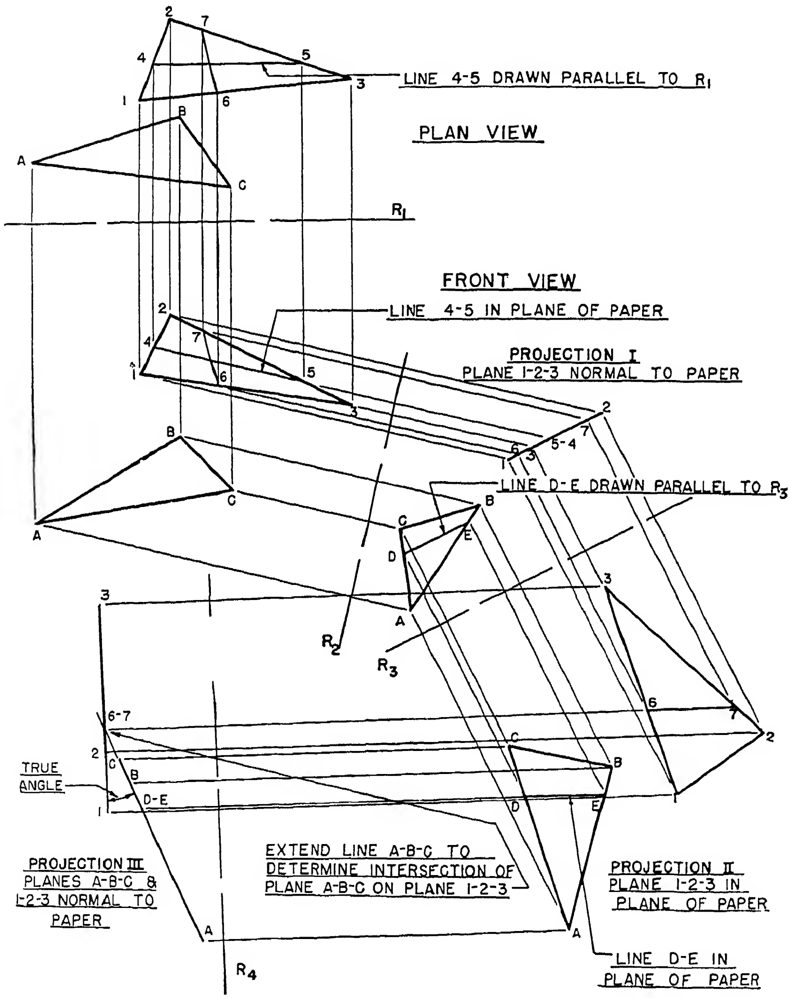


FIG. 2:14*a*.

tion I. The distances of these points for projection I from R_2 are the same as the respective distances from R_1 in the plan view.

6. In projection I, the line 5-4 is normal to the paper and the plane 1-2-3 is also perpendicular to the paper. The line 5-4, having served its purpose, will not be carried any further in this illustration.
7. In projection I of plane *A-B-C* draw the line *D-E* parallel to the line 1-3-2.
8. At any point below projection I, draw the reference line R_3, parallel to line 1-3-2 of projection I.
9. Draw construction lines perpendicular to reference line R_3 through all the points of the two planes *A*, *B*, *C*, *D*, *E*, and 1, 2, 3; and on these lines locate the points for projection II. The distances of these points from R_3 for projection II are the same as the respective distances from R_2 in the front view.
10. In projection II, the plane 1-2-3, as well as the line *D-E*, is in the plane of the paper.
11. At any point to the left of projection II, draw the reference line R_4, perpendicular to the line *D-E* of projection II.
12. Draw construction lines perpendicular to R_4 through all the points of the two planes, and on these lines locate the points for projection III. The distances of these points from R_4 for projection III are the same as the respective distances from R_3 in projection I.
13. In projection III, both planes are normal to the paper; it is here that the plane *A-B-C* may be extended until it intersects the plane 1-2-3, forming the line 6-7. Also, the true angle between these planes may be measured in projection III.
14. Trace the line 6-7 back through the projection II and I until it is located in the front and plan views.

References

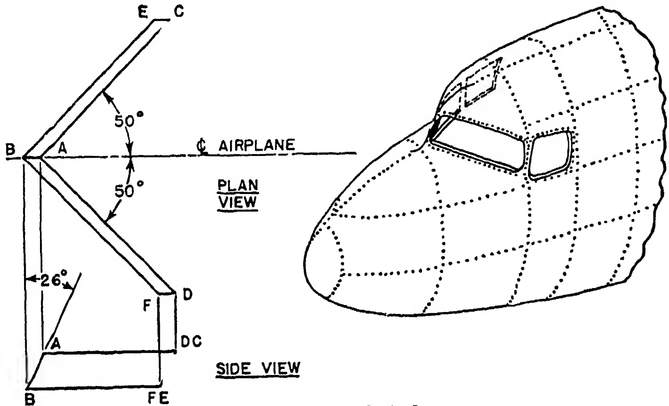
"Applied Descriptive Geometry," by Frank M. Warner, Art. 23.3, pp. 52 and 53.

"Descriptive Geometry," by F. A. Smutz and R. F. Gingrich, Art. 75, p. 84.

Problems

2:6:1. Determine the true angle between the plane of the front spar and the plane of the nose rib shown on the diagram for Prob. 2:4:3.

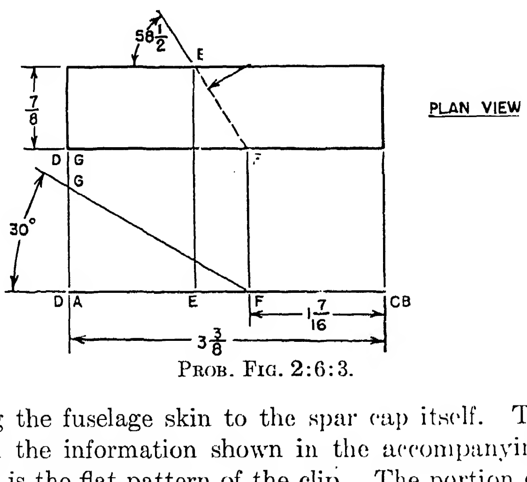
2:6:2. The loft, when laying out lines around the cockpit of a transport airplane, determined the angles given in the accompanying diagram. The layout man who was assigned the windshield had to determine the true angle between the panes of



Prob. Fig. 2:6:2.

glass, so that he could design the supports, etc. Find the true angle between the two front panes of glass in the cockpit enclosure shown in this diagram.

2:6:3. For sealing the spar caps at the side of the fuselage of a high altitude transport, it was necessary to provide clips



Prob. Fig. 2:6:3.

attaching the fuselage skin to the spar cap itself. The layout man had the information shown in the accompanying sketch. *A-B-C-D* is the flat pattern of the clip. The portion of the clip

bounded by points F, D, A, and E is to be bent up in the plan view along the line E-F until the line F-G lies in the extended plane of the clip. Note that F-G is merely a line in space that determines the amount of bend.

a. Find the true angle of bend.

b. Draw three orthographic views of the clip after it is bent.

For this problem, the thickness of the clip and the bend radius will be neglected. Work through this problem using only the planes given. This is to be done full size.

2:6:4. Determine the true angle between the planes of Prob. 2:5:1.

2:6:5. Determine the true angle between the planes of Prob. 2:5:3.

2:6:6. The following is an actual problem from a commercial transport airplane. On this airplane, the main electrical distribution panel was inside the fuselage and the generator was in the nacelle with the engine. A heavy aluminum bar, known as a "bus bar," connected the generator with the main panel. It was necessary to mount a junction box on this bar so that the planes of the two sides of the box were normal to the center line of the bus bar; at the same time, the box had to be rotated about the bus bar as an axis to a predetermined angle to line up with a certain opening in the nacelle nose rib. Also, 1¾-in. clearance had to be provided between the rib and the nearest corner of the box. The accompanying diagram shows the information the layout man had, in order to provide a suitable bracket for this box.

a. Determine the distance as measured along the center line of the bus bar between the rib plane and the nearest side of the junction box.

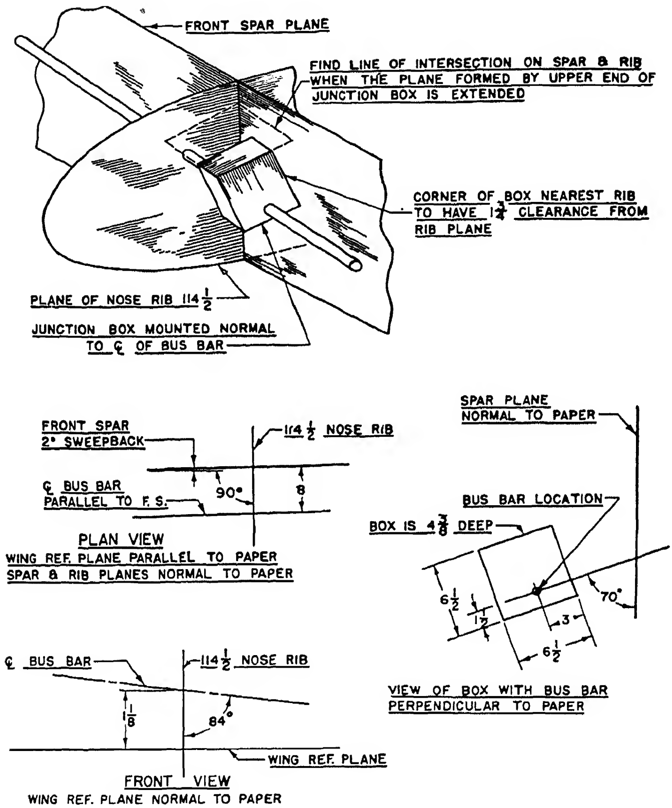
b. 1. Find the line of intersection on the spar plane when the plane formed by the upper side of the box is extended. Locate this line of intersection with respect to the wing reference plane by a dimension in the plane of the nose rib.

2. What is the angle as measured on the spar plane between this line of intersection and the rib plane?

c. 1. Find the line of intersection on the rib plane when the plane formed by the upper side of the box is extended. Locate this line of intersection with

respect to the wing reference plane by a dimension in the plane of the spar.

2. What is the angle as measured on the rib plane between this line of intersection and the spar plane?



PROB. FIG. 2:6:6.

2:7. Determining the Intersections of Planes and Cylinders. Three conditions will be considered in this article:

2:7:*a*. Center line of cylinder in plane of paper. Plane normal to paper.

2:7:*b*. Center line of cylinder oblique to plane of paper. Plane normal to paper.

2:7:*c*. Center line of cylinder and plane oblique to paper.

Since the layout man frequently encounters problems where various air ducts, conduits, etc. intersect bulkheads, ribs, and

other planes, he should be able to determine their intersections on these planes in order to provide suitable clearance holes. At times, he must also design a collar that will attach the conduit or air duct to the bulkhead; this necessitates his determining the true angle between the conduit and the plane.

Illustrations

2:7:a. Center line of cylinder in plane of paper. Plane normal to paper.

Given: Two orthographic views of a plane *A-C-B-D* intersecting a cylinder (see Fig. 2:15).

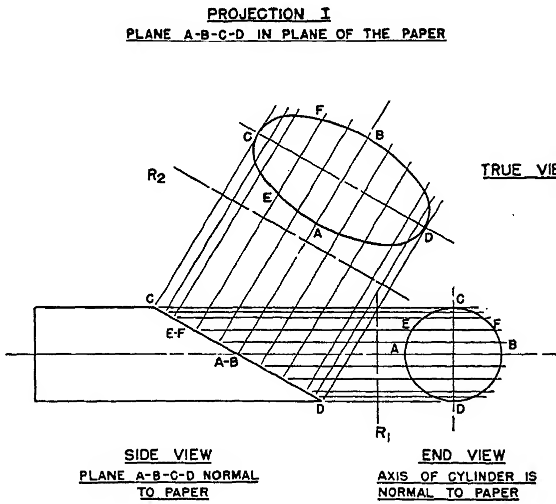


FIG. 2:15.

To find: The true view of this oblique cut.

Discussion: To see the true view of this oblique cut, a view in which plane *A-C-B-D* will be in the plane of the paper must be drawn.

Solution: Since plane *A-C-B-D* is normal to the paper in the side view, a view projected directly off the line *C-D* will be the required view.

Operations:

1. At any point between the side and end views, draw a reference line R_1, perpendicular to the construction lines.

2. At any point above the side view, draw a reference line R_2, parallel to the line C-D of the side view.
3. Draw projection lines perpendicular to R_2 from points A, B, C, and D of the side view.
4. Locate points A, B, C, and D in projection I from the reference line R_2. The distances of these points from R_2 are the same as the distances from R_1 in the end view.
5. Draw any number of lines in the end view parallel to line A-B, such as line E-F, and project through the side view into projection I.
6. Connecting the points A, E, C, F, B, D, etc. in projection I with a curved line gives the true view of the line of intersection of the plane and the cylinder.

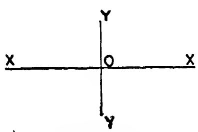


FIG. 2:16.

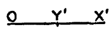


FIG. 2:17.

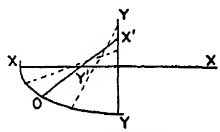


FIG. 2:18.

This true view is an ellipse, which may be constructed immediately after the points A, B, C, and D are located in projection I. Knowing the line C-D is the major axis and line A-B is the minor axis, the student may construct the ellipse from the geometrical method given in many standard engineering reference books,[1] or if an ellipsograph is available much time may be saved.

Probably one of the simplest and quickest ways for the layout man to construct an ellipse after he knows the lengths of the major and minor axes is as follows:

In Fig. 2:16, line X-X is the major axis of the ellipse and Y-Y is the minor axis. Draw a straight line and lay out the distance O-Y' on the straight line and the distance O-X', as shown in Fig. 2:17. The distances O-Y' and O-X' in Fig. 2:17 are identical with the distances O-Y and O-X in Fig. 2:16. This straight line should be drawn on a scrap of vellum or the edge of a sheet of scratch paper.

Now superimpose the straight line on the two axes, as shown in Fig. 2:18. Points X' and Y' on the straight line are placed over the two axes and point O is marked. This gives one point on the ellipse. Repeat this for any number of points, always

[1] See "Machinery's Handbook," 10th ed., p. 279.

keeping X' and Y' on the two axes. Connecting these points forms the ellipse.

The method outlined in operations 5 and 6 is sometimes referred to as the "element method"; it is presented here since it is basic and its use should be understood. However, as pointed out previously, it frequently takes longer to use than other methods.

2:7:b. Center line of cylinder oblique to paper. Plane normal to paper.

Given: A plane *A-B-C-D* normal to the paper in the side view (see Fig. 2:19) and the center line 1-2 of a cylinder which is oblique to the paper in both orthographic views.

To find: 1. The true view of the line of intersection between the plane of the cylinder.

2. The true angle between the plane and the cylinder.

Discussion: To find the line of intersection of the cylinder on the plane, it will be necessary to have a view in which the axis of the cylinder is in the plane of the paper and the plane is normal to the paper.

If the student thinks of the center line of the tube only as a line, the second part of this problem becomes merely finding the true angle between a line and a plane, the method for which was discussed in Art. 2:4.

Solution: Follow the procedure outlined in Art. 2:4 for determining the true angle between a line and a plane. This gives a view showing the line (axis of cylinder) in the plane of the paper and the plane normal to the paper. This view having been obtained, the problem is reduced to the condition described in Art. 2:7:*a*.

Operations: The following operations are not so detailed as is customary in this text since the student may refer to Art. 2:4 if he experiences any difficulty. In Fig. 2:19, the plan and side views are given.

1. Project normal to the line *A-D* in the side view to obtain projection I. In projection I the plane *A-B-C-D* is in the plane of the paper.
2. Project normal to the center line of the tube, line 1-2 in projection I, for projection II. Here the plane is normal to the paper and the tube is in the plane of the paper. In this

projection, the true angle between the plane and the tube is measured.

3. In projection II, where the tube intersects the plane, call these points 3, 4, 5, and 6.
4. Carry these four points back into projection I, establishing the major and minor axes of the elliptic intersection.

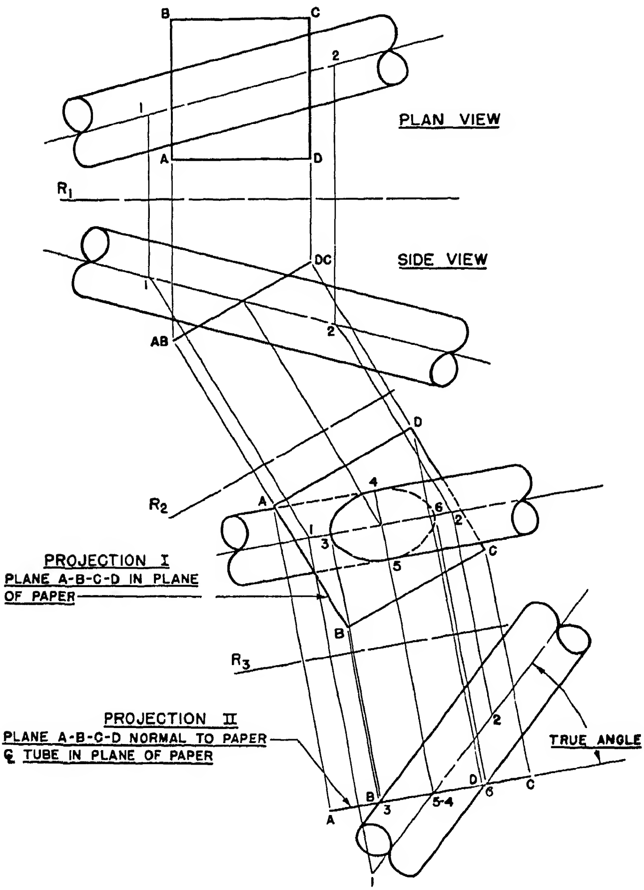


FIG. 2:19.

5. Construct the ellipse by any one of the methods outlined previously.
6. If it is desired to see this line of intersection in the plan view, the points forming the ellipse in projection I may be carried through the side view into the plan view.

2:7:c. Center line of cylinder and plane oblique to paper.

Given: Two orthographic views of a plane *A-B-C* and a cylinder whose center line is 1-2; both are oblique to the paper in both given views (see Fig. 2:20).

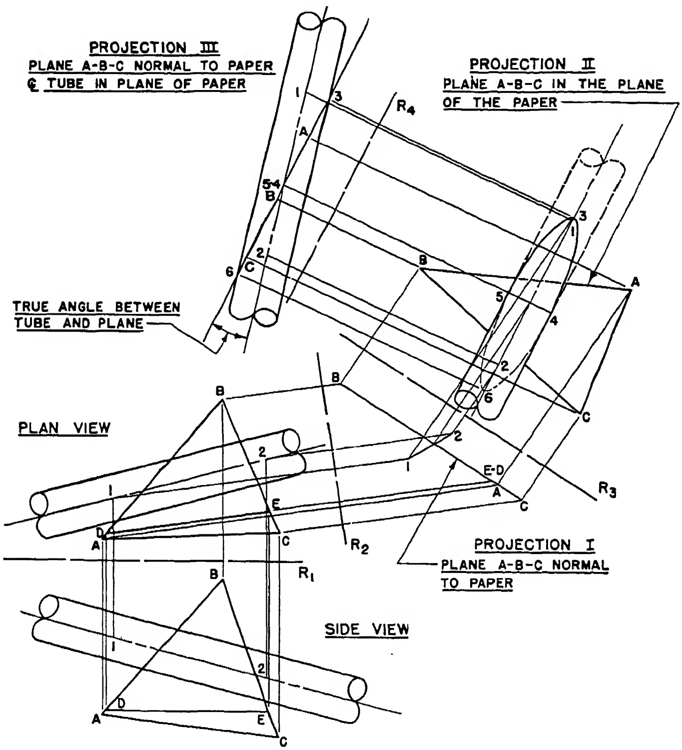


FIG. 2:20.

To find: 1. The true view of the line of intersection between the plane and the cylinder.

2. The true angle between the plane and the cylinder.

Discussion: This case is very similar to that discussed in Art. 2:7:*b*, since the same views are required here as in the previous condition.

Solution: One more projection will be required than in Art. 2:7:*b*, since the plane is oblique to the paper in both given views. The method used to obtain the first projection is clearly out-

lined in Art. 2:4, operations 5, 6, and 7. The balance of the projections are similar to Art. 2:7:*b*.

Operations: Here again the operations will not be so detailed as is customary since they have all been explained previously.

1. Draw the line *D-E* in the side view parallel to reference line R_1 and project into the plan view where *D-E* will be in the plane of the paper.
2. Project off line *D-E* of the plan view to obtain projection I where plane *A-B-C* will be normal to the paper.
3. Project off line *B-C* of projection I for projection II, where plane *A-B-C* will be in the plane of the paper.
4. Project perpendicular to line 2-3 in projection II for projection III, where the plane *A-B-C* will be normal to the paper and the center line of the tube 1-2 will be in the plane of the paper. Here the true angle between the plane and the cylinder may be scaled.
5. Extend the plane in projection III and mark the points where the tube intersects the plane 3, 4, 5, and 6.
6. Carry these points back into projection II, establishing the major and minor axes of the line of intersection, which is an ellipse.
7. Construct the ellipse by any method outlined previously.
8. If it is desired to obtain the line of intersection in the two given views, the points forming the ellipse may be carried back through projection I into the plan and side views.

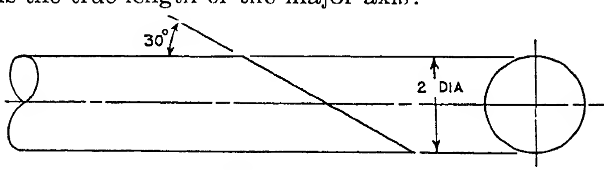
References

"Applied Descriptive Geometry," by Frank M. Warner, Arts. 18.6 to 24.6, pp. 101–104; also Art. A1, p. 221.

"Descriptive Geometry," by F. A. Smutz and R. F. Gingrich, Art. 161, pp. 181–183.

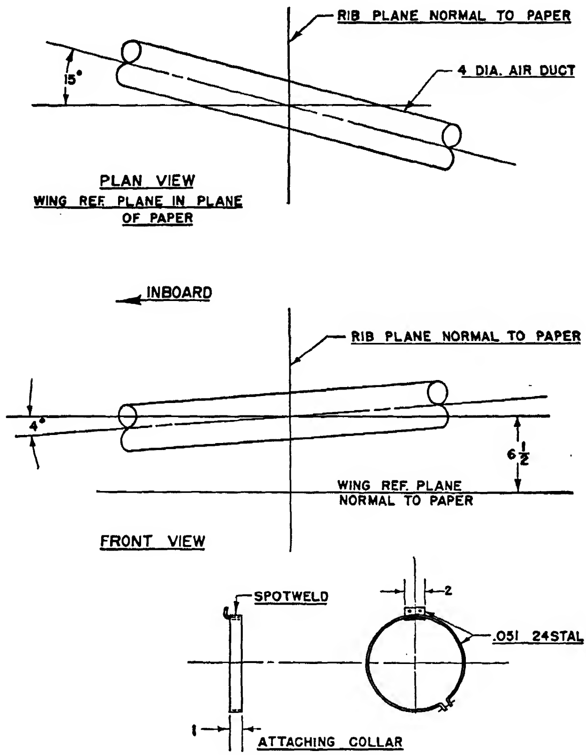
Problems

2:7:1. Using the element method, lay out, full size, the true view of the oblique cut shown in the accompanying figure. What is the true length of the major axis?



PROB. FIG. 2:7:1.

2:7:2. An air duct from the air intake in the nacelle to the fuselage passes through a wing rib, and it is found necessary to provide a collar to attach the duct to the rib. In the following sketch will be found the information furnished to the layout man.



PROB. FIG. 2:7:2.

a. Plot the intersection of the air duct on the rib, using any desired method.

b. What is the length of the major axis of the ellipse?

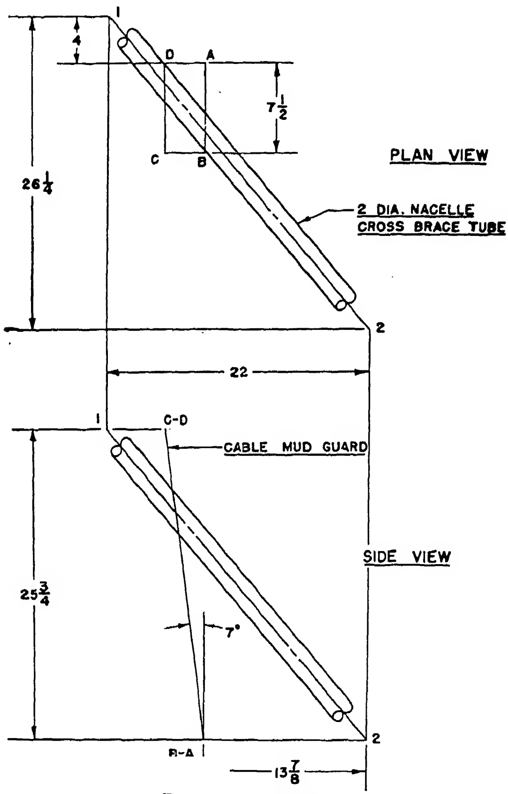
c. What is the true angle between the rib and center line of the air duct?

d. Draw a view with the axis of the tube normal to the paper showing the collar in place. Also show in this view the line of intersection of the wing reference plane on the rib plane (usually called the "wing reference line" on rib drawings). Draw a line in

this view through the center of the tube bisecting the clip. What is the apparent angle between this line and the wing reference line?

e. Lay out the collar, full size, giving the angle of bend for the clip.

2:7:3. A 2-in. diameter nacelle cross-brace tube passes through an 0.064 24ST Alclad sheet guard shielding the engine



PROB. FIG. 2:7:3.

control cables from mud and rocks thrown by the main landing wheels. It is necessary to provide an opening in the mud guard that will give ¼-in. clearance around the tube. The accompanying diagram provides the data necessary to layout this problem.

a. What is the length of the major axis of the clearance hole?

b. What is the true angle between the tube and the mud guard?

c. Plot the clearance hole, using any desired method.

d. Locate the hole in the mud guard by dimensions with respect to the lines *C-D* and *A-D* and give the angle between the major axis and the line *A-D*.

2:8. Determining the Intersections of Curved Surfaces. Cylinders will be used to illustrate the principle involved in determining the line of intersection of curved surfaces. The same principle, however, may be applied to *any* curved surfaces.

Two conditions will be discussed:

2:8:*a*. Center lines of cylinders in plane of paper.

2:8:*b*. Center lines of cylinders oblique to paper in both given views.

Illustrations

2:8:a. Center lines of cylinders in plane of paper.

Given: The plan and end views of two intersecting cylinders whose respective axes are in the plane of the paper and normal to the paper in the plan view (see Fig. 2:21).

To find: The line of intersection as seen in the side view.

Solution: Construct planes parallel to the axes of the cylinders in both given views. The points where these planes intersect both cylinders are points on the line of intersection.

Operations:

1. Draw reference line R_1 at any convenient distance to the left of the end view.
2. Draw reference line R_2 below the plan view the same distance from the center line as R_1 is from the center line in the end view.
3. Draw a series of planes in the end view parallel to R_1 and call these planes 1-1, 2-2, 3-3, etc.
4. Lay out these planes in the plan view, the distances from R_2 being the same as the distances from R_1 in the end view.
5. Draw construction lines perpendicular to R_1 through the points where these planes 1-1, 2-2, 3-3, etc. cut the horizontal cylinder in the end view, and extend these construction lines into the side view.

6. Draw construction lines perpendicular to R_2 through the points where the planes 1-1, 2-2, 3-3, etc. of the plan view cut the vertical cylinder, and extend these construction lines into the side view.
7. Where the construction lines from plane 1-1 cross in the side view, will be point 1. Where the construction lines from plane 2-2 cross, will be point 2, etc.

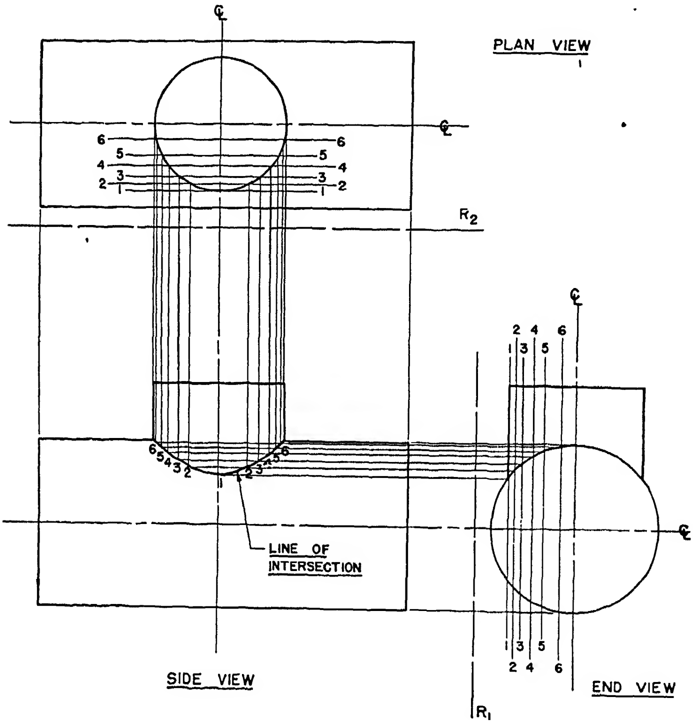


FIG. 2:21.

8. After all the points are located, draw the line of intersection in the side view using a French curve or spline.

2:8:b. Center line of cylinders oblique to paper in both given views.

Given: Two orthographic views of two intersecting tubes which are oblique to the paper in both given views (see Fig. 2:22).
To find: The line of intersection of these tubes.

Discussion: In Fig. 2:22, only the center lines of the tubes *A-B* and *C-D* and their intersection at point *O* are shown in the two given views in order to simplify the drawing.

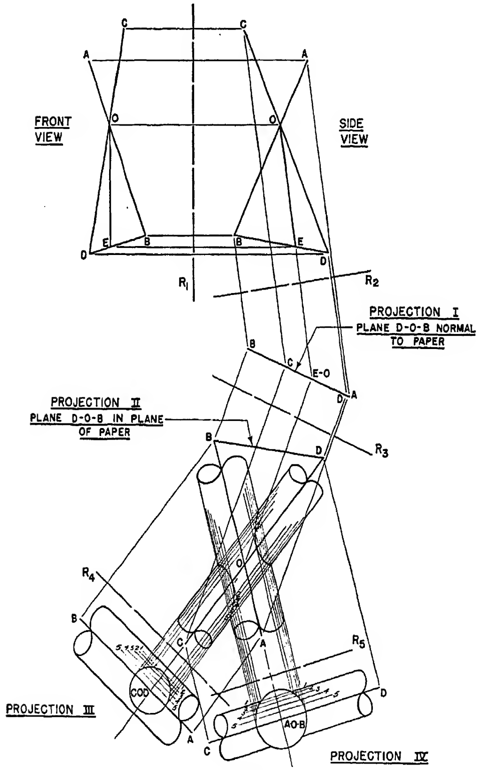


FIG. 2:22.

Solution: Let the lines *D-O* and *O-B* be the boundaries of a plane *D-O-B*; this plane contains both axes of the tubes. By obtaining a view in which the plane *D-O-B* is in the plane of the paper, this problem is reduced to the condition discussed in Art. 2:8:*a*. After obtaining the view with both center lines in the plane of the paper, the balance of the solution is similar to Art. 2:8:*a*.

Operations: The following operations are not so detailed as is customary in this text, since the student may refer to Art. 2:4, operations 4 and 7, if he experiences any difficulty in obtaining a view showing the plane *D-O-B* in the plane of the paper.

1. Draw the line *O-E* in the front view parallel to reference line R_1 and project into the side view where line *O-E* is in the plane of the paper.
2. Project off line *E-O* in the side view for projection I where line *E-O* and the plane *D-O-B* are perpendicular to the paper.
3. Project off line *A-B* in projection I for projection II where the plane *D-O-B* and the axes of the tubes are in the plane of the paper.
4. Lay in projections III and IV looking down the axes of the tubes.
5. Draw a series of planes in these two end views numbered 1, 2, 3, 4, etc.
6. Draw projection lines through the points where these planes intersect the cylinders as was done in operations 5 and 6 of Art. 2:8:*a*.
7. The points at which these construction lines intersect in projection II are on both tubes; hence are points of intersection between these tubes. These points are numbered 1, 2, 3, etc.
8. Connecting these points with a curved line gives the required line of intersection.
9. If it is desired to obtain the line of intersection in the two given views, draw in the tubes and project the points 1, 2, 3, etc. back through projection I into the side and front views.

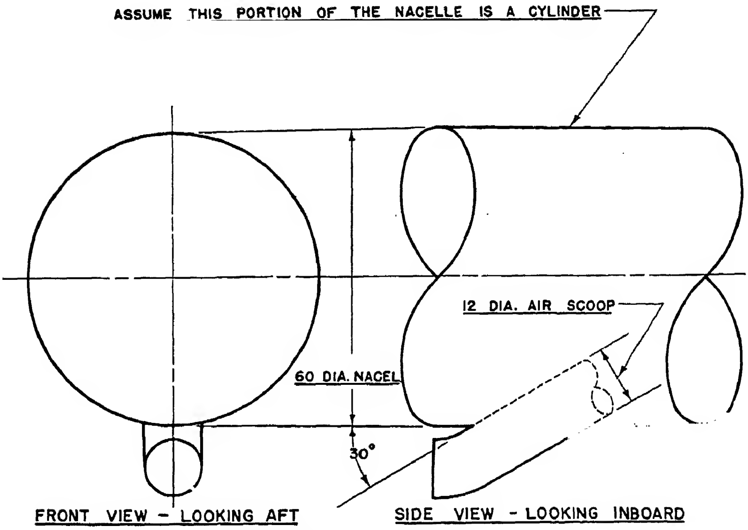
References

"Applied Descriptive Geometry," by Frank M. Warner, Arts. 4.7 and 5.7, pp. 135–137.

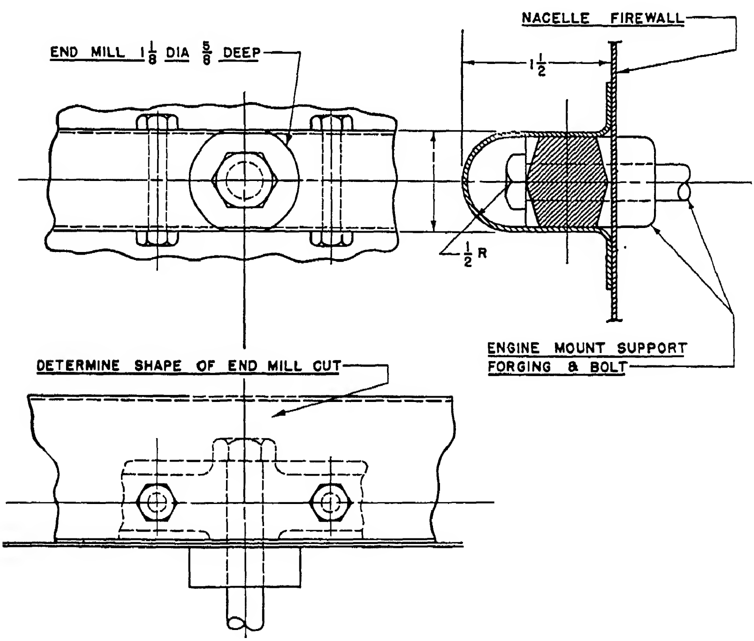
"Descriptive Geometry," by F. A. Smutz and R. F. Gingrich, Art. 165, pp. 190–192.

Problems

2:8:1. In the accompanying diagram is shown a portion of a nacelle with the oil cooler air scoop. Plot the line of intersection



PROB. FIG. 2:8:1.



PROB. FIG. 2:8:2.

with the air scoop on the nacelle in the side view. Also show a plan view of this line of intersection.

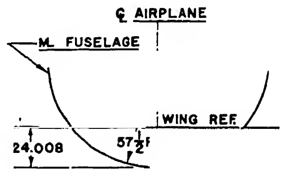
2:8:2. A hat section brace on a transport fire wall requires an access hole for tightening the engine mount bolt. Complete the view shown in the diagram at the bottom of page 57.

2:8:3. The accompanying diagram *A* gives the data necessary to lay out the plan view of the intersection of the wing on a

TABLE OF WING OFFSETS

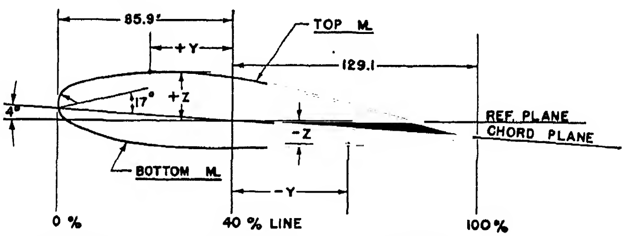
%*	TOP ML		BOTTOM ML	
	Y	Z	Y	Z
0	85.9	6.1	85.9	6.1
1.25	83.2	13.1	83.3	2.1
2.50	80.5	15.3	80.6	-0.3
5.0	75.1	18.3	75.3	- 2.0
7.5	69.7	20.2	69.9	- 3.7
10.	64.3	21.5	64.5	- 5.2
15	53.6	23.0	53.8	- 7.7
20	42.9	23.2	43.0	- 9.8
30	21.4	22.1	21.5	-12.5
40	0	19.7	0	-13.8
50	- 21.5	16.3	- 21.5	-14.3
60	- 42.9	12.2	- 43.0	-14.2
70	- 64.4	7.6	- 64.6	-13.6
80	- 86.0	2.5	- 86.1	-12.5
90	-107.5	-3.0	-107.6	-11.1
100	-129.1	-9.1	-129.1	- 9.1
LEADING EDGE RADIUS = 5.982				
TRAILING EDGE RADIUS = 0.19				

* % OF CHORD LENGTH MEASURED IN REFERENCE PLANE.



SECTION THRU FUSELAGE IS CONSTANT AT WING ATTACHMENT

NOTE: THE SYMBOL ML DESIGNATES THE MOULD LINE, A TERM USED TO DESCRIBE THE INNER SURFACE OF THE SKIN OR THE OUTER EDGES OF THE INTERNAL STRUCTURAL MEMBERS.



WING SECTION REMAINS CONSTANT THRU FUSELAGE

PROB. FIG. 2:8:3 *A*.

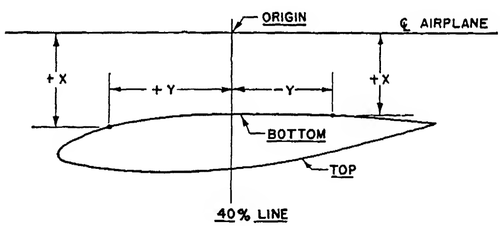
fuselage. The table of wing offsets is usually furnished to the layout man by the loft. After determining this line of intersection, prepare a table similar to the one shown in diagram *B*, and fill in the offsets. The offsets as seen in the plan view are to be scaled from the layout.

2:9. Pulley Bracket Geometry. The geometry of pulley brackets may at times become quite complicated. In present-

day aircraft, with so many systems using cable controls, this is one of the frequent problems confronting the layout man and, consequently, should be given serious study. This article will therefore discuss the application of some of the principles set forth in previous articles to the layout of pulley brackets.

COORDINATES — WING TO FUSELAGE INTERSECTION

%	TOP		BOTTOM	
	X	Y	X	Y
0				
1.25				
2.5				
5.0				
7.5				
10.0				
15.0				
20				
30				
40				
50				
60				
70				
80				
90				
100				



PLAN VIEW OF LINE OF INTERSECTION

PROB. FIG. 2:8:3 *B*.

It is assumed that the designer has certain data. He knows the location in space of the surface on which his bracket is mounted, and knows the desired path of the cable. Having these data, he must then work out a suitable bracket that will support the cable in the required position. The actual design of the bracket is not discussed as that is covered elsewhere in this text; it is the *geometry* of the bracket that is considered here.

Three conditions will be considered:

2:9:*a*. Mounting surface in plane of paper in one given view, normal to paper in other given view.

2:9:*b*. Mounting surface normal to paper in one given view, oblique to paper in other given view.

2:9:*c*. Mounting surface oblique to paper in both given views.

The student is cautioned to remember that it is the center line of the cable that is used for layout purposes. It is very easy to overlook this fact when drawing in the actual pulley.

Illustrations

2:9:a. Mounting surface in plane of paper in one given view, normal to the paper in other given view.

Given: Two orthographic views showing the path of the cable 1-2-3 and the location and direction of the mounting channel (see Fig. 2:23). The location of the cables with respect to mold line 18 is also given.

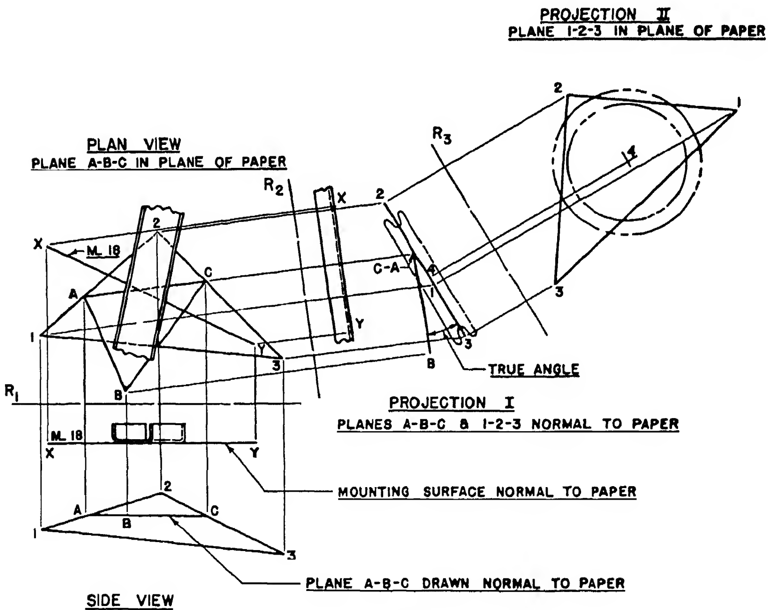


FIG. 2:23.

Find: 1. The true angle between the plane of the mounting surface and the plane formed by the cables.

2. The location of the pulley with respect to mold line 18.

Discussion: In most actual problems, the layout man must locate the pulley bracket with respect to some known station, such as a fuselage or wing station, or to some mold line (usually abbreviated M.L., or on drawings by the symbol ML). Hence, in this problem line *X-Y*, which is called ML 18, has been introduced. The path of the cable, lines 1-2 and 2-3 in Fig. 2:23, may be thought of as being the boundaries of a plane, which is known as "the plane of the cable."

Solution: The true angle between the plane of the cable and the plane of the mounting surface is first determined; then a view is drawn where the plane of the cable lies in the plane of the paper. It is here that the pulley may be laid in. The line *X-Y* (ML 18) is carried through the necessary projections so its relation to the center of the pulley may be determined.

Operations:

1. Draw a reference line R_1 at any convenient point between the plan and side views.
2. In the side view, construct any plane *A-B-C* parallel to the mounting surface.
3. Project this plane *A-B-C* into the plan view where the line *A-C* is the line of intersection between plane *A-B-C* and the plane of the cables 1-2-3, and is in the plane of the paper.
4. At any convenient point to the right of the plan view, draw reference line R_2, normal to line *A-C* of the plan view.
5. Draw construction lines perpendicular to reference line R_2 from the points 1, 2, and 3 and A, B, and C for the planes and from points *X* and *Y* for the location of ML 18.
6. Locate these points for projection I from reference line R_2 the same distances as they are from R_1 in the side view. In projection I, the planes *A-B-C* and 1-2-3 are normal to the paper and it is here that the true angle between them may be scaled. The line *X-Y* should be parallel to line *C-B* in this projection.
7. At any convenient point above projection I, draw reference line R_3, parallel to line 2-3 in projection I.
8. Draw construction lines perpendicular to R_3 from the point 1, 2, and 3 of projection I.
9. Locate these points for projection II from R_3 the same distances as they are from R_2 in the plan view. In projection

II the plane of the cable is in the plane of the paper; hence, the pulley is also in the plane of the paper and the pulley axis is normal to the paper.

10. When the diameter of the pulley to be used is known, the pulley itself may now be laid out in projection II. It must be remembered that the lines 1-2 and 2-3 represent the center line of the cable and that proper allowances for the size of the cable to be used must be made when locating the center of the pulley. Call the center of the pulley point 4.
11. Project the pulley back into projection I, where the line 2-3 bisects the pulley as shown in Fig. 2:23. The axis of the pulley is normal to the line 2-3 and point 4 will fall on line 2-3. Here we may locate the center of the pulley with respect to the mounting channel.

2:9:b. Mounting surface normal to paper in one given view, oblique to paper in the other given view.

Given: Two orthographic views showing the path of the cable 1-2-3 and the location and direction of the mounting channel

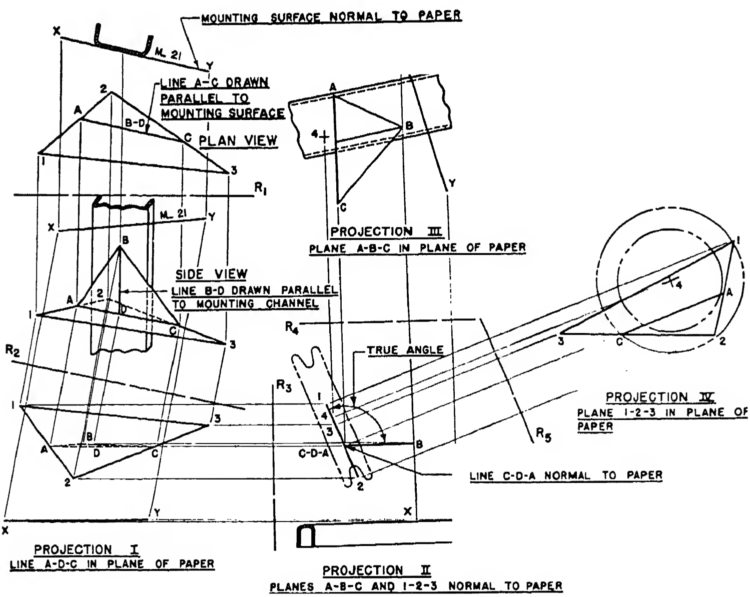


FIG. 2:24.

(see Fig. 2:24). Also known is the location of the cable with respect to ℄ 21.

To find: 1. The true angle between the plane of the cable and the plane of the mounting surface.

2. The location of the pulley with respect to ℄ 21.

Discussion: This problem is similar to that discussed in Art. 2:9:*a*; however, here an extra projection is involved since the plane of the mounting surface is oblique to the paper in one given view.

Solution: The true angle between the plane of the mounting surface and the plane of the cable is determined; then a view is drawn showing the plane of the cable in the plane of the paper. In this view the pulley may be laid in. Line *X-Y* (℄ 21) is carried through the various projections so its relation to the center of the pulley may be determined.

Operations: The following operations are not so detailed as is customary, since they follow closely the operations of Art. 2:9:*a*.

1. Construct any plane *A-B-C* in the plan view parallel to the face of the mounting channel and project it into the side view. The line *A-C* is the intersection of plane *A-B-C* on plane 1-2-3 and is parallel to line *X-Y* in the plan view.
2. In the side view draw the line *B-D* parallel to the mounting channel.
3. Project normal to line *A-C* in the side view for projection I where line *A-C* is in the plane of the paper.
4. Project off line *A-C* in projection I for projection II where line *A-C* and planes *A-B-C* and 1-2-3 are normal to the paper. Here the true angle between the planes may be scaled.
5. Project normal to line *C-B* of projection II for projection III where plane *A-B-C* and the plane of the mounting surface are in the plane of the paper.
6. Project normal to line 1-2 of projection II for projection IV where the plane 1-2-3 is in the plane of the paper.
7. Lay in the pulley in projection IV and mark its center point 4.
8. Project point 4 and the pulley back into projection II.
9. Carry point 4 into projection III where it may be located with respect to line *X-Y* (℄ 21).

2:9:c. Mounting surface oblique to paper in both given views.

Given: Two orthographic views showing the path of the cable 1-2-3 and the location and direction of the mounting surface

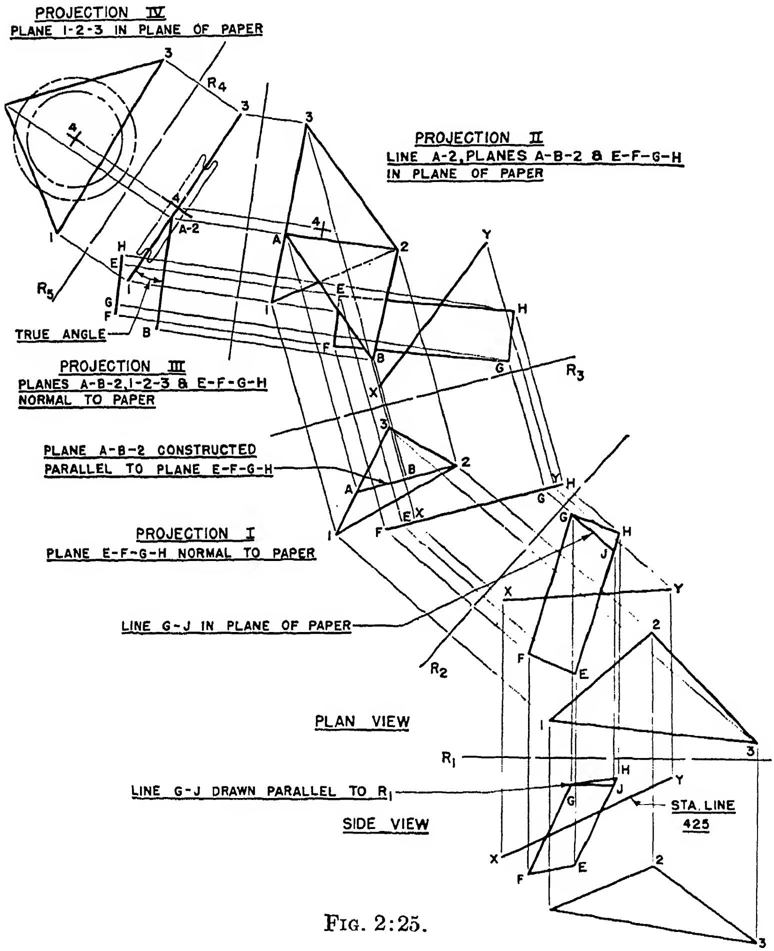


FIG. 2:25.

E-F-G-H (see Fig. 2:25). Also given is the location of the cable with respect to station line 425.

To find: 1. The true angle between the plane of the mounting surface and the plane of the cable.

2. The location of the center of the pulley with respect to station line 425.

Discussion: This problem is similar to those discussed in Arts. 2:9:*a* and 2:9:*b*; however, since the mounting surface is oblique to the paper in both given views, this is the most difficult of the three conditions. The student should experience no trouble if he understands Arts. 2:9:*a* and 2:9:*b* and the principles involved in determining the first projection.

Solution: Here two oblique planes are defined and it is required to find the line of intersection of these planes. This may be done either by the method described in Art. 2:5:*d* or by an adaptation of the method described in operations 5-7 of Art. 2:4. In Fig. 2:25, the latter method has been chosen as it is a principle often used both in this text and by the layout man in actual practice. After determining the line of intersection of the two planes, the balance of the solution is similar to Arts. 2:9:*a* and 2:9:*b*.

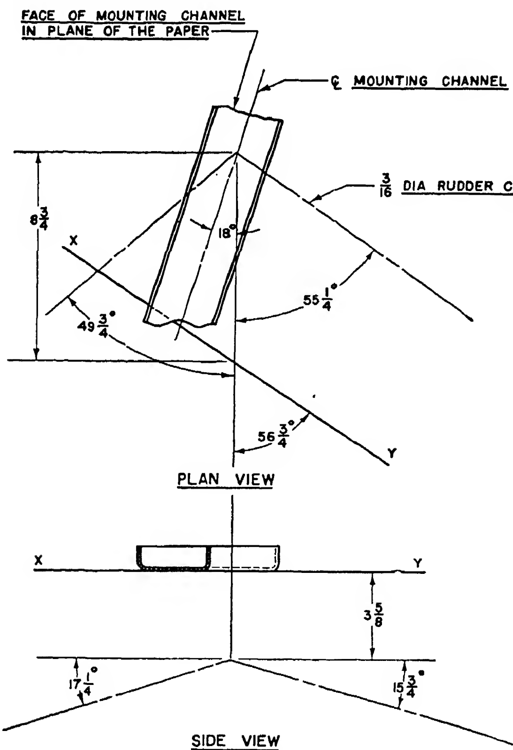
Operations: In the following operations some of the steps have been omitted, as the student by this time should have become fully acquainted with the reference line method and the principle involved in locating points thereby.

1. In the side view, draw the line *G-J* parallel to R_1 and project it into the plan view where it lies in the plane of the paper.
2. Project normal to line *G-J* in the plan view for projection I, where the plane of the mounting surface *E-F-G-H* is perpendicular to the paper.
3. In projection I, construct plane *A-B*-2 parallel to plane *E-F-G-H*.
4. Projection II is taken perpendicular to line *A*-2 of projection I. In projection II line *A*-2 is the intersection of plane *A-B*-2 on plane 1-2-3 and is in the plane of the paper.
5. Project off line *A*-2 of projection II for projection III, where line *A*-2 is normal to the paper. Here the planes 1-2-3 and *A-B*-2 are normal to the paper and the true angle between the planes may be scaled.
6. Carry the plane of the mounting surface *E-F-G-H* through projection II into projection III, where it should be seen as a line *H-F*, parallel to line *A-B*.
7. Project normal to line 1-3 of projection III for projection IV, where the plane of the cable 1-2-3 is in the plane of the paper.

8. Lay in the pulley in projection IV and locate its center. Call this point 4.
9. Project the pulley back into projection III and locate its center, point 4.
10. Station line 425, line *X-Y*, was not carried into projection III from projection II owing to lack of space in Fig. 2:25. However, since line *X-Y* is in the plane of the mounting surface, it is obvious that it will coincide with the line *H-F* of projection III.
11. Carry point 4 back into projection II so that its relation to *X-Y* may be determined.

Problems

2:9:1. In the diagram for this problem, the center line of the path of a $\frac{3}{16}$-in.-diameter rudder cable is given, as well as the

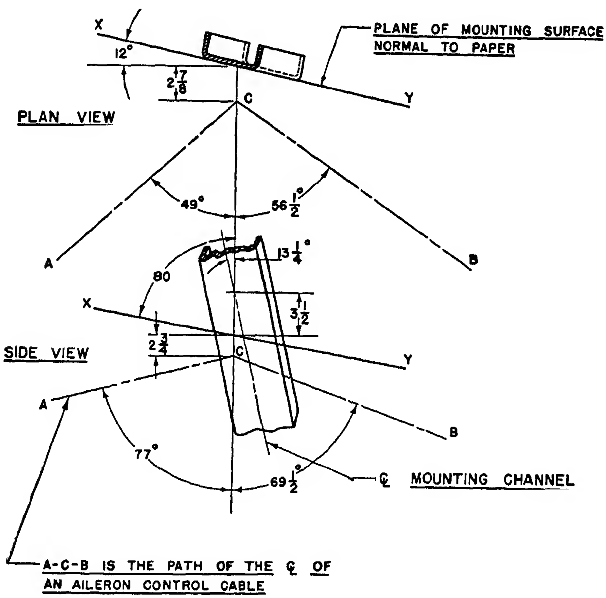


PROB. FIG. 2:9:1.

location and direction of the plane of the mounting surface. Line *X-Y* is a station line in the plane of the mounting surface. An AN210-6A pulley is to be used.

Determine the true angle between the plane of the pulley and the plane of the mounting surface. Also determine the distance from the center of the pulley to the line *X-Y* in the projection where the plane of the mounting surface is normal to the paper. This would be the projection that corresponds to projection I in Fig. 2:23. This distance is to be measured perpendicular to line *X-Y*.

2:9:2. In the diagram for this problem, the center line of the path of a 3⁄16-in.-diameter aileron control cable (line *A-C-B*), as



PROB. FIG. 2:9:2.

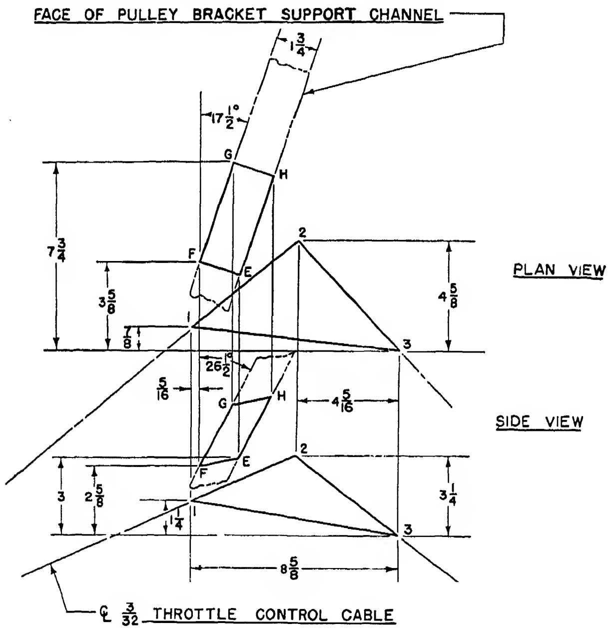
well as the location and direction of the plane of the mounting surface, are given. Line *X-Y* is a mold line in the plane of the mounting surface. Use an AN210-10A pulley.

Determine the true angle between the plane of the pulley and the plane of the mounting surface. Also determine the distance from the center of the pulley to line *X-Y* in the projection where the plane of the mounting surface is in the paper (projection III

of Fig. 2:24). This distance is to be measured normal to line *X-Y*.

2:9:3. The diagram for this problem shows the center line of a $\frac{3}{32}$-in.-diameter throttle control cable, line 1-2-3, and a plane *E-F-G-H* which is a portion of the face of a mounting channel. Use an AN210-2A pulley.

Determine the true angle between the plane of the cable and the plane of the mounting surface. Also determine the distance



PROB. FIG. 2:9:3.

from the center of the pulley to the plane of the face of the channel measured normal to the plane of the mounting surface.

2:10. Streamline Strut Geometry. Quite frequently in aircraft it is necessary to have structural members exposed to the slip stream. For example, an airplane whose landing gear is not retractable must have the structure supporting the wheel and axle exposed. Usually these structural members are tubes; however, since it would be poor design aerodynamically to have round tubes exposed to the slip stream, they are "streamlined." This

is often done by rolling the basic round tube into a streamlined shape similar to that shown in Fig. 2:26. The basic cross-sectional dimensions for streamline tubing may be obtained from page 8-3 of ANC-5.

Other examples of the use of streamlined tubes in aircraft are the lift struts of a wing, the antenna mast, etc.

The large end of the strut is always forward, and it is necessary to have the long axis of the tube (referred to as the "major axis") in a plane parallel to the line of flight. It is best to think of this major axis as a plane extending the full length of the tube. Then it may be said that the *plane* of the major axis must be parallel to the line of flight. If the major axis is turned across the line of flight, that is, across the slip stream, any desirable characteristics of the streamlined section would be lost and an unnecessarily large amount of drag would be introduced.

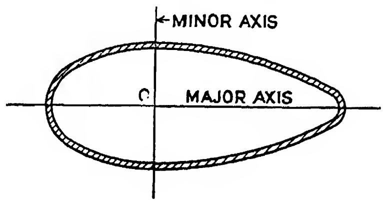


FIG. 2:26.

The term "streamlined strut" is applied to any member that has been streamlined, whether the structural tube has been so shaped or whether an auxiliary fairing has been attached. In either case, *the plane of the major axis must be parallel to the line of flight.* The center line of a streamlined section is the line of intersection of the planes of the major and minor axes, and is marked *O* in Fig. 2:26. The minor axis is at the widest point of the tube which, in a standard streamline section, occurs at 35 per cent of the length. See ANC-5, page 8-3, for dimensions and diagrams explaining this more fully.

Illustration

Given: The plan and side views of a streamline strut and attaching bolt (see Fig. 2:27). Line 0-1 is the center line of streamline strut. Line 2-3 is drawn parallel to the line of flight. Line 4-5 is the center line of the attaching bolt.

To find:

1. The true length of the strut.
2. The direction of the plane of the major axis of the strut (always parallel to line of flight).

3. The apparent angle of twist between the axis of attaching bolt and the plane of the major axis of the strut when the strut center line is normal to the paper.
4. The true angle between the center line of the attaching bolt and the strut center line.

Discussion: It was stated that line 2-3 is parallel to the line of flight. In all aircraft drawings, the airplane is normally drawn

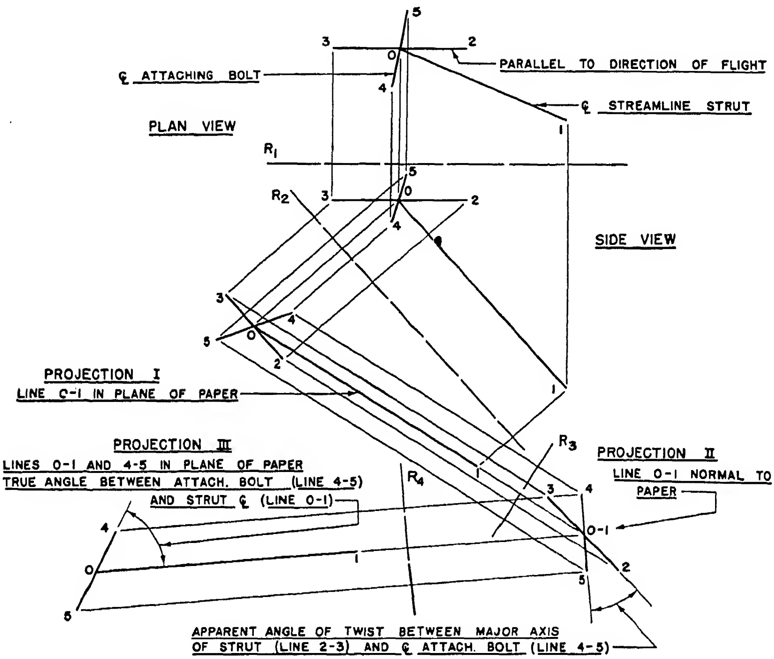


Fig. 2:27.

so that it is headed towards the left edge of the paper. Hence in our problem, the direction of motion would be from right to left, or from 2 to 3 along line 2-3.

Operations:

1. At any point between the plan and side view, draw a reference line R_1, parallel to line 2-3 of the plan view.
2. At any point below the side view, draw a reference line R_2, parallel to line 0-1 of the side view.

3. Draw projection lines perpendicular to the reference line R_2 from the points 0, 1, 2, 3, 4, and 5 of the side view.
4. Locate the points 0, 1, 2, 3, 4, and 5 for projection I from the reference line R_2. The distances of these points from R_2 are the same as the distances from R_1 in the plan view. In projection I the line 0-1 is in the plane of the paper, which gives the true length of the streamline strut.
5. Draw the reference line R_3, perpendicular to the line 0-1 of projection I.
6. Draw projection lines from the points 0, 1, 2, 3, 4, and 5 perpendicular to the reference line R_3.
7. These points are located for projection II from the reference line R_3 the same distances as they are from R_2 in the side view. In projection II, the line 0-1 is normal to the paper.
8. It is in projection II that the apparent angle of twist between the axis of the attaching bolt (line 4-5) and the plane of the major axis of the stream-line strut (line 2-3) is seen. Note that originally line 2-3 was called the "line of flight"; but since the plane of the major axis of the strut is always parallel to the line of flight, this line can also be called the "major axis."
9. Draw reference line R_4, parallel to line 4-5 of projection II.
10. Draw projection lines from the points 0, 1, 4, and 5 perpendicular to the reference line R_4.
11. For projection III, these points are located from R_4 the same distances as they are from R_3 in projection I.
12. In projection III the center line of the strut (line 0-1) and the axis of the attaching bolt (line 4-5) are both in the plane of the paper. Therefore, the true angle between the attaching bolt and the strut center line is seen in this view.

Problems

2:10:1. Refer again to the landing gear shown in Prob. 2:2:2 and find the following for the front brace strut attachment to the fuselage, assuming that the struts are streamline tubes:

a. The direction of the axis of the fuselage attaching bolt.[1]

[1] This is in the nature of a "catch question" and is purposely inserted in order to make the student reason out a fundamental fact. When the shock

b. The true length of the front brace strut.

c. The apparent angle of twist between the major axis of the streamline strut and the axis of the attaching bolt.

d. The true angle between the center line of the strut and the center line of the bolt.

2:10:2. For the rear brace strut of this landing gear, find the same things as were determined in Prob. 2:10:1.

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absorbers deflect, the wheel and brace tubes move. There can be only one line about which this movement occurs, and the axis of the attaching bolts must lie on this line. What is the line about which a landing gear such as the one in this problem must pivot?

CHAPTER 3

AIRFOILS

3:1. Introduction. The National Advisory Committee for Aeronautics (N.A.C.A.) has developed and standardized a system of mathematically constructed airfoil contours, based on the results of actual wind-tunnel and flight research. Most manufacturers rely entirely on the published characteristics of the N.A.C.A. derived airfoil sections, instead of assuming the expense and time for individual investigations.

3:2. Description. Airfoils may be considered as being made up of various symmetrically shaped sections plotted about a center line, known as the "mean line." This mean line may be straight, giving a symmetrical section such as is used on most empennages, or it may be curved, giving a cambered section such as is used on practically all wings (see Fig. 3:1). The amount of curvature of the mean line is known as the "camber." The thickness form and the curvature of the mean line are of particular importance: the former as pertaining to structural properties, the latter in regard to aerodynamic characteristics. Related airfoil profiles are derived by systematic changes of these shape variables. Mean lines are obtained by variable mathematical formulas (with which we are not concerned in this text) which allow a controlled variation in the camber and in the position of maximum camber.

3:3. Numbering Systems. Airfoils are numbered systematically in the form of a code, which permits ready identification. Those airfoils in common use are known as the four-digit series or the five-digit series. An explanation of this code follows:
Example of the four-digit series: 1415

1 = the camber designation, that is, the maximum ordinate of the mean line, in this case approximately 1 per cent of the chord.

4 = the maximum camber location, or the location of the maximum ordinate of the mean line from the leading

edge (abbreviated L.E.), in this case 40 per cent of the chord.

15 = the maximum over-all thickness of the profile, in this case 15 per cent of the chord.

Example of the five-digit series: 23019 (see Fig. 3:1).

2 = the camber designation, that is, the maximum ordinate of the mean line, in this case approximately 2 per cent of the chord.

30 = the maximum camber location times 2, in this case 15 per cent of the chord.

19 = the maximum over-all thickness of the profile, in this case 19 per cent of the chord.

The leading-edge radius and its slope are determined by separate mathematical formulas as functions of the maximum

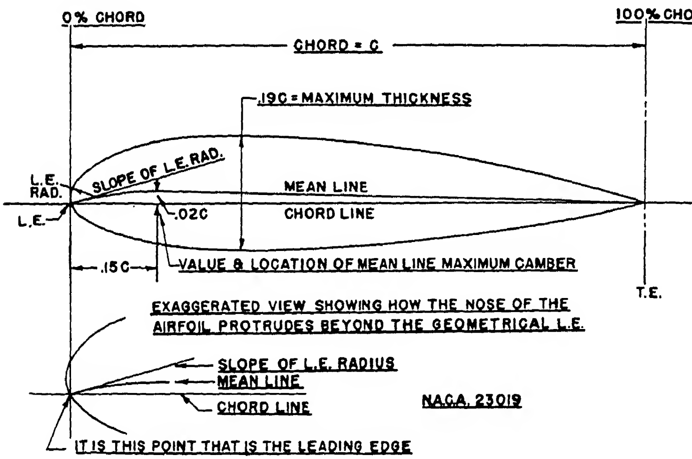


FIG. 3:1.

thickness ordinate. However, certain modifications of the leading-edge radius may be required for aerodynamic purposes, and an auxiliary series of designations has been established as dash numbers to indicate the extent of the modification, both as to the radius itself and to the resulting change in the position of the maximum ordinate. For example, in -54 the first digit represents one of the following:

0 = sharp leading edge
3 = $\frac{1}{4}$ normal radius
4 = $\frac{1}{2}$ normal radius
5 = $\frac{3}{4}$ normal radius
6 = normal radius
9 = 3 or more times normal radius

The second digit represents the position of maximum thickness in relation to the chord, in this case 40 per cent.

3:4. Source of Data. In the N.A.C.A. tables, the profile and mean line data are condensed and appear as a series of ordinates and abscissas in respect to the chord line, in terms of percentage of the chord. In addition, the leading-edge radius is also given in the same terms. These are data which the layout man must learn how to use.

The report used in this article is *N.A.C.A. Technical Note* 567, "Tests of N.A.C.A. Airfoils in the Variable-Density Wind Tunnel, Series 230." However, the principles used may be applied to *any* N.A.C.A. airfoil. The number of the airfoil to use, the length of the wing chord, the required incidence, and the spar locations are usually determined by the preliminary design group for the layout man.

3:5. How to Lay Out Airfoils from N.A.C.A. Reports When a Table of Ordinates Is Given.

Given:

1. Table of ordinates for the N.A.C.A. 23021 airfoil (Table 3:1).
2. Wing chord = 72 in.
3. Incidence = +6 deg. rotated about the 35 per cent line
4. Front spar = 15 per cent chord; center spar = 35 per cent chord; rear spar = 65 per cent chord
5. All spar planes to be perpendicular to the wing reference plane

To find:

1. Plot the airfoil.
2. Rotate for incidence and lay in the wing reference plane.
3. Lay in the spars.

Discussion: By an inspection of *N.A.C.A. Technical Note* 567, it will be seen that the ordinates and characteristics of the following airfoils are given:

N.A.C.A. 23006
N.A.C.A. 23009
N.A.C.A. 23012
N.A.C.A. 23015
N.A.C.A. 23018
N.A.C.A. 23021

Refer to Table 3:1, which is the table of ordinates given in Fig. 6, "N.A.C.A. 23021 Airfoil," *N.A.C.A. Technical Note* 567. The figures in this table are in terms of the *per cent of chord.* Column I, headed "Station," is the horizontal distance where 0 per cent is the leading edge and 100 per cent is the trailing edge. "Upper" and "Lower" are the distances of the airfoil above and below the chord line at those particular stations. Mathematically speaking, column I is the abscissa and columns 2 and 3 are the ordinates.

Operations:

1. Multiply (usually by slide rule) *each* figure in Table 3:1 by the chord length (72 in.), producing Table 3:2. The one exception is "Slope of radius through end of chord = 0.305," which is given at the bottom of Table 3:1 and is discussed in operation 4.
2. Draw a base line, such as line *A-B* in Fig. 3:2, and call it the "chord line."
3. Lay out the horizontal distances from column I of Table 3:2 beginning at the left side for station 0, and draw construction lines of indefinite length through the points just laid out.
4. Through the 0 per cent point of the chord line, construct the line 0-*C* at an angle of 17 deg. to the chord line. Notice that the tangent of 17 deg. is 0.305, which is the figure given in Table 3:1 for "Slope of radius through end of chord"; that is, the N.A.C.A. report gave the tangent or slope of this line.
5. On this line lies the center of the leading-edge radius. Therefore, set a compass for 3.49-in. radius; place the point

on the line 0-C and draw the radius through point 0, which is the 0 per cent point. Point D is the center of the L.E. radius.

Lay out all the points for the upper and lower contour lines from the ordinates given in columns 2 and 3 of Table 3:2.

TABLE 3:1

STA	UPPER	LOWER
0	–	0
1.25	4.87	-2.08
2.5	6.14	-3.14
5.0	7.93	-4.52
7.5	9.13	-5.55
10.0	10.03	-6.32
15	11.19	-7.51
20	11.80	-8.30
25	12.05	-8.76
30	12.06	-8.95
40	11.49	-8.83
50	10.40	-8.14
60	8.90	-7.07
70	7.09	-5.72
80	5.05	-4.13
90	2.76	-2.30
95	1.53	-1.30
100	0.22	-0.22
100	—	0
L.E. RADIUS 4.85 SLOPE OF RADIUS THROUGH END OF CHORD :0.305		

TABLE 3:2

DISTANCE	UPPER	LOWER
0	0	0
0.90	3.51	-1.50
1.80	4.42	-2.26
3.60	5.71	-3.25
5.40	6.58	-4.00
7.20	7.23	-4.55
10.80	8.07	-5.41
14.40	8.50	-5.98
18.00	8.68	-6.31
21.60	8.69	-6.45
28.80	8.28	-6.36
36.00	7.50	-5.86
43.20	6.41	-5.09
50.40	5.11	-4.12
57.60	3.64	-2.97
64.80	1.99	-1.66
68.40	1.10	-0.94
72.00	0.15	-0.15
—	—	—
LEADING EDGE RADIUS = 3.49 SLOPE = 17°		

TABLE OF OFFSETS COPIED FROM FIG.6 N.A.C.A TECHNICAL NOTE 567 FOR AIRFOIL N.A.C.A. 23021

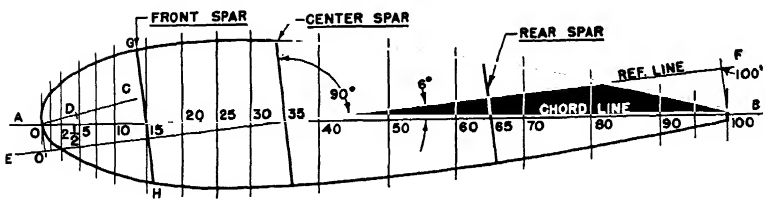


FIG. 3:2.

7. By means of a spline or French curves, connect all these points with a fair line, which gives the airfoil section.
8. To rotate this airfoil for incidence, draw a line E-F through the 35 per cent point at 6 deg. to the chord line. This line is known as the wing reference line and is actually the end view of the wing reference plane where the reference plane is normal to the paper. When the incidence is positive, the leading edge of the wing is elevated above the

reference line; when the incidence is negative, the leading edge is depressed below the reference line. The reference plane is always parallel to the normal line of flight.

9. Through the 15 per cent point on the chord line, draw a line *G-H* normal to the reference line. This line is the intersection of the front spar plane on the rib plane. Likewise, construct the center spar and the rear spar lines through the 35 per cent and 65 per cent points.

If it is desired to obtain a table of offsets using the reference line as a datum line, which is frequently done by the loft, the procedure is as follows:

10. Draw construction lines normal to the reference line through the station points along the chord line.
11. The 0 per cent point will move to 0′, the 100 per cent point will move to 100′, and all intermediate points will move similarly. However, the leading-edge radius does not move. It is now just a matter of scaling the offsets from the reference line to obtain the tables of offsets as furnished by the loft to the engineering department.

3:6. How to Lay Out Airfoils from N.A.C.A. Reports When a Table of Ordinates Is Not Given.

Given: The table of ordinates for the N.A.C.A. 23018 airfoil (Table 3:3).

To find: The table of ordinates for the N.A.C.A. 23017 airfoil.

Discussion: Since the last two numbers represent the thickness of the airfoil in the 230 family, the ordinates in the 23018 table may be multiplied by $^{17}/_{18}$ to obtain an approximate table of ordinates for the 23017. Although a slight error is introduced by this method, it is accurate enough for all practical purposes provided the layout man follows these instructions.

Operations:

1. Determine the constant for the multiplication

$$^{17}/_{18} = 0.944$$

2. Multiply each figure in column 2, Table 3:3, by 0.944 to obtain column 2, Table 3:4.

$$4.09 \times 0.944 = 3.86$$
$$5.29 \times 0.944 = 5.00$$
$$6.92 \times 0.944 = 6.54, \text{ etc.}$$

TABLE 3:3[1]

Station	Upper	Lower
0	0	0
1.25	4.09	−1.83
2.50	5.29	−2.71
5.0	6.92	−3.80
7.5	8.01	−4.60
10	8.83	−5.22
15	9.86	−6.18
20	10.36	−6.86
25	10.56	−7.27
30	10.55	−7.47
40	10.04	−7.37
50	9.05	−6.81
60	7.75	−5.94
70	6.18	−4.82
80	4.40	−3.48
90	2.39	−1.94
95	1.32	−1.09
100	0.19	−0.19

L.E. radius................ 3.56
Slope of radius through end of chord................ 0.305

[1] From the table in Fig. 5, *N.A.C.A. Technical Note* 567, airfoil 23018.

TABLE 3:4[1]

Station	Upper	Lower
0	0	0
1.25	3.86	−1.73
2.50	5.00	−2.56
5.0	6.54	−3.59
7.5	7.56	−4.35
10	8.34	−4.93
15	9.31	−5.84
20	9.79	−6.48
25	9.98	−6.87
30	9.97	−7.06
40	9.49	−6.96
50	8.55	−6.44
60	7.32	−5.61
70	5.84	−4.55
80	4.16	−3.29
90	2.26	−1.83
95	1.25	−1.03
100	0.18	−0.18

L.E. radius................ 3.20
Slope of radius through end of chord................ 0.305

[1] Table 3:3 multiplied by 17/18 to obtain ordinates for airfoil 23017.

3. Multiply each figure in column 3, Table 3:3, by 0.944 to obtain column 3, Table 3:4.

$$-1.83 \times 0.944 = -1.73$$
$$-2.71 \times 0.944 = -2.56$$
$$-3.80 \times 0.944 = -3.59, \text{ etc.}$$

4. The leading edge radius must be determined by direct proportion between the N.A.C.A. 23015 and the N.A.C.A. 23018 airfoils.

L.E. radius for 23018 = 3.56
L.E. radius[1] for 23015 = 2.48

Therefore,

L.E. radius for 23017 = 3.20

[1] *N.A.C.A. Technical Note* 567.

Caution:

1. Do not multiply the slope 0.305 by the constant. It remains the same for both tables since it is the tangent of an angle.
2. In using this method for converting a known airfoil into the unknown, always use the table of ordinates of the airfoil *nearest* the one desired. For example: The ordinates for 23015 and 23018 are known. To obtain the 23017 ordinates, it should be said that

$$23018 \text{ ordinates} \times {}^{17}\!/_{18} = 23017 \text{ ordinates}$$

and never that

$$23015 \text{ ordinates} \times {}^{17}\!/_{15} = 23017 \text{ ordinates}$$

To obtain 23016 ordinates, it should be said that

$$23015 \text{ ordinates} \times {}^{16}\!/_{15} = 23016 \text{ ordinates}$$

and never that

$$23018 \text{ ordinates} \times {}^{16}\!/_{18} = 23016 \text{ ordinates}$$

After the table of ordinates for the N.A.C.A. 23017 airfoil has been calculated (usually by slide rule), proceed to lay it out exactly as in Art. 3:5. After the layout man understands the meaning of the terms used in the N.A.C.A. reports, he can lay out any airfoil.

References

N.A.C.A. Reports 460 and 610.
N.A.C.A. Technical Note 567.

3:7. How to Lay Out Intermediate Airfoils When the Two End Airfoils Are Given. It is customary, in tapered wings, to use one airfoil at the root of the wing and another airfoil at the tip; hence it becomes necessary to determine intermediate airfoils for the various ribs and bulkheads. Usually this is done by the loft; however, it frequently happens that some layout man must determine these intermediate contours, particularly if he is in preliminary design, where the general characteristics of a new airplane are being determined.

For empennage surfaces, the procedure is much simpler than outlined here. In the first place, a symmetrical airfoil such as the N.A.C.A. 0012 is used; and usually there is incidence only. To determine empennage intermediate airfoils, a procedure similar to the following is used, except that the chord plane is used as the datum plane in all cases, and only half breadths are shown.

Two airfoils for a tapered wing will be chosen and one intermediate airfoil determined. By this method any number of intermediate airfoils may be similarly determined.

Illustration

Given:

1. Root airfoil = N.A.C.A. 23017 (see Fig. 3:3)
   Chord = 132 in.
   Incidence = +5 deg. about the 35 per cent line
   (For table of ordinates see Table 3:4.)
2. Tip airfoil = N.A.C.A. 23009
   Chord = 48 in.
   Incidence = +3 deg. about the 35 per cent line
   (For table of ordinates see Table 3:6.)
3. Root airfoil is at wing station 0
   Tip airfoil is at wing station 528
   The 35 per cent line is normal to the plane of symmetry of the airplane

To find: The contour of the airfoil at wing station 221.

Operations:

1. Calculate the ordinates for the root airfoil, plot it to any convenient scale and rotate it for incidence as described in Art. 3:5 (see Fig. 3:3).
2. Repeat operation 1 for the tip airfoil, and place the tip airfoil over the root airfoil so that the respective intersections of the 35 per cent lines and the chord lines coincide, as shown in Fig. 3:3.
3. Draw construction lines perpendicular to the wing reference line through the various per cent points as outlined in Art. 3:5, operations 10 and 11. This gives the new per cent points along the contour using the reference line as a base line.

4. Connect these common per cent points of the root and tip airfoils with straight lines.
5. Scale the length of these lines and prepare a table as shown in columns 1, 2, 3 of Table 3:5.
6. To obtain the contour of the rib at wing station 221, these per cent lines must be divided proportionately. For

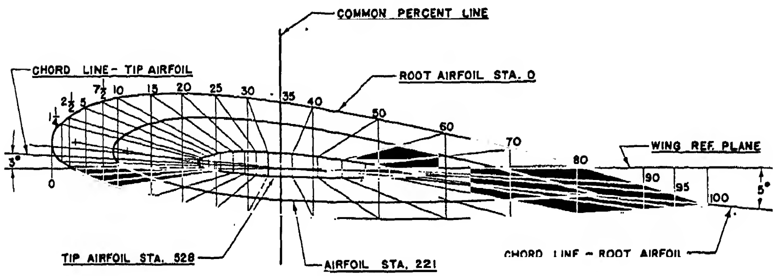


FIG. 3:3.

example, the upper 10 per cent line is 23.5 in. long between stations 0 and 528. Therefore, the length from station 0 to station 221 will be

$$23.5 \times {}^{221}\!/_{528} = 9.8 \text{ in.}$$

Divide every common per cent line in this manner; in so doing, columns 4 and 5 of Table 3:5 will be filled in.

7. The centers of the leading-edge radii for the root and tip airfoils are also connected by a straight line, which line is divided proportionately as outlined in operation 6.
8. The leading-edge radius must also be determined proportionately; the calculations are shown beneath Table 3:5.
9. After these values have been determined, plot these points on the common per cent lines.
10. If the work has been correctly and accurately done, it should be possible to draw a smooth curve through these points, this being the airfoil shape at wing station 221. It is necessary to repeat these operations at each wing station where it is desired to plot a contour.

TABLE 3:5

1	2	3	4	5
Per cent chord	Total length of per cent lines from station 0 to station 528		Length of per cent lines from station 0 to station 221	
	Upper	Lower	Upper	Lower
0	29.6	29.6	12.4	12.4
1¼	29.3	28.3	12.3	11.8
2½	28.4	27.4	11.9	11.5
5	26.6	25.1	11.1	10.5
7½	25.2	23.3	10.5	9.7
10	23.5	21.4	9.8	8.9
15	20.3	17.5	8.5	7.3
20	17.0	14.1	7.1	5.9
25	13.9	10.9	5.8	4.6
30	11.5	8.5	4.8	3.6
35	10.3	7.6	4.3	3.2
40	10.4	8.8	4.4	3.7
50	14.5	14.6	6.1	6.1
60	21.5	22.2	9.0	9.3
70	29.0	29.6	12.1	12.4
80	37.5	38.2	15.7	16.0
90	45.5	46.0	19.0	19.3
95	49.7	49.9	20.8	20.9
100	54.2	54.2	22.7	22.7
Radius centers	26.0		10.9	

To find points on per cent lines for rib 221, multiply each of the values in columns 2 and 3 (Table 3:5) by $^{221}/_{528} = 0.418$.

$$\begin{aligned}
\text{L.E. radius at station 0} &= 4.44 \text{ in.}\\
\text{L.E. radius at station 528} &= \underline{0.43 \text{ in.}}\\
\text{Difference in radius} &= 4.01 \text{ in.}
\end{aligned}$$

Therefore,

$$\begin{aligned}
\text{L.E. radius at station 221} &= 4.44 - (4.01 \times 0.418)\\
&= 4.44 - 1.68 = 2.76
\end{aligned}$$

3:8. The N.A.C.A.[1] It has been shown in this chapter what an important part the National Advisory Committee for Aeronautics has played in the development of airfoils. However, the

[1] For a well-illustrated and interestingly written article on the N.A.C.A., see "From the Wind Tunnels of Langley," in the March, 1941, issue of *Fortune Magazine*. Much of the information in this article was obtained from the above-mentioned magazine.

work of the N.A.C.A. is not limited to airfoils, but covers practically all branches of the aeronautical sciences. Whether the layout man realizes it or not, practically all of his design has been affected in some manner by the findings of this committee.

It was in 1915 that Congress established the N.A.C.A. with an appropriation of $5000 a year to "supervise and direct the scientific study of the problems of flight." As a measure of its growth and of the importance of its work, it should be noted that the appropriations for 1941 exceed $11,000,000.

The President appoints the committee of fifteen to serve without pay. Six are specially qualified civilians, two each from the Army, the Navy, and the Civil Aeronautics Administration, and one each from the Weather Bureau, the Smithsonian Institution, and the National Bureau of Standards. There are four major technical committees: (1) Aerodynamics, (2) Power Plants, (3) Aircraft Materials, (4) Aircraft Structures, whose titles define the scope of their work. The actual work is done by a staff of nearly a thousand employees.

The original laboratories are at Langley Field, Va., where practically all the research has been carried on in the past. However, a new engine research laboratory is being constructed at Cleveland, Ohio; at Sunnyvale, Calif., a new aerodynamic laboratory, the Ames Aeronautical laboratory, is being built at the old Navy dirigible base.

Fortune Magazine in its article on the N.A.C.A. stated that a perfect definition of the aircraft designer's goal is "the reduction of drag." This statement is no exaggeration for the aircraft designer must strive for better performance and more speed, and there are many things he may do to help attain that goal.

Since the reduction of drag is so important, the various types of drag and their meaning should be discussed. "Drag" is the term applied to those forces which resist the forward movement, or the lifting capacity, of an airplane. Probably everyone has tried holding his arm out of the window of a moving automobile and has experienced that invisible force of the air trying to push the arm back. As an airplane flies, the air is resisting its forward movement, just as the air resisted the forward movement of the autoist's arm. The unfortunate thing about drag is that it increases as the square of the speed. An airplane flying at 200 m.p.h. will have four times as much drag as at 100 m.p.h. At 300 m.p.h. it will have nine times the drag of 100 m.p.h.

Some drag can be reduced, some cannot. Induced drag is the inevitable price paid for lift and cannot be reduced except very slightly. It is extremely difficult to define "induced drag" and to show clearly why it cannot appreciably be reduced. A simple analogy of an object moving through water may be drawn: the water is parted by the moving object and flows together in its wake, which sets up eddies. Energy, which can never be recovered, was imparted to the water to set it in motion. The airplane as it passes through the air moves this air; energy was expended to set it in motion. The imparting of this motion to the air manifests itself in the form of lost energy which is known as induced drag.

Parasite drag includes the various kinds of drag that can be reduced. "Profile drag," one form of parasite drag, is a term applied to a wing to describe the effect of turbulence in the thin layer of air close to its surface and in its wake. Another form of parasite drag is interference drag which is due to eddies set up by the proximity of two structural members. Skin friction, another type of parasite drag, results when particles of air are forced along the exterior surface of the wing. When any liquid flows through a tube or pipe, the friction between the liquid and the walls of the tube resists the flow of the liquid. Since air is a liquid, there is a similar action between the air and the skin of the airplane.

The N.A.C.A. has led the world in the attack on parasite drag. Some of the more important contributions may be listed as: airfoils, engine cowling, location of nacelles, location of the wing in relation to the fuselage, and a general cleanup of struts, landing gear, etc.

Until 1934, the Clark Y was considered a very good airfoil, but in that year the N.A.C.A. brought forth their "twenty-three-oh-twelve" (23012) which proved so superior to previous airfoils that approximately three-fourths of the world's airplanes today are using versions of this scientifically famous airfoil. Among other things, it is distinguished for its high ratio of lift to drag. This ratio (usually expressed as L/D or in conversation as "L over D") is similar to an efficiency factor; the higher the ratio, the greater the efficiency. In the 23012 airfoil, L/D goes up to 24, which is considered very good. This airfoil has an unfortunate tendency to stall suddenly, that is, it suddenly loses its lifting power. However, ways have been discovered to alleviate this tendency to stall, which will be discussed later.

Until about 1931, air-cooled engines had no cowling; the radial cylinders were fully exposed to the air stream. The N.A.C.A.

At the speed of sound (approximately 1100 ft. per second or 750 m.p.h.), there occurs a peculiar form of drag known as "compressibility burble" which imposes a prohibitive drag, as far as efficient aircraft design is concerned. If it were not for compressibility burble, the range of gun-fired projectiles would be greatly increased; in this case, by increasing the powder charge, enough additional energy can be imparted to the projectile to overcome this drag. Obviously, such methods are impossible with aircraft, so that, unless some new means of propulsion is devised or unless means of controlling compressibility burble is discovered, the speed of an airplane will be limited by the speed of sound.

The N.A.C.A. has done much work on lift. The present-day flaps for landing which are standard on all airplanes result from N.A.C.A. tests and research. When the flaps are lowered at relatively slow speeds, they increase both the lift and drag, which not only slows the speed of the airplane, but also enables the airplane to remain aloft at a lower rate of speed. At times, in order to provide more flap area, the flaps are extended to the wing tips. This presents a problem of where to place the ailerons. One solution is to mount them on top of the wing on short vertical masts, in which case the aileron is known as a "spoiler." Other work of the N.A.C.A. relates to stall—that peculiar characteristic of a wing where, at certain attitudes of flight, the lift suddenly drops. This stalling characteristic of the N.A.C.A. 23012 and related airfoils was mentioned previously in this article. To alleviate this tendency to stall suddenly, the N.A.C.A. recommends giving the wing tips "wash out" which is no more than twisting the wing so that the incidence at the tip is less than that at the root. This explains why in the original design of a wing, the incidence so often decreases toward the tip. (See Fig. 3:3 where the incidence changed from 5 deg. at the root to 3 deg. at the tip.)

Although the model to be tested in a wind tunnel is usually mounted in the air stream on delicate measuring devices, the N.A.C.A. may use their free flight tunnel in which an exact model correctly balanced to agree with the actual airplane is actually flown. A tiny electric motor usually drives the propeller while the operator, by means of fine trailing wires, may operate the various control surfaces. Another operator controls the speed of the air stream so that the model will not smash itself against the walls of the tunnel.

The free spin tunnel may be used if the spinning characteristics are being investigated. Here the model, controls set for a spin, is tossed into a vertical tunnel having an uprushing blast of air. Then the operator moves the controls by means of the fine trailing wires and the recovery is studied. Before the N.A.C.A.'s spin tunnel, it was necessary for a pilot to take the airplane aloft, throw it in a spin and then hope for the best. If it did not come out of the spin, it was necessary for the manufacturer to build another airplane, making whatever changes were deemed advisable, and try it again. Now it is possible to make these changes in the model, thus saving much time and expense. The N.A.C.A. has found that the design of the tail is nearly everything in spin recovery. The vertical stabilizer and rudder should be large, with the horizontal stabilizer and elevator mounted rather high so as not to blanket the vertical surfaces while the plane is spinning.

Everyone who has ridden in an airplane is familiar with "bumps" which are usually caused by uprising drafts of air. If the N.A.C.A. is investigating the behavior of an airplane in these vertical drafts, they use their gust tunnel. Here a model is catapulted into uprising currents of air, and its behavior is studied by means of cameras, etc. They have developed what is known as "V-G recorders," measuring instruments that record the severity of the bumps. They are carried on many planes in flight to measure actual flight conditions; very small ones are also carried by the models in the gust tunnel. There are many other tunnels and laboratories used by the N.A.C.A. in their research; however, it is impossible to discuss them all in this text. From the foregoing, the student can gain some idea of the magnitude and the complexity of the work. Much investigation has been done on materials, and many of them have been improved as a result of N.A.C.A. research and tests.

One of every service model of an Army or Navy airplane automatically goes to Langley Field for test, and invariably leaves 20 to 60 m.p.h. faster. The Bell Airacobra gained approximately 60 miles by changing air scoops, supercharger location, wheel wells, etc. When the manufacturers submit designs of a new model to the N.A.C.A., an opportunity is offered to eliminate some problems before they arise. The new Vought Navy fighter had the air ducts for the oil cooler changed, and the list could go on indefinitely.

Problems

3:1. Table 3:6 is the table of ordinates in Fig. 2, *N.A.C.A. Technical Note* 567, for the N.A.C.A. 23009 airfoil.

The following are also known:

Wing chord = 48 in.
Incidence = +3 deg. rotated about the 37½ per cent point
Front spar = 12 per cent chord
Center spar = 37½ per cent chord
Rear spar = 65 per cent chord
Spar planes are normal to the wing reference plane

From the above data:

a. Prepare a table of ordinates similar to Table 3:2.
b. Rotate for incidence and lay in the spars.

TABLE 3:6
(N.A.C.A. 23009)

Station	Upper	Lower
0	0	0
1.25	2.04	−0.91
2.5	2.83	−1.19
5.0	3.93	−1.44
7.5	4.70	−1.63
10	5.26	−1.79
15	5.85	−2.17
20	6.06	−2.55
25	6.11	−2.80
30	6.05	−2.96
40	5.69	−3.03
50	5.09	−2.86
60	4.32	−2.53
70	3.42	−2.08
80	2.41	−1.51
90	1.31	−0.86
95	0.72	−0.50
100	0.10	−0.10

L.E. radius. 0.89
Slope of radius through end of chord. 0.305

The table of ordinates as calculated from the data given in this problem is to appear on the layout.

3:2. Prepare a table of ordinates for the N.A.C.A. 23011 airfoil from the data given in Table 3:7.

TABLE 3:7
(N.A.C.A. 23012)

Station	Upper	Lower
0	0	0
1.25	2.67	−1.23
2.50	3.61	−1.71
5.0	4.91	−2.26
7.5	5.80	−2.61
10	6.43	−2.92
15	7.19	−3.50
20	7.50	−3.97
25	7.60	−4.28
30	7.55	−4.46
40	7.14	−4.48
50	6.41	−4.17
60	5.47	−3.67
70	4.36	−3.00
80	3.08	−2.16
90	1.68	−1.23
95	0.92	−0.70
100	0.13	−0.13

L.E. radius............. 1.58
Slope of radius through
end of chord......... 0.305

Plot the airfoil, rotate for incidence, and lay in the spars for the following conditions:

Wing chord = 60. in.
Incidence = +4 deg. rotated about the 100 per cent point
Front spar = 20 per cent chord
Rear spar = 45 per cent chord
Spar planes are normal to the wing reference plane

The table of ordinates as calculated from the data given in this problem is to appear on the layout.

3:3. Plot the airfoil and prepare a table of offsets for rib 585 when the following data are given:

Airfoil at wing station 421 = N.A.C.A. 23014.8
Chord = 135 in.
Incidence = +4 deg. rotated about the 40 per cent line

Airfoil at wing station 710.3 = N.A.C.A. 23012
Chord = 70½ in.
Incidence = +1 deg. rotated about the 40 per cent line

The 40 per cent line is normal to the plane of symmetry of the airplane.

For table of ordinates for N.A.C.A. 23015 airfoil, see Table 3:8, which is the table of ordinates in Fig. 4, *N.A.C.A. Technical Note* 567. For table of ordinates for N.A.C.A. 23012 airfoil see Table 3:7.

After the rib contour at station 585 is plotted, a table of offsets similar to Table 3:9 is to be shown on the layout. This table is to be prepared using the 40 per cent line and the reference lines as the datum planes.

TABLE 3:8
(N.A.C.A. 23015)

Station	Upper	Lower
0	0	0
1¼	3.34	−1.54
2½	4.44	−2.25
5	5.89	−3.04
7½	6.91	−3.61
10	7.64	−4.09
15	8.52	−4.84
20	8.92	−5.41
25	9.08	−5.78
30	9.05	−5.96
40	8.59	−5.92
50	7.74	−5.50
60	6.61	−4.81
70	5.25	−3.91
80	3.73	−2.83
90	2.04	−1.59
95	1.12	−0.90
100	0.16	−0.16

L.E. radius.............. 2.48
Slope of radius through end of chord.............. 0.305

3:4. *a.* What is the meaning of the initials N.A.C.A.?
b. How many members are on this committee?

3:5. Explain the meaning of the coding for the
a. N.A.C.A. 1415 airfoil.
b. N.A.C.A. 23012 airfoil.

TABLE 3:9

Per cent	Y*	Z* (top)	Z* (bottom)
40			
38¾			
37½			
35			
32½			
30			
25			
20			
15			
10			
0			
10			
20			
30			
40			
50			
55			
60			

L.E. radius....

* For definitions of *Y* and *Z* see Prob. Fig. 2:8:3*B*.

3:6. The data for airfoils in N.A.C.A. reports are in what terms?

3:7. What is considered to be a perfect definition of the aircraft designer's goal?

3:8. Define briefly the term "drag."

3:9. Can the following forms of drag be reduced:
a. Parasite drag?
b. Induced drag?

3:10. Discuss briefly
a. Induced drag.
b. Three kinds of parasite drag.

3:11. What are some of the leading contributions of the N.A.C.A. in their attack on parasite drag?

3:12. *a.* What is one of the things the N.A.C.A.'s 23012 airfoil is distinguished for?
b. What is one undesirable characteristic?
c. What is the maximum lift/drag ratio of this airfoil?

3:13. What is the most efficient position of
a. Engine with respect to the wings?
b. Wings with respect to the fuselage?
c. Propeller with respect to the wings?

3:14. Name the two ways to reduce parasite drag. Discuss each briefly.

3:15. *a.* What affliction is common to all existing airfoils? Describe briefly.
b. What kind of drag is imposed by this affliction?

3:16. *a.* What is remarkable about the laminar flow wing?
b. What is the main objection to this new airfoil?

3:17. When parasite drag is fully under control, what will be the limitations on aircraft? Why?

3:18. By what means do wing flaps reduce the speed of an airplane?

CHAPTER 4

GENERAL DESIGN CONSIDERATIONS

It would be well to discuss some general design considerations before taking up specific design practices. There are many things an experienced designer knows—things which are hard to classify; hence we shall group and discuss this miscellaneous knowledge at this point.

4:1. Standard Parts.[1] Standard parts must be used wherever possible. Practically all engineering departments have their standards group, or standards engineer, who usually issues what is commonly known as the "Standards Book." In it are listed all the miscellaneous items that have been standardized, such as extruded sections, bolts, nuts and nut plates, clips, clamps, etc. The Army-Navy standard parts are usually in a separate book known as the "AN Book." Those parts with the prefix AN have been approved by the Army and Navy, such as AN295-2, oil cup for ⅛ pipe thread. Those parts with the prefix AC have been approved only by the Air Corps, such as AC366-D1032, dural nut plate with 10-32 thread. If or when these nut plates are approved by the Navy, the prefix will become AN. It is obvious that the designer should familiarize himself with such books and use these standard parts wherever possible.

4:2. Standard Purchased Sizes. The designer should see that the material necessary for his part falls within the standard purchased sizes. Usually the material group in the engineering department issues lists showing standard purchased sizes of all sheet, tube, bar, etc. It is true that in isolated cases it is necessary to order a special size, but the layout man should make every effort to keep his design within the limitations of the standard purchased size. This is because of the time element and cost in securing special size stock from the mill; although it frequently takes months to secure special stock, standard stock may be obtained in a few days or weeks. Special sizes always

[1] See C.A.B. 27, pp. 38–41, for a discussion of AN standard parts.

cost more per pound than standard sizes. If the designer feels that a special size is imperative, he should consult his supervisor or a member of the production design group.

4:3. Tolerances and Limits. Tolerances must be made as liberal as possible. The shop admittedly can hold close tolerances; however, close tolerances increase rejections, which are highly undesirable. Many designers do not understand the problems of the shop and underestimate the difficulties in holding very close tolerances. If ± 0.001 would be satisfactory on a machine part, ± 0.0005 should not be specified just because it might be thought to work a little better. It is wise to check with the supervisor or with the chief checker on such problems, for they have had more experience on such matters and are in a better position to decide the required tolerance for the particular problem.

Many designers use the terms "tolerance" and "limits" loosely, without regard to their correct meaning. It would be well to clear up this misunderstanding so that the correct term will always be used. Limits indicate the maximum and minimum dimensions of a part within which they are acceptable and beyond which they are not. In other words, the limits show the size of the part. For example, a dimension reading $\substack{0.750 \\ 0.746}$ indicates the maximum permissible size is 0.750, while the minimum is 0.746. If the part is larger than 0.750 or smaller than 0.746, it would not be acceptable.

The tolerance is the difference between the limits. In the above example, the tolerance is $0.750 - 0.746 = 0.004$. This dimension might have been written $0.750^{+0.000}_{-0.004}$ where the tolerance is $^{+0.000}_{-0.004}$. It should be remembered that the tolerance does not show the size of the part, but does indicate the accuracy with which it should be made.

4:4. Possible Adjustments. Parts should be so designed as to permit adjustment for small shop errors by means of shims, lap splices, etc., wherever possible. By this means, the tolerances of individual parts may be quite liberal; in many cases $\pm 1/32$ is satisfactory.

4:5. Minimum Number of Parts. Always keep the number of parts down to a minimum. Many designers do not realize the cost of handling an order, from the time it is issued until the part is in the proper stock bin. By keeping down the number of

parts, the number of orders is held to a minimum, which in turn effects a considerable saving in dollars. By a little ingenuity, the designer can often make one part do the work of two, or two parts do the work of three. The use of forgings or castings often eliminates a number of parts; a bead pressed in a rib may eliminate a stiffener. Since there are many ways in which a designer can eliminate parts, he should be constantly alert for such opportunities.

4:6. Prevention of Relative Movement between Parts. All stringers and stiffeners should be fastened at both ends. For example, when a wing stringer stops adjacent to a rib, this stringer should be attached to the rib by means of a clip or other suitable fastener. This is to prevent relative motion between the end of the stiffener and the rib, which may cause the skin to crack. This fact has been often overlooked in the design of the airplane, only to have cracks develop at these points after the airplane is in service. Then the airplane must be sent to a repair depot where often expensive repairs are necessary owing to the inaccessibility of these points. This holds true not only for the wings, but for any flat sheet on the airplane where relative motion between the parts is transmitted only by the skin. On spars and other thin web beams having vertical stiffeners, these stiffeners are usually securely fastened to the caps themselves. The design usually consists of cap material in the form of angles or tees, with a sheet web. The stiffeners usually overlap the vertical leg of the angle or tee, and are attached securely by means of rivets. Such a design is shown in Fig. 11:9.

4:7. Bend Radii. Bend radii for sheet stock should be as large as possible, to allow for an increase in gage. It is common practice for many designers when working with sheet stock to specify the minimum bend radii for their particular material and gage, with a bolt or rivet head just tangent to the radius point. All too frequently, the gage must be increased owing to increased loads, which in most cases increases the minimum bend radius. Then the bolt head that was just tangent to the radius is now riding the radius, which is not recommended practice. So the bolt must be moved, which in turn usually requires a change in the part the bolt was attaching. If the designer had not specified the minimum radius in his original design, an increase in gage probably would not have affected the layout at all. Tables of

minimum bend radii are usually in the drafting room manual and are included in the Appendix of this text for the use of the student; these tables are reproduced from the drafting room manual of a large aircraft manufacturer.

The undesirable practice of some designers in guessing bend radii should be discussed. A layout man should never *guess* at the bend radius, assuming that the detailer will look it up and note the correct value on his drawing. All too often his guess is incorrect, which means in many cases that a part shown on the layout with proper clearance will actually ride the radius.

4:8. Preferred Sheet Stock.[1] The most commonly used sheet stock at the present time is 24ST Alclad, often abbreviated as 24STAL. The 24ST is the designation of the Aluminum Company of America for a particular alloy in the hard state; 24SO would be the same alloy in the soft or annealed state. Alclad is the term applied to aluminum alloy (duralumin) sheet, which has a very thin layer of aluminum on each side. This thin coating of aluminum is obtained by placing an ingot of aluminum alloy between two ingots of aluminum as in a sandwich, and rolling the ingots out into sheet or plate. The rolling process forces the aluminum and aluminum alloy molecules together so that there is a permanent bond; in reality, they become one sheet.

Alclad is preferable because of the superior corrosion resistant properties of aluminum. There is some stainless steel and some magnesium sheet used in aircraft, but in the majority of cases, the layout man will be working with Alclad sheet.

4:9. Joggles. Joggles are usually omitted for steps of 0.032 or less, except in the outer skin where it is of paramount impor-

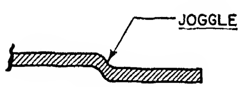


FIG. 4:1.

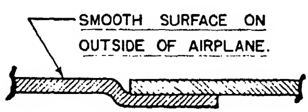


FIG. 4:2.

tance to maintain a perfectly smooth surface for aerodynamic reasons. Figure 4:1 shows a joggle in a sheet; Fig. 4:2 shows how this joggle is used to present a perfectly smooth surface; Fig. 4:3 shows the same joint when the joggle is omitted. On

[1] See TM 1–435, pp. 46–53, and C.A.B. 27, pp. 20–26, for a thorough discussion of the aluminum alloys and their heat-treatment.

any interior construction, as was mentioned previously in this paragraph, it is not customary to joggle for 0.032 or less. This is because any unevenness caused by the overlapping of such thin sheets as shown in Fig. 4:3 would be negligible and usually would not justify the cost of the required operation.

FIG. 4:3.

4:10. Limitations of Aluminum Alloy Mating Parts. Dural mating parts should not be used in bearings, or in removable or adjustable parts. For example, a dural shaft should never be designed to rotate in a dural bushing. Two dural parts should never be specified where there is any sliding motion between them owing to the tendency to seize and gall. Although sheets or bars of aluminum alloy feel smooth to touch, if there is a sliding action between them accompanied by forces that tend to squeeze them together, the molecules seem to seize one another and literally tear the surface so that the parts are unfit for service. This is why the Air Corps specifies that dural nuts and bolts should not be used in installations where frequent removal is necessary. In the fuel and oil systems, dural mating fittings are not ordinarily used in places where they must be frequently removed.

4:11. Primary and Secondary Structure. A term often encountered by the layout man is "primary structure," which is defined in C.A.R. (Civil Aeronautics Regulations) 04.131 as follows: "Those portions of the airplane, the failure of which would seriously endanger the safety of the airplane." This definition is rather broad and leaves much to the judgment of the individual; in many borderline cases it is difficult to say definitely whether or not a certain part of the airplane should be classed as primary structure. Obviously a fitting attaching a wing to the fuselage would be primary structure, as its failure would assuredly "endanger the safety of the airplane." On the other hand, a seat support could hardly be classed as primary structure since its failure would do no more than inconvenience the passenger. "Secondary structure" is all structure not classed as "primary structure"; hence the seat support just mentioned would be classified as secondary structure.

4:12. Control Surface Clearances. Control surface clearances must be designed to prevent fouling by elastic deformation. It is

impossible to set forth any definite rules regarding clearance in a control surface, such as an aileron, for it will vary with the design, size, etc. If it is to be fabric covered, the application of the fabric to the frame with all the tape and dope tends to make the surface grow, that is, the covered surface will be larger than that shown in the layout. When deflection occurs owing to the flying loads, the gap between the control surface and the airplane tends to decrease in many cases. On large airplanes having large control surfaces, the clearance should be more than on small airplanes having small control surfaces. No definite figure can be given for this; however the supervisor should be consulted on this important question.

4:13. Drainage. Provision for drainage of wings, fuselages, and control surfaces should be made to prevent the collection of rain or condensation water within the airplane. Drain holes should be at the low points where water would collect; they need not be very large, $\frac{1}{4}$ to $\frac{3}{8}$-in. diameter being considered sufficient by many designers.

4:14. Weight vs. Strength and Rigidity. The importance of keeping weight at a minimum must be emphasized, since each pound saved in the structure means a pound gained in the useful load that the airplane can carry. A part should never be heavier than is consistent with strength or rigidity. This is why it is so important for a layout man to strength-check his own design, in order that a part will not be stronger than necessary. He should never design "by eye," which is no more than guessing at the size required. After a man has had years of experience in aircraft design, he knows fairly well what will be required, but he should strength-check his design to be sure it is as light as possible. Another factor to be considered is rigidity. At times a part may not carry a very high load and from a stress standpoint could be extremely light; yet that part, if designed for stress alone, would be flimsy to touch and may vibrate excessively; so the designer cannot lose sight of this fact in his efforts to keep weight down to a minimum.

4:15. Sources of Outside Information. A good layout man constantly seeks advice, yet he should not bother his neighbor. There are specialists in charge of all the engineering branches, such as stress, weights, aerodynamics, standards, etc., and these men should be consulted when advice is needed, as well as the supervisor.

The layout man should learn all he can about shop tools and machines, their uses and operations; in this way he will understand shop problems and become a more competent designer. Many aircraft factories allow members of their engineering department free access to the shop during their lunch hour, and, since the shop lunch period is frequently at a different time from that of the engineering department, they are able to watch the men and machines at work. They should be careful not to get in the way of the operators or ask too many questions; much can be learned by just watching the men and the machines.

4:16. Fittings.[1] In the design of a fitting, one of the first things for a layout man to do is to determine the loads. He should make no attempt to calculate the loads himself, but should secure that information from the stress department, and find out at that time what margin of safety he should maintain for that particular fitting, using the given loads (see Art. 1:12).

Fittings should be made up of the fewest possible parts to reduce welding, riveting, etc., to a minimum. Important and heavy fittings should not be made up of welded sheet stock but should be manufactured from a single piece of material and attached by means of threads, bolts, or rivets. At times, it is advantageous to make fittings from extrusions. The outside shape may be formed by the extrusion process, and thus save considerable machining time and expense. This is usually done only on aluminum alloy fittings, for very few extrusions other than aluminum alloy are used in aircraft.

For experimental airplanes, fittings are usually machined from bar stock. When the experimental fitting is being laid out, provision should be made for the draft (see Art. 6:1) on the forged part for production. The draft, in many instances, is great enough to cause interference if not provided for in the original design. This provision should be made, since many fittings are converted into forgings if the experimental airplane is approved and is put in production.

In secondary structure fittings, sand and permanent mold castings are considered good practice and should be used wherever the design and production value permits.

Figure 4:4 shows a lug on a fitting with an unbushed hole. When no bushing is used in a hole, the design must be such that it will be possible to enlarge the hole and install the next larger

[1] See C.A.B. 27, p. 75, for a definition and brief discussion of fittings.

size of bolt. The layout man must remember that although the AN series of bolts increase by $\frac{1}{16}$-in. increments in many cases, in the larger sizes the increments are $\frac{1}{8}$ in. If in doubt as to the next larger size of AN bolt, look in the AN book. When a strength check is made on a design where there is no bushing, the calculations should be made for the *next* larger size of bolt, for it is imperative to have the required strength after the larger bolt is installed.

The use of an eyebolt, AN42 to AN49, as part of a fitting is usually not recommended. It is considered better design to have the eye as an integral part of the fitting, such as the lug shown in Fig. 4:4.

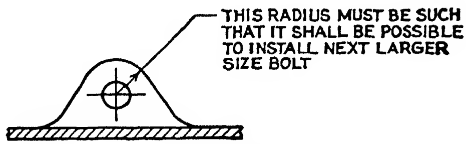


FIG. 4:4.

In an effort to save weight, an inexperienced layout man may tap a fitting to eliminate the use of a steel nut; this often is done when the design requires a large steel bolt and nut. Although this practice may save a little weight, it is not usually recommended because of the difficulty in locking the head, that is, fastening the head so that the bolt will not rotate and become loose. Another disadvantage is the possibility of the threads being stripped in the fitting by an inexperienced or careless workman. If this should happen, it would probably involve costly delays while the damaged fitting was being removed and a new one installed. In Fig. 4:13, a male and female type of connection is shown with a through bolt, and a nut on the outside. This illustrates a design where it would be possible (although not recommended) to thread one side of the female fitting in order to replace the nut; it should not be done for the reasons already mentioned. As pointed out in Art. 1:9, the layout man must provide sufficient wrench clearance for all bolts and nuts; the required clearances are usually found in the drafting room manual.

4:17. Eccentricity. This is the term applied to a condition where the force and its reaction are not in line, and which pro-

duces a very undesirable loading condition. A good axiom to remember is: *Eccentricity must be kept to a minimum.* Many illustrations could be drawn showing various places where eccentricities enter into design; however only one condition will be shown to illustrate the term. Figure 4:5 shows a connection between a wing spar and fuselage. The load P may be considered as being applied by the bolt from the fuselage attachment, and is resisted by a reaction along the spar cap as shown. The

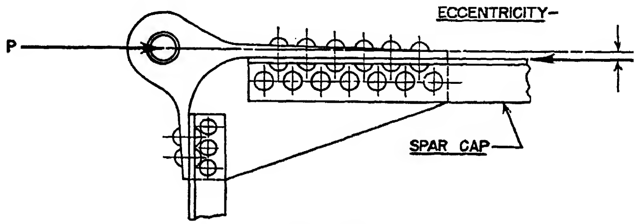


FIG. 4:5.

distance between these lines of force is known as the "eccentricity." In many cases, it is impossible to have zero eccentricity, that is, to have the force and its reaction in line with one another; but the designer should make every effort to keep the eccentricity to a minimum.

4:18. Stress Concentrations. As this term implies, there may be places where stresses concentrate because of faulty design. The layout man, by adhering to a few simple principles, can do

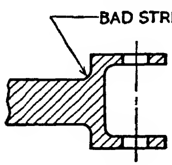


FIG. 4:6.

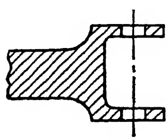


FIG. 4:7.

much to eliminate these stress concentrations. Figure 4:6 shows a simple yoke where a small radius is used, causing bad stress concentrations; Fig. 4:7 shows the same yoke correctly designed with a large fillet effecting a gradual change in section and eliminating these stress concentrations.

It is because of possible stress concentrations that the designer should always have a radius on inside corners. For example, Fig. 6:31 shows an extruded angle with a sharp inside corner.

Although it is possible to extrude this section, a sharp inside corner is never recommended. It is preferable to have a radius as shown in Fig. 6:32, which will eliminate this undesirable condition. The amount of radius depends largely on the part; for extruded sections, $\frac{1}{16}$-in. radius should be a minimum unless it is a large section, then the radius should be $\frac{1}{8}$-in. or more. If it is a machined fitting, $\frac{3}{32}$-in. radius is considered by many designers to be a good minimum, with 0.005 to 0.015 as an absolute minimum. A turned part, such as the shaft shown in Fig. 4:8, should have 0.005 to 0.015 as an absolute minimum with a larger radius preferable. It is impossible to give any preferred minimum radius for a shaft as it will vary with the design; in any case, keep it as large as is consistent with good design.

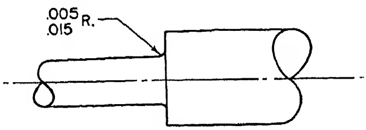


FIG. 4:8.

The important thing for the layout man to remember is to avoid sharp inside corners because of possible stress concentrations. It is impossible to set forth definite minimum radii for all problems; hence the layout man should consult his supervisor if in doubt.

An abrupt change in section is considered poor design, owing to possible stress concentrations. Figure 4:9 shows a simple

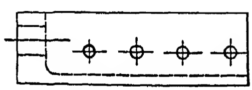


FIG. 4:9.

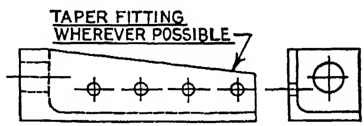


FIG. 4:10.

fitting incorrectly designed since there is an abrupt change in section. Figure 4:10 shows this same part tapered which reduces the abruptness of the change in section and at the same time provides an ideal means of saving weight. A fitting should be designed to have a gradual transfer of stress, that is, it must begin to pick up load from a member at a point where the fitting attaches, then transfer the load gradually from the member into the fitting. If there is a too abrupt change in section at the point where the fitting attaches, bad stress concentration may result;

hence fittings should be tapered wherever possible to reduce these stress concentrations.

4:19. Shear Conditions. Fittings should be designed, where possible, to carry the attaching bolts and rivets in shear, preferably in double shear. Rivets should not be used in tension. In order to avoid any misunderstanding in terminology, Fig. 4:11 shows a simple fitting where the load produces tension in the bolt; in cases such as this, rivets should not be used. Figure 4:12 shows a simple joint in single shear where a rivet or rivets would be acceptable. Figure 4:13 shows a simple male and female joint where the bolt is in double shear; here again rivets would be permitted since the connection subjects the rivet or

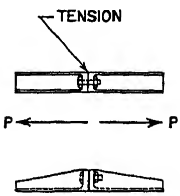


Fig. 4:11.

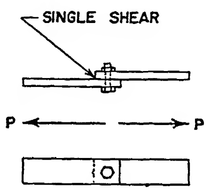


Fig. 4:12.

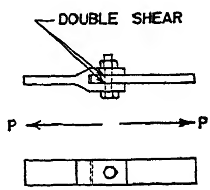


Fig. 4:13.

rivets to shear only. The advantage in designing for double shear is that the bolt (or rivet) is being sheared in two places simultaneously; in Fig. 4:12 it is being sheared in only one place.

4:20. Tension Fittings. This term is applied to a fitting where the principal loads produce tension such as shown in Fig. 4:11. When this is true, the designer should check the bearing area of the bolt head or nut on the fitting itself, particularly if it is an aluminum alloy fitting. This point has occasionally been overlooked, only to have the dural crushed under the head of the bolt owing to the high loads producing stresses greater than the metal could withstand. This was true in the case of an experimental transport where the undersides of the wings were attached to the fuselage structure by large tension fittings. After a few hours of flight, it was noticed that these bolts through the tension fittings had loosened; when this had repeated itself a time or two, it was discovered that the dural fitting under the bolt head was being crushed, causing the bolts to be loose when the airplane was at rest on the ground. This condition was remedied by placing heavy steel washers between the head of

the steel bolt and the dural fitting, to increase the bearing area on the fitting.

Special considerations should be given to these tension fittings, in order that they may satisfactorily carry the complex loads within themselves. Figure 4:14 shows a simple channel fitting of the type used in the connection shown in Fig. 4:11. Fittings such as this are frequently used to carry tension (or compression) loads across a bulkhead or spar. The eccentricity e must be kept to a minimum, as pointed out in Art. 4:17. The width of the end

FIG. 4:14.

W is designed by empirical rules depending on the conditions of loading.

In tension, $W = 1.25 \times$ diameter of bolt

In compression, $W = 0.75 \times$ diameter of bolt

The dimension A should never be less than the diameter of the bolt. Many designers prefer to use a 1.25 or 1.5 diameter for this dimension where possible.

The foregoing rules apply only to those fittings that are just wide enough for the bolt head and nut. For fittings that are considerably wider than this, these rules should be modified and the stress department consulted about the design.

4:21. SAE Steel Numbering System. The Society of Automotive Engineers (SAE) has developed numbering systems for various materials, by which they may be identified. Since this system is used in aircraft for steel designations, such as SAE 1025 or SAE X4130, the meaning of these numbers should be explained. The SAE steel numbering system is a code forming a four- or five-digit number; the first digit on the left indicates the type to which the steel belongs, where

1 = carbon steel
2 = nickel steel
3 = nickel chromium steel
4 = molybdenum steel
5 = chromium steel
6 = chromium vanadium steel
9 = silicon manganese steel

The second digit indicates the percentage of the predominant alloying element. The last two digits indicate the average carbon content in "points" or hundredths of 1 per cent.

Thus, SAE 1025 steel is a carbon steel containing no alloying element and having approximately 0.25 per cent carbon. SAE 2330 steel is a nickel steel of approximately 3 per cent nickel and 0.30 per cent carbon.

The prefix X is used in numerous instances to denote variations in the quantity of uncoded alloying elements. For example, SAE X4130 allows a greater proportion of chromium and a smaller proportion of manganese than the SAE 4130 steel.

This explanation does not cover the entire system but gives enough information so that the aircraft designer will know the principal alloying elements in the steel being specified. For complete information on this numbering system, refer to the 1941 SAE handbook, page 303.

Problems

4:1. *a.* What is the AN book?
b. What does the prefix AN mean?
c. What does the prefix AC mean?

4:2. *a.* Where is information usually obtained pertaining to standard purchased sizes?
b. Why is it desirable to use standard purchased sizes wherever possible?

4:3. *a.* Why are close tolerances on any part undesirable?
b. Explain fully the difference in meaning of the terms "limits" and "tolerances."

4:4. *a.* What can the designer do to provide for small shop errors?
b. Why is it desirable to keep the number of parts down to a minimum?

4:5. *a.* What may be done to prevent movement between parts?
b. Why is relative movement between parts undesirable, particularly when the parts are joined by flat sheet?

4:6. Is it desirable to use minimum bend radii for sheet stock on a layout? Explain your answer.

4:7. *a.* What material is most commonly used for flat sheet in airplanes?

b. Explain the meaning of the terms used in the answer to *a*.

c. What properties in Alclad sheet make it preferable to aluminum alloy sheet?

4:8. Draw a sketch showing a joggle, and explain when joggles are desirable and when they are not.

4:9. What limitations are placed on aluminum alloy mating parts? Explain fully.

4:10. *a*. Define "primary" and "secondary" structure.

b. Would an engine mount brace tube be primary or secondary structure?

4:11. *a*. What provision must be made at all low points for water that may accumulate within the airplane?

b. Will deflections of a wing under flying loads cause the clearance between wing and aileron to decrease? Explain fully, making any necessary sketches to illustrate your answer.

4:12. *a*. What characteristic besides strength must be considered in the design of any part?

b. What are some of the sources of information a layout man must learn to rely on?

c. How can he learn the uses and operations of shop machinery and tools?

4:13. *a*. What is one of the first things for a layout man to do when starting to design a fitting?

b. On experimental airplanes, how are fittings usually made?

c. If the airplane goes into production, how are the fittings usually redesigned?

4:14. *a*. What provision must be made if a bolt hole in a fitting is unbushed?

b. Are eyebolts recommended as part of a fitting?

4:15. When would threading a fitting in order to save the weight of a large steel nut be recommended? Explain fully.

4:16. *a*. Sketch a fitting showing lines of force that would produce the loading condition known as "eccentricity." In this sketch, show where the eccentricity is.

b. How much eccentricity is desirable?

4:17. *a*. What are stress concentrations? Sketch two conditions where they would occur.

b. Why are sharp inside corners objectionable?

c. What is considered an ideal minimum radius for machined fittings?

d. What is the absolute minimum radius for turned parts?

e. What is the preferred minimum radius for extruded sections?

4:18. *a.* Why are abrupt changes of sections not recommended?

b. Sketch two fittings, one having an abrupt change in section, the other having a gradual change.

4:19. *a.* Rivets should not be used in tension; single shear; double shear. State which is correct.

b. Make three sketches, showing a bolt in tension, single shear, and double shear.

4:20. *a.* Explain why the bearing area under the head of the principal bolt in a tension fitting should be checked.

b. Sketch a tension fitting giving the more important dimensions in terms of the principal bolt diameter.

4:21. *a.* What is known about SAE 1020 steel from the numbers 1020?

b. Describe what may be known from the coding for SAE 2320 steel.

4:22. *a.* What does the prefix X indicate?

b. Where may complete information on the SAE steel numbering system be obtained?

company in which he is employed, before indiscriminately calling for type D or DD rivets. A comparison of strengths of these three alloys for various diameters of rivets is on page 5-20 of ANC-5. It is interesting to note that the type of head does not affect the allowable strength of the rivet.

There are other practical considerations to bear in mind when selecting rivet sizes. Rivets of 1/16-in. diameter are extremely difficult to handle and are used very seldom. Rivets of 3/32-in. diameter, although not so difficult to handle as the 1/16-in. diameter, are considered by some companies to be too small to be practical and their use is often discouraged. The sizes most commonly used are the 1/8-, 5/32-, and 3/16-in. diameters. Up in the larger sizes, say 1/4-in. diameter and greater, the designer should remember that they are difficult to drive. Some companies believe that 1/4-in.-diameter rivets are as large as can be conveniently driven with present equipment; hence the layout man should consult his supervisor before specifying 5/16- or 3/8-in.-diameter rivets on his layout.

No attempt should be made to drive 1/4-in. rivets or larger through relatively thin sheet, since the force exerted by the rivet in the driving operation stretches the sheet. After the sheet has been stretched out of shape by this riveting operation, it tends to form what is known as a "can," since it behaves like the bottom of an oilcan when pressure is applied—snapping back and forth. No definite information can be given as to exactly what size of rivets should be used with any given gage of sheet; therefore, if in doubt, the layout man would do well to consult his supervisor or a member of the production design group.

Aluminum sheet should not be riveted with aluminum alloy rivets, since they crush the sheet during the riveting operation. Aluminum rivets, referred to as "type A" in the AN book, should be used with aluminum sheet.

Since the strength requirements for riveted connections is discussed in Chap. 8, that important factor will be omitted at this point.

5:3. Bolts and Nuts.[1] In aircraft, no commercial steel bolts or nuts are used, since it is necessary to use special high-strength steel for them. The Army and Navy have standardized on a series of special bolts, AN3 to AN16, which are heat-treated to

[1] See C.A.B. 27, p. 55, for a discussion of bolts and nuts.

a minimum of 125,000 pounds per square inch ultimate tensile strength (usually abbreviated 125,000 psi), and which are made to closer tolerances than the average commercial bolt. It is this AN series of bolts that are standard for the aircraft industry. Whenever bolts are mentioned in this text, reference is made to AN bolts unless otherwise stated. The nuts are so designed that the bolt will break before the threads strip.

No steel bolts smaller than $\frac{3}{16}$ and no aluminum alloy bolts smaller than $\frac{1}{4}$-in. diameter should be used in the primary structure. Aluminum alloy bolts less than $\frac{1}{4}$-in. diameter may be used in locations not classed as primary structure. Many $\frac{3}{16}$-in.-diameter (10-32) bolts are broken when an inexperienced or careless workman tightens the nuts. For this reason, many shop supervisors advocate using nothing smaller than $\frac{1}{4}$ in.; however there are times when the loads are such that $\frac{3}{16}$-in. bolts are sufficiently strong, and the substitution of a larger bolt would be impossible owing to interferences. The allowable strengths of the AN bolts may be obtained from the AN book.

In no case should aluminum alloy nuts be used for tension bolt applications. Aluminum alloy nuts may be used on steel bolts for landplanes, provided the bolts are cadmium plated.[1] Aluminum alloy nuts are not permitted under any condition on seaplanes, because of possible corrosion resulting from salt spray. Aluminum alloy bolts or nuts are not permitted in places where they will be repeatedly removed for purposes of maintenance or routine operation of the airplane (see Art. 4:10).

On primary structure, the threaded portion of a bolt should not be used to take shear loads, and threads should not be used in bearing. The length of the shank (the unthreaded portion of the bolt) should be such that not more than one thread is below the surface, that is, not more than one thread may be in bearing inside a fitting.

When a draftsman is selecting the length of bolt to use, he frequently finds that a certain size will have more than one thread in bearing inside a fitting. For example, suppose he is using a $\frac{3}{8}$-in.-diameter bolt and finds an AN6-14 bolt will have more than one thread inside the fitting, yet an AN6-15 bolt will have a portion of the shank outside the fitting. In this case, he should

[1] See TM 1-435, pp. 135-143, for a discussion of the plating process, equipment used, and solutions for testing.

call for the AN6-15 bolt and standard spacer washers (AN960-616) to fill up the gap between the nut and fitting. He should never specify cutting additional threads, in cases such as this, for the bolt would have to be cadmium plated again, and would become a special bolt, which is undesirable. Always use standard AN bolts wherever possible.

Bolts connecting members having relative motion should not be drilled for lubrication. Provisions for lubricating should be incorporated in the surrounding parts, such as a lubricator on the fitting itself.

Bolts connecting parts having relative motion on stress reversals should have close tolerances with no preceptible shake. Each aircraft company has its standard limits covering this case, and the designer should follow the practice of the company in which he is employed.

In landing gear and other major fittings, no principal bolts smaller than 3/8-in. diameter are recommended.

On layouts and shop drawings, all bolts should be shown with their heads uppermost, as that is the way they will be installed on the airplane. Wherever possible, the inspectors require bolts to be so placed that in the event a nut works loose and falls off, gravity will hold the bolt in place. Although this possibility is remote, the alert designer will show bolts on his layout as they will be installed on the airplane.

All nuts that require removal should be castellated (AN310) and locked with cotter pins (AN380), or else be self-locking of an approved type, such as AC365. Since the most commonly used self-locking nuts have fiber inserts, their limitations should be mentioned:

a. They should not be used at joints that subject the bolt or nut to rotation.

b. They should not be used where the temperature exceeds 250°F.

c. They should not be used on bolts that have been drilled for cotter pins.

Clevis bolts (AN23 to AN36) and shear nuts (AN320) are used only where there is no tension load, such as in the male and female fitting shown in Fig. 4:13.

Internal wrenching bolts are special bolts made from high-tensile strength steel, and are so called because of the head,

which has an opening similar to a wrench socket. The nuts are shaped similarly. To tighten these internal wrenching bolts, special wrenches are inserted in the heads. These special bolts have two advantages:

(1) Since they are heat-treated to 180,000 psi minimum, they may be used where standard AN bolts are not strong enough for the tension loads.

(2) Owing to the internal wrenching feature, they may be used where it would be impossible to get wrench clearance around a standard AN bolthead.

The disadvantages to these bolts are their weight and their cost. In all cases, internal wrenching nuts should be used with internal wrenching bolts.

In general, bolts should not be used in cantilever bending; an example of this type of loading is shown in Fig. 5:1.

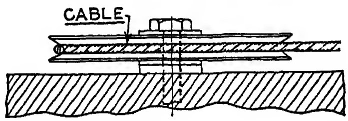


Fig. 5:1.

5:4. Pins.[1] Clevis pins (AN392 to AN406) may be used in tie rods only; their heads must be up and they should be safetied with standard cotter pins (AN380).

Taper pins (AC386) may be used in all permanent connections where absence of play is essential; however, it is considered preferable by many designers to use either drive pins or rivets. When taper pins are used, they should project into spacer washers, (AC975) to permit the nuts to be drawn up tight. Either castellated (AN320) or self-locking (AC364) nuts may be used.

Drive pins are actually oversize bolts ground to very close tolerances, and are designed to be driven in standard reamed holes and to be used only where it is necessary to fill the hole completely. Drive pins should not be used in tension without first consulting the stress department. They are designed to be used with AC365 self-locking nuts.

5:5. Bearings and Bushings. In fittings where there is a single bolt in double shear for attachment to another fitting, the hole frequently is bushed. If there are a number of bolts or rivets attaching the fitting to the structure, it is unnecessary to bush these holes. Figure 4:5 shows a fitting attaching the wing to the fuselage: the type of fitting that has a cluster of

[1] See C.A.B. 27, p. 56, for a discussion of clevis and taper pins.

bolts or rivets through the wing spar, but only one large bolt in double shear into the mating fuselage fitting. The holes for the bolts and rivets into the spar should not be bushed; however, the hole for the large single bolt in most cases should be bushed. If the fitting is subjected to sudden or shock loads, as in landing gears, or if it is subjected to rotation, it definitely should be bushed. If it is desirable to increase the bearing area of the bolt on a dural fitting, it is customary to provide a steel bushing (see Art. 7:4).

It is difficult to set up rules governing all cases and to say definitely when a bushing should or should not be used; when there is doubt, the supervisor should be consulted. If no bushings are called for, the designer should provide sufficient metal around the hole to permit boring out for bushings, or for the installations of the next larger size bolt.

If rotation occurs, either a patented oil-impregnated bushing or a plain bushing and lubricator should be provided. The lubricator should be incorporated in the fitting itself; never drill a bolt for lubrication (see Art. 5:4).

Since bushings may be purchased in a wide variety of materials and sizes, always attempt to use standard purchased bushings whenever possible without reworking. If it is impossible to use a standard purchased bushing in the design, use some bushing from the standards book.

Bearings and bushings should not be spun or staked in magnesium alloy castings because of the tendency of this material to work-harden. Frequently this spun-over lip will break off, permitting the bearing or bushing to slide out (see Art. 6:2).

Antifriction bearings are used on all movable surfaces and their controls, as well as on engine controls. The outer race should be secured in a tight-fitting housing by staking or some other safe manner. The inner race need not fit tightly the bolt or shaft which passes through it, but should be clamped securely endwise to eliminate play between the bolt or shaft and the inner race.

Piano hinges are to be avoided in places that are subjected to continuous wear and high loading.

Problems

5:1. *a.* What types of rivet heads are used on exterior surfaces?

b. Which of these types are preferred? Why?

5:2. *a*. What types of rivet heads are used in interior structure?

b. Give advantages of each type.

5:3. *a*. What is the difference between types AD, D, and DD rivets?

b. Which of the above three types is the Army standard rivet? Why?

5:4. *a*. Which of the types of rivets mentioned in Prob. 5:3*a* require special handling? Explain the reason for this special handling and what it consists of.

b. Where may strengths of rivets be obtained?

5:5. Discuss the practical considerations in the selection of sizes of rivets as far as ease of handling and driving are concerned.

5:6. What are type A rivets? When is their use recommended? Why?

5:7. *a*. What are the smallest steel bolts to be used in primary structure?

b. Aluminum alloy bolts?

5:8. Are the following statements true or false?

a. Aluminum alloy nuts may be used on bolts in tension.

b. Aluminum alloy nuts may be used on landplanes.

c. Aluminum alloy nuts may be used on seaplanes.

d. Aluminum alloy nuts and bolts should not be used where they will be repeatedly removed.

e. Commercial machine screws and bolts should not be substituted for the AN standard screws and bolts.

f. In primary structure, not more than two threads may be below the surface.

g. Never drill a bolt for lubrication.

h. Bolts should be shown on layouts with the heads down.

i. Shear nuts and clevis bolts should never be used when the bolt is subjected to tension loads.

j. Washers may be used under nuts where necessary.

5:9. What are the restrictions on self-locking nuts with fiber inserts?

5:10. *a*. When is it considered good design to use the high-strength internal wrenching bolts?

b. What is the minimum recommended size bolt for landing gear and other principal bolts?

5:11. *a.* What are drive pins and when should they be used?
b. When may clevis pins be used?
c. If taper pins are used, how should they be secured?

5:12. *a.* When in doubt as to whether or not a hole should be bushed, what should be done?
b. When should holes definitely be bushed?

5:13. *a.* If no bushing is used, what provision should be made in the design?
b. When rotation occurs, what should be provided for?

5:14. Where should antifriction bearings always be used?

5:15. *a.* How are bearings secured in fittings?
b. Where would the use of a piano hinge be recommended?

CHAPTER 6

FABRICATION METHODS

6:1. Forgings. Forgings, often referred to as "drop forgings," are parts made by placing hot metal in a plastic state between reciprocating dies, and working this metal until the cavities in the dies are completely filled. See Fig. 6:1 for a photograph of a forge hammer in operation. The art of cutting the cavities in the special steel die blocks is known as "diesinking." The excess material squeezed out between the die blocks during the forging operation is known as "flash," and is subsequently trimmed off normally leaving a flat from $\frac{1}{32}$ to $\frac{1}{4}$ in. in width, depending on the size of the piece. The two die blocks must necessarily fit tightly together, and the contacting surface between them is known as the "parting plane."

In order to remove the forged part from the die, it is necessary to have the width of the cavity smaller at the bottom of the die block than at the surface. The angle formed by the sloping sides is the draft angle, commonly referred to as "draft."

The forged part as delivered by the manufacturer to the customer is known as the "forging blank." The customer must then bore holes, install bushings or bearings, and machine the various faces before it is ready for installation in the airplane.

In production airplanes, the use of aluminum alloy or steel forgings is recommended wherever possible as they lend themselves readily to high production. The information contained in this text applies to either material.

One advantage of the forging is that the "flow lines" of the metal follow the contours of the part. See Fig 6:2, which is an unretouched photograph of a forged socket head capscrew showing these flow lines. If the forging blank is machined extensively, these flow lines no longer parallel the contour but may terminate at critical sections. For highly stressed parts, the forging should be used with a minimum amount of machining.

If a forging is designed properly, it has a high "strength-weight" ratio, and there is a uniformity of parts that is desirable from an assembly standpoint.

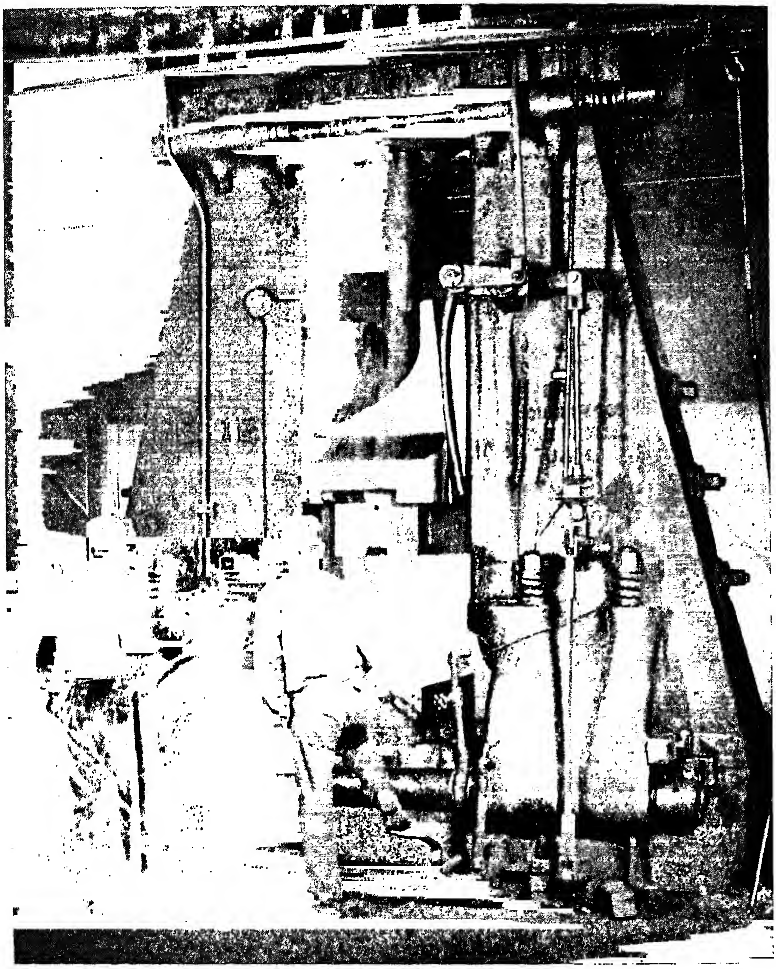


FIG. 6:1.—Forge hammer in operation. *(General Metals Corporation, Los Angeles, California.)*

Since there are certain disadvantages to the use of forgings, the layout man should never decide to make any part a forging without first consulting his supervisor. Unfortunately, forgings require expensive dies and tools, often costing thousands of

dollars. Also, there is necessarily a delay of many weeks after releasing the drawings, in which time bids are invited, contracts for the dies are let, the dies are made, and lead casts from these dies are returned to the customer and checked for accuracy.

The layout man should remember that it is often impossible for the forge shop to hold the tolerances as specified in the title block of the drawing. For forging tolerances, use those adopted

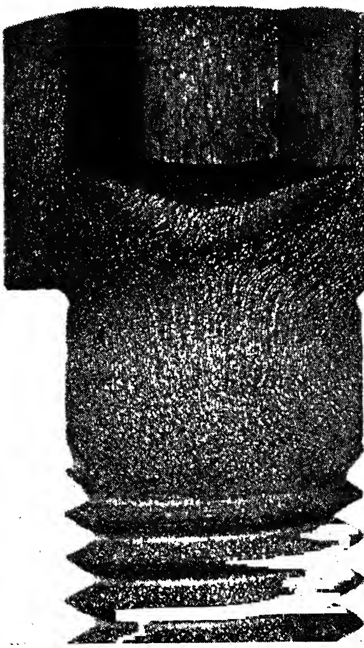


FIG. 6:2.—Unretouched photograph of etched cutaway forged socket-head screw. (*The Holo-Krome Screw Corporation, Hartford, Connecticut.*)

by the Drop Forging Association, Feb. 11, 1937, which are included in many drafting-room manuals.

The following design recommendations are made with the purpose of enabling the layout man to create a design that can be made in the shop with a minimum amount of expense and time, and are to be followed in the absence of more specific recommendations.

The parting plane must be kept as straight as possible. Often it must change from one level to another, which is permissible

providing the dies do not interlock with the part in place. It is imperative that nothing prevent the separation of the die blocks, or the lifting of the forged part from either block. That is why it is not practicable to have intricate cores or undercuts in forgings as are permitted in castings.

Usually the draft is 7 deg. and starts at the parting line and extends to both sides (see Fig. 6:4). Avoid rapid change in section by using large fillets.

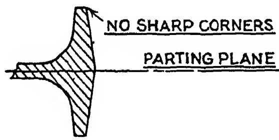


Fig. 6:3.

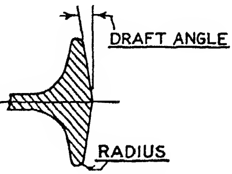


Fig. 6:4.

Sharp corners, as shown in Fig. 6:3, should always be avoided; in. should be considered a minimum radius for corners. Investigate the possibility of combining the left-hand and right-hand parts in one forging blank.

Parts of the forgings to be subsequently machined off must have at least $\frac{1}{16}$ in. allowed for finishing; on very large forgings $\frac{1}{8}$ in. or more should be allowed for this operation.

Narrow and deep depressions in the forging should be avoided, since in the die these depressions are weak male protrusions which do not stand up under the heat and pressure of the hot metal while forging. If depressions are necessary, they must have 10- to 15-deg. draft angle and generous fillets. The depth of a depression or forged hole should not exceed two-thirds of the least width of the depression (see Fig. 6:5).

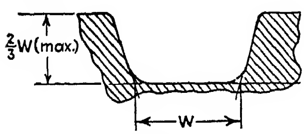


Fig. 6:5.

The minimum thickness of webs should be $\frac{3}{32}$ in. for small forgings; for those weighing several pounds, the minimum web should be not less than $\frac{1}{8}$ in. The hot metal chills and does not work properly when the sections are too thin.

When the material cools to the point where it is no longer in a plastic state and the forging operation is continued, cracks often form along the flanges, these cracks being termed "cold shuts." Cold shuts may be caused by not having the material hot enough; more frequently, they result from faulty design in

which the depth of the flange was too great for its width and caused the material to cool before the cavity was filled.

The minimum radii for fillets depend on the material to be forged and on the forging itself. In general the minimum radius of the fillet should not be less than one-third the height of the flange, as shown in Fig. 6:6; this large radius assists the flow of plastic metal into the die cavity.

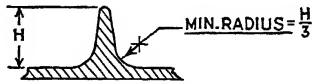


FIG. 6:6.

Lugs should be designed more generously than would be required by theoretical considerations. In Fig. 6:7, *R* should be $\frac{1}{16}$ in. larger than normally required for strength, so that die shift, warpage, or misalignment of the forging will not throw the hole too near the edge of the lug.

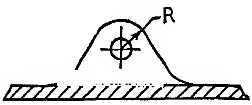


FIG. 6:7.

No special edge distance is required for rivets as is true for nonferrous castings (see Art. 6:2). Standard edge distances for rivets and bolts may be applied to forgings.

The forged surface (without draft) may be used against bolt-heads or nuts, without spot facing for unimportant fitting attachments.

References

Drop Forgings—Why They Should Be Streamlined, *Product Engineering*, Vol. 9, 1938, pp. 16–18.

Drop Forgings—Design Fundamentals and Factors Affecting Casts, *Product Engineering*, Vol. 9, 1938, pp. 110–112.

"Forging Ahead with Forgings," pamphlet by The Steel Improvement and Forge Co., Cleveland, Ohio, 1938.

Problems

6:1:1. *a.* What is a forging?
b. What is diesinking?

6:1:2. *a.* Describe "flash" and explain what becomes of it.
b. What is the parting plane and why is it important to keep it as straight as possible?

6:1:3. *a.* What is draft and why is it used?
b. How much draft is recommended for average forgings, and where does it start?

6:1:4. *a.* What is the forging blank?
b. How does the customer use this blank?

6:1:5. *a.* What materials are recommended for aircraft forgings?

b. What tolerances are recommended for forgings?

6:1:6. *a.* Name the most important advantages in the use of forgings.

b. What are the most important disadvantages in their use?

6:1:7. *a.* What is considered the minimum radius for outside corners?

b. What is the minimum recommended radius for inside corners or fillets?

6:1:8. *a.* What provision should be made if the forging is to be machined?

b. What is considered a minimum thickness of web?

6:1:9. Discuss depressions in a forging, their limitations, etc., illustrated by freehand sketches.

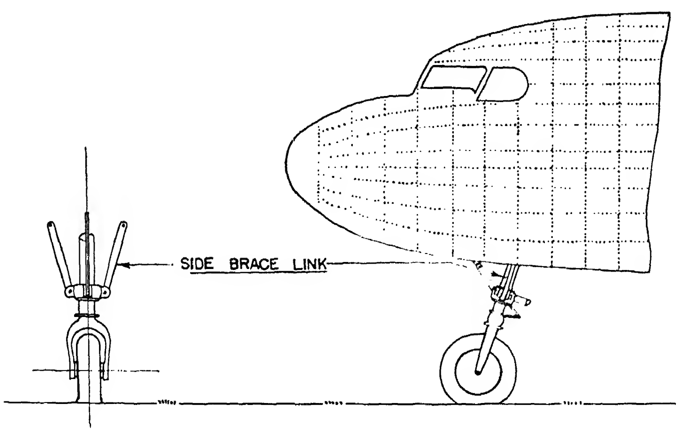
6:1:10. *a.* What is a cold shut and how is it caused?

b. What provision should be made in the design of lugs?

6:1:11. *a.* Is special edge distance necessary for rivets or bolts?

b. Should forged surfaces always be spot-faced for boltheads or nuts?

6:1:12. The accompanying diagram is the front end of a transport showing the retractable nose wheel. Design a forged



PROB. FIG. 6:1:12.

aluminum alloy side brace link to suit the following conditions:

27.370 in. center to center of bolts
AN12 bolts, used both ends
$\begin{matrix}1.248 \text{ in.}\\ 1.243 \text{ in.}\end{matrix}$ thickness of link at bolt
1.72 sq. in. area required at center of link

6:2. Sand and Permanent Mold Castings. These castings are parts produced by pouring molten metal into a previously prepared mold, allowing the metal to solidify or "freeze," then removing the part. If the mold is made of sand, the part is a sand casting; if it is a metallic mold (usually cast iron), the part is a permanent mold casting. It should be remembered that sand and permanent mold castings are produced by *pouring* liquid metal into the mold; that is, the metal flows under the force of gravity alone. The "gate" is where the liquid metal enters the mold; the "risers" are chimney-like vents permitting easy escape of the gas and providing convenient reservoirs for the collection of slag and other impurities. They also supply liquid metal to the casting as it shrinks on setting.

The mold is usually made in two parts, the contacting surface between the two halves being known as the "parting plane." The patterns for sand castings are usually made of wood and must have suitable draft to facilitate removal from the sand. The mold for a permanent mold casting is usually machined in the metallic block, approximately the same amount of draft being provided for both sand and permanent mold castings. However, the amount of draft is small and is neglected when the casting is designed; that is, the layout man does not show draft on a casting as he must do with a forging.

Cores may be set in the mold to provide holes in the casting. If the casting is such that complicated coring is necessary, it will probably be a sand casting since complicated coring is not practical in a permanent mold casting. In general, cores for sand castings are of sand; for permanent mold castings they are of metal. Sand cores are removed after the part has cooled, by breaking up the sand and removing it from the inside of the casting, which is obviously impossible when a metallic core is used.

The permanent mold process is a fairly recent development of the sand-casting process. Permanent mold castings are gated

and poured similarly, the major difference being in the material from which the molds are made.

The advantage of this process is that there is less porosity than in sand castings. The sand and binder (a molasses-like substance that is mixed with the sand to make it stick together) give off a certain amount of gas in a sand casting which tends to make for porosity. With the elimination of the sand and binder in a permanent mold casting, much of this porosity is avoided; thus the strength is increased. Permanent mold castings are approximately one and a half times as strong as sand castings. Permanent mold castings have a very smooth finish which may eliminate some machining, and closer tolerances may be held than with sand castings.

There are certain disadvantages to a permanent mold casting. The dies are quite expensive and are used only where production warrants such an expense. There is also the time element to be considered, since it takes many weeks to have the dies made and checked. Complicated coring is impractical in permanent mold castings. At times, sand cores are set in the mold; however, this is not very desirable. In general, permanent mold castings are not used where complicated coring is required.

The same fundamentals of good design apply equally well to sand and permanent mold castings. The minimum webs, fillets, etc. apply to both types. The layout man need not attempt to decide whether the part should be a sand or permanent mold casting, as he may design it the same for either process.

Castings should not be used in the primary structure as lugs for tension members or for long struts which are likely to vibrate. Castings are usually confined to secondary structure; however, they are frequently used for flight control pulley brackets. These brackets are primary structure, yet castings are often specified, owing to their rigidity.

Castings may be welded, except those used in any structure or in control systems. An example of a nonstructural casting that may be welded would be a cast sump for a fuel tank. These sumps are frequently contoured to fit the bottom of the tank, and are welded to the tank to prevent any possibility of leaks.

A layout man should never design a part as a casting without first consulting the stress department, to be sure that a cast-

ing would be acceptable and to determine the latest strength requirements.

The following are general rules that should be adhered to by the layout man in order to produce a casting of good design. They apply to aluminum alloy or magnesium alloy castings. Although steel castings are permitted under certain conditions, comparatively few are used in aircraft; hence they are not discussed in this article. No attempt has been made to explain foundry practice as many textbooks have been written on that subject alone. It is recommended that the student secure from a library the references listed at the end of this article.

Avoid abrupt changes in cross sections. Figure 6:8 shows a heavy flange joining a thin web. Owing to uneven cooling and shrinkage, cracks and porosity often occur at the point of abrupt change. Figure 6:9 shows the part as it should be designed.

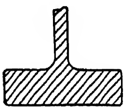


Fig. 6:8.

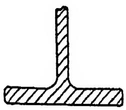


Fig. 6:9.

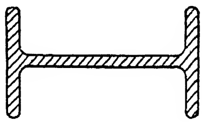


Fig. 6:10.

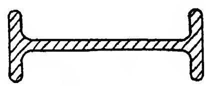


Fig. 6:11.

The pattern should not extend too deeply in the sand because of the difficulty in removing it. Figure 6:10 shows a part with deep flanges that may cause trouble when the pattern is removed. Figure 6:11 shows the part with smaller flanges that would be considered better design. It is impossible to give definite figures on the permissible width of a flange; however, it is recommended that the layout man consult his supervisor, or a member of the production design group, on this important question.

Generous fillets should be provided for all inside corners; the minimum radius for aluminum and magnesium alloy castings is $\frac{3}{16}$ in. The same radius should be used for all fillets in the same casting, since it facilitates the work of the pattern shop. To obtain fillets on a pattern, it is common practice to use preformed leather strips, as shown in Fig. 6:12. As these strips are obtainable for various radii, the task of the patternmaker is simplified when the same radius is used throughout a pattern.

The minimum practical web thickness is considered to be $\frac{5}{32}$ in. Thinner webs may be cast; however they are not recommended, since the metal often chills before completely filling the

cavity and leaves imperfect castings. For the same reason, large thin areas should be avoided.

Avoid a design that requires machining unsupported projecting portions of the casting. Figure 6:13 shows such a design where the upright flanges would have to be machined to provide a smooth surface for the bearing shown in phantom. If no braces were provided to support these flanges during the machining operation, the flanges would deflect to one side, owing to the

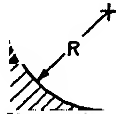


Fig. 6:12.

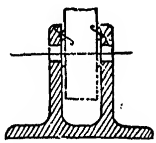


Fig. 6:13.

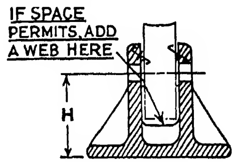


Fig. 6:14.

pressure of the cutter, and might even break off. Supports should be provided similar to those shown in Fig. 6:14 to prevent this deflection during the machining operation. It is considered good design to keep the dimension H in Fig. 6:14 to a minimum; hence there is seldom room to add a web between the lugs as shown.

If a bearing is to be staked or spun into a casting, the thickness of the casting should be the same as the width of the bearing, as shown in Fig. 6:15. Staking is the operation of using a center punch or similar tool, and indenting the casting adjacent to the hole for the bearing, usually in four places equally spaced around the hole. This indenting causes the casting to spread at these points, reduces the size of the opening, and provides a satisfactory method of holding the bearing in place. Spinning is a method by which a rotating tool with a small wheel, similar to the wheel of a glass cutter, is forced into the casting adjacent to the hole. As this tool rotates, forcing the wheel into the casting, the material between the groove and the bearing is forced over, reducing the diameter of the hole, thus locking the bearing in place. As pointed out in Art. 5:5, bearings should not be spun or staked in magnesium alloy castings, but should be secured in some other safe manner.

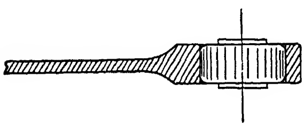


Fig. 6:15.

Lugs should be designed more generously than would be required by theoretical considerations. *R* should be $\frac{1}{16}$ in. larger than normally required for strength, so that shrinkage and warpage of the casting will not throw the drilled hole too near the edge of the lug. Figure 6:16 shows a conventional cast lug.

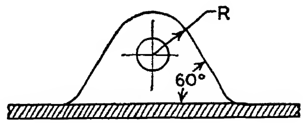


FIG. 6:16.

Avoid a design that requires machining a large flat area as shown in Fig. 6:17 where the entire base must be machined. It is considered better design to cast bosses on the base and machine only these bosses as shown in Fig. 6:18. Although the weight

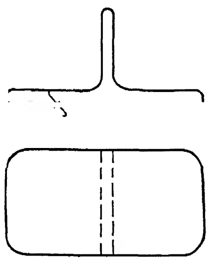


FIG. 6:17.

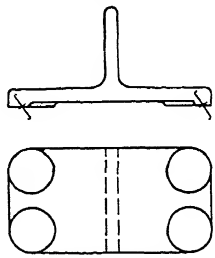


FIG. 6:18.

is increased slightly, the reduction in machining time is considered more important than this slight increase in weight.

It was mentioned earlier in this article that the number of cores should be kept to a minimum. Figure 6:19 shows a bracket where cores may have to be set to form the bosses on the outside

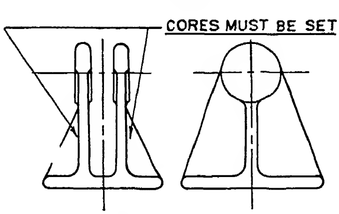


FIG. 6:19.

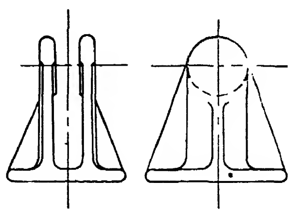


FIG. 6:20.

faces of the bracket. By extending these bosses to the base as shown in Fig. 6:20, some cores may be eliminated. This type of boss adds a small amount of weight but, if cores can be eliminated by this means, it is considered good practice to do so. It is usually necessary to have the type of boss shown on the inside face, to

provide clearance for the part supported by the bracket. Note the method of bracing the web, and how clearance is provided for the cutters as the faces are milled.

The minimum edge distance for rivets should be three times the diameter of the rivet. When a rivet is driven, it swells and tends to break out the casting unless there is more than the usual edge distance. Edge distance is measured from the center of the rivet to the edge of the casting. No special edge distance is needed for bolts, as there is no swelling action when they are tightened.

References

"The Technology of Aluminum and Its Light Alloys," by A. von Zeerleder, pp. 132–138, Nordemann Publishing Co., Amsterdam, 1936.

Castings in Design, *Product Engineering*, Vol. 9, 1938, pp. 19–21.

Questions

6:2:1. *a.* What is a sand or permanent mold casting?
b. Describe the difference between the sand ana permanent mold processes.

6:2:2. *a.* What are gates and risers?
b. Explain their uses.

6:2:3. *a.* Why is draft necessary for the pattern?
b. What allowance for draft is made by the layout man on his drawings?

6:2:4. *a.* What is the parting plane?
b. How is the die depression obtained for permanent mold castings?

6:2:5. *a.* Assuming that a casting is complicated and will require extensive coring, should it be a sand or permanent mold casting?
b. What material is generally used for cores in sand castings? In permanent mold castings?

6:2:6. *a.* Name the more important advantages of permanent mold castings over sand castings.
b. What are their disadvantages?

6:2:7. *a.* If a part has been designed to be a sand casting, and then it is decided to make it into a permanent mold casting, what alterations will be necessary in the design of the part?
b. Whom should the layout man consult before designing a part as a casting?

6:2:8. *a.* Are castings recommended as lugs for tension members? For flight control pulley brackets?

b. The use of castings is generally confined to what structure?

6:2:9. *a.* Can castings be welded? Discuss fully, giving an illustration of a casting that can be welded.

b. What material is used most frequently for aircraft castings?

6:2:10. Discuss abrupt changes of section in castings, making any necessary freehand illustrations.

6:2:11. *a.* Why is it objectionable to have the pattern extend too deeply into the sand?

b. Why is it desirable to have the same radius on all cast fillets?

6:2:12. *a.* What is considered the minimum web thickness?

b. What is the minimum recommended fillet radius?

6:2:13. Discuss fully the machining of unsupported projecting portions of the casting, making any necessary freehand sketches.

6:2:14. *a.* What is staking?

b. What is spinning?

c. Why are these operations used?

6:2:15. What provisions should be made in the design of cast lugs? Discuss fully.

6:2:16. *a.* Why is it desirable to avoid machining large flat areas?

b. Discuss one method of overcoming this problem, drawing any freehand sketches that are necessary.

6:2:17. Discuss the elimination of cores and show one way they may be eliminated.

6:2:18. *a.* What is the minimum edge distance for nonferrous castings? Why?

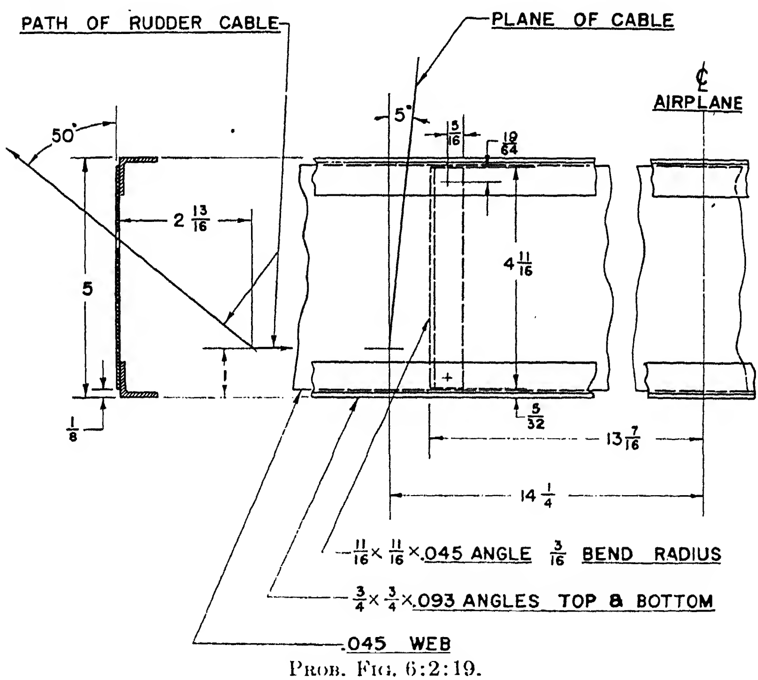
b. What special edge distance is necessary for bolts?

6:2:19. In the accompanying diagram is shown a beam designed to support a number of empennage cable pulley brackets. The path of the cable and the surrounding structure are shown for the rudder cable.

Lay out a sand casting to mount on this beam that will guide the cable along the path shown. Design for the following conditions:

AN210-6A pulley
$3/16$-in.-diameter cable guard pins
AN3 attaching bolts
Design to minimum sections
Use $3/16$-in.-diameter cable

It is recommended that the student study Art. 6:8 before attempting to solve this problem.



PROB. FIG. 6:2:19.

6:3. Die Castings.[1] In aircraft, die castings are usually aluminum alloy or magnesium alloy, the choice of material being largely a matter of personal opinion. If weight is of primary importance, probably magnesium alloy should be used, since it is slightly lighter than aluminum alloy. On the other hand, since aluminum alloy is a little stronger than most magnesium

[1] The line drawings and much of the text material in this article are used by special permission of the copyright owner, New Jersey Zinc Co., New York.

alloys, it may be advisable at times to specify aluminum alloy. The layout man is not expected to decide what material to use; he should consult his supervisor. The information contained in this article applies only to these light metals, and does not apply to zinc die castings which have been so successfully used in many industries. Zinc alloy die castings are seldom used in aircraft because of their weight, zinc weighing approximately 2.3 times more than aluminum alloy.

A die casting is a part that has been produced by forcing molten metal under pressure into a metallic die and allowing it to solidify, after which the die is opened and the part removed. The basic difference between permanent mold castings and die castings is that in the permanent mold process, the metal flows into the metallic die under gravity only; in the die-casting process, the metal is forced in under very high pressure. It was pointed out in Art. 6:2 that the elimination of sand in the permanent mold process lessened the tendency towards porosity. There is a certain amount of gas trapped in the liquid metal, and this small amount of gas can ascend the risers and gates of a permanent mold casting; in the die-casting process, there is no place for this trapped gas to go and an undesirable porosity in many die castings is created.

It should be noted that in the first sentence of the last paragraph, it was stated that the molten metal is allowed to solidify in the die. The casting is not allowed to cool in the die; just as soon as the liquid metal "freezes," that is, reverts to the solid state, the die is opened and the part is removed and allowed to cool outside. In this way, a high production rate is obtainable with the die-casting process.

According to the Air Corps, die castings are class III castings, where stresses should not exceed 7½ per cent of the ultimate stress, and are not to be used in primary structure. Owing to the severe limitations placed on them, they are recommended for use only in secondary structure or for nonstructural parts such as interior trim, etc. Die castings should not be used where large concentrated alternating loads are present, such as wire pull lugs. The layout man should never design a part as a die casting, without first consulting his supervisor.

There are certain advantages of die castings over sand castings. They may be summarized as follows:

1. High uniformity of parts facilitates interchangeability.
2. Very smooth surface is produced except at parting lines and at ejector pins.
3. Die castings can be made with a minimum wall thickness of 0.062. Thinner webs have been cast but are not recommended for general practice.
4. There is less warpage in die castings.
5. Close tolerances may be obtained.

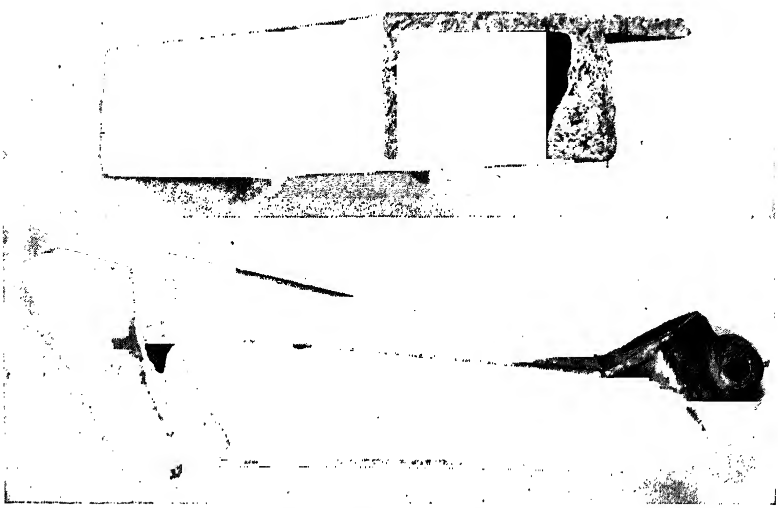


FIG. 6:21.—(*Douglas Aircraft, Inc.*)

There are also certain disadvantages of die castings as compared to sand castings:

1. Machining should be avoided since it removes the outside hard shell which furnished the strength. One of the peculiarities of die castings is that the metal may be quite porous except for the outside shell of approximately $\frac{1}{32}$-in. thickness.
2. Die castings have poor elongations, that is, they are very brittle. Lugs and flanges break easily under impact loads.
3. The porosity is higher in a die casting than in a sand casting for the reason given in the second paragraph of this article.
4. A change in section is worse in a die casting than in a sand casting, owing to this tendency toward porosity. Usually the porosity is intensified at the point of change in section.

A very good example of concentration of porosity is shown in Fig. 6:21, which is a photograph of a die-cast aluminum alloy cowl support. This support had caused trouble in service, so several tests were made. The part shown in the figure failed under a vibration test after 3½ hr. The interesting thing to note is that it failed through the fillet joining a thin web with the heavy base, because of the concentration of porosity at this point. As a result of the tests it was apparent that this part should have been designed as a forging.

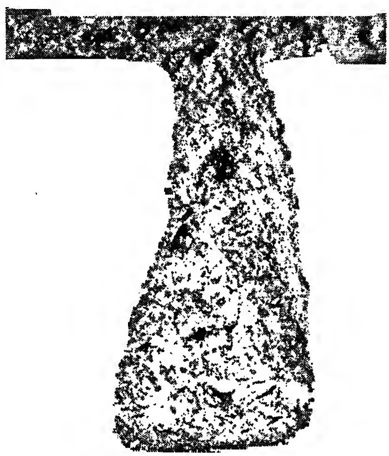


FIG. 6:22.—(*Douglas Aircraft, Inc.*)

Figure 6:22 is a greatly enlarged photograph of this fracture through the fillet. Notice that at the edges of the fracture the grain is much finer than at the center. The black spots in the photograph are cavities resulting from the trapped gas bubbles. Though the picture does not show it clearly, the entire cross section, with the exception of the outer edges, was extremely porous; however it should be pointed out that this example is considered an exceptional case, not typical of all die castings.

There are certain fundamental good design principles that should be adhered to in all die castings. By following them, a high production rate may be maintained and the cost per part kept to a minimum. In any die casting, whether aluminum

or magnesium alloy, the following design recommendations should be considered:

If the part is to be used where appearance is important and where blemishes in the surface would be objectionable, such as for interior trim, the designer should observe the following rules:

1. Plain flat areas of any considerable size which must be cast with a perfectly smooth surface, often referred to as a "hard-

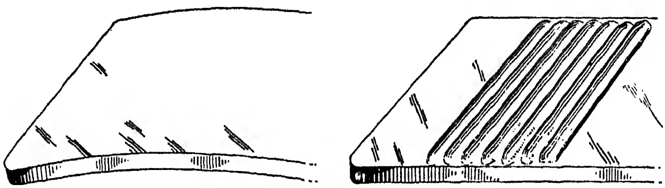


FIG. 6:23.

ware finish," lead to high rejections and hence increased costs. To eliminate this problem, it is desirable that the flat area be modified by slightly curving the flat portions, or by breaking the surface with some simple design such as shown in Fig. 6:23.

2. "Shadow marks" are blemishes occurring on smooth surfaces of thin section where bosses, ribs, or studs are on the back. These blemishes show directly opposite the protrusions and are caused by the uneven cooling of the metal. They can be avoided

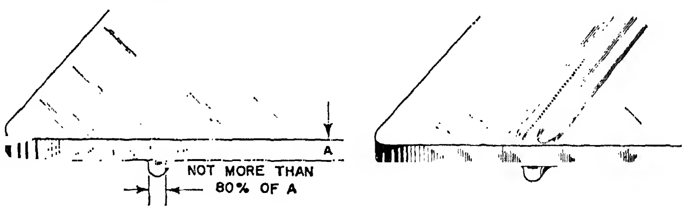


FIG. 6:24.

if the thickness of the section is kept above 0.10 in., or if the width of these ribs, bosses, etc., is kept to 80 per cent of the wall thickness. If it is impossible to design the part within these limitations, the defects may be covered by adding some simple design such as is shown in Fig. 6:24.

If studs are necessary in the design, incorporate an unthreaded stud cast integral with the balance of the casting. Separate studs cast in place are undesirable as the production rate of the casting is seriously impaired when such small parts must be fastened in hot dies prior to each cast. The stud should have at

least a $\frac{1}{8}$-in. radius fillet at the base, and the threaded portion should not extend into the fillet (see Fig. 6:25). If it is absolutely necessary to have a separate stud for some special purpose, such

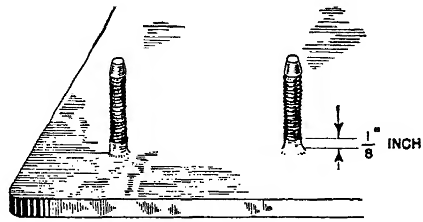


FIG. 6:25.

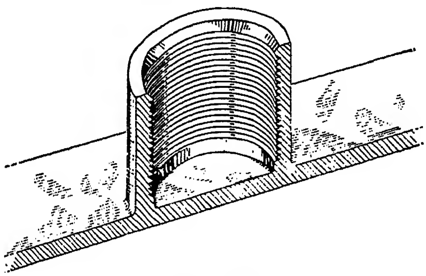


FIG. 6:26.

as for strength, the thread should not extend closer than $\frac{3}{32}$ in. to the casting.

Bosses are stronger than studs and should be used in their stead wherever possible. The design should be such that at least $\frac{1}{8}$ in. is left at the bottom of the tapped hole for chip clearance. The hole of proper size for tapping as well as the countersink may be incorporated in the casting (see Fig. 6:26).

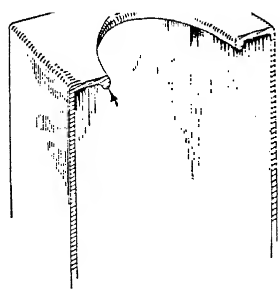


FIG. 6:27.

The possibility of adding ribs or beads to thin-walled castings for strength purposes should not be overlooked. These ribs or beads assist in keeping the casting from deforming when hot, and supply a reservoir of molten metal to allow for the contraction of the cooling metal without causing cracks. These beads also reduce trimming costs in many cases. The arrow in Fig. 6:27 points to such a bead.

Holes should be eliminated in a die casting unless necessary, as they require a trimming operation which it is desirable to avoid. To clarify this statement, imagine a part where the drawing calls for a hole in the die casting, yet this hole merely serves as a pilot hole for assembly purposes. That is, on assembly, a drill will be run through this cast hole for purposes of attachment. It would be better design to call for a depression to serve as a center-punch

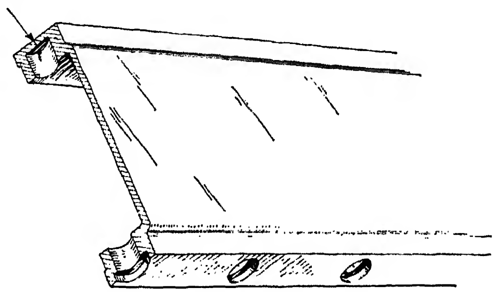


Fig. 6:28.

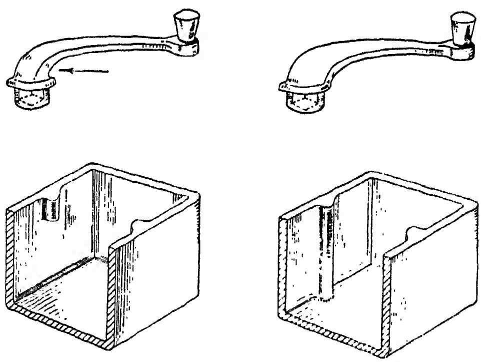


Fig. 6:29.

mark for the drill. Never have a hole all the way through the casting if one part of the way through will do, in order to save this trimming operation. The arrow in Fig. 6:28 points to a blind hole.

Undercuts complicate die construction and increase the cost of the parts. Undercuts and complicated coring should be avoided wherever possible. Figure 6:29 shows two examples of correct and incorrect design. Although a slight amount of weight is added by the elimination of undercuts, the saving in die cost will usually justify the additional weight.

Sharp inside corners should be avoided, as they set up shrinkage strains that tend to develop into cracks. An 0.015-in. radius should be the absolute minimum used on any die casting, with 0.062-in. radius preferable. A slight radius on outside corners reduces die costs, and so should be used.

Draft is necessary on die casting; however, less draft is required for die castings than for sand castings. On outside walls usually 0.010 per inch is sufficient. For internal cores, the following empirical formula may be used:

$$A = L\left(0.005 + \frac{2D + L}{1000}\right)$$

where A = total amount of draft on diameter of core

L = length (or depth) of core in inches

D = diameter of core in inches

For rectangular cores, a draft of 0.008 per inch on each side is suitable. Since in aircraft work most die castings are relatively small—parts weighing no more than a pound or two—the layout man need not concern himself with the draft required. Die castings are laid out in the same manner as sand and permanent mold castings: parallel lines are drawn representing the various surfaces and no mention of draft is necessary on the layout for most die castings. This is usually taken care of by the manufacturer.

For tolerances expected in the finished part, it would be logical to use the tolerances as given in the bids submitted by a prominent die casting manufacturer. These read as follows: "All drawing tolerances will be restricted by the following minimum tolerances of ± 0.002 per running inch; but not less than ± 0.004. Where dimensions are affected by a parting line $+0.010 - 0.000$ additional will be required. On drilled holes, the minimum tolerances will be $+0.003 - 0.001$ on the nominal diameter of the drill."

References

"Manual of Engineering Design for Die Castings," by Harvill Aircraft Die Casting Corporation, Los Angeles, Calif.

"Designing for Die Casting," by the New Jersey Zinc Co., New York.

"Die Castings," by Charles O. Herb, Industrial Press, New York, 1936.

Problems

6:3:1. *a.* What alloys are generally used for aircraft die castings?
b. What affects the choice of material?

6:3:2. *a.* Why are zinc alloy die castings seldom used in aircraft?
b. Should the designer make his own decision as to what material to use?

6:3:3. *a.* What is a die casting?
b. Describe the basic difference between die and permanent mold castings.

6:3:4. *a.* Are the castings allowed to cool in the dies? Explain your answer fully.
b. What limitations are placed on die castings by the Air Corps?

6:3:5. For what uses are die castings recommended? Explain fully.

6:3:6. What are the principal advantages of die castings over sand castings?

6:3:7. What are the chief disadvantages of die castings as compared to sand castings?

6:3:8. What provision should the designer make if a stud is necessary? Are separate studs recommended? Why?

6:3:9. Should threaded bosses be provided instead of studs where possible? Explain fully.

6:3:10. Fully discuss holes in die castings.

6:3:11. *a.* What are undercuts? Sketch a design having undercuts.
b. Is this condition desirable? Why?

6:3:12. *a.* What is absolutely the minimum radius for inside corners?
b. What is the preferred minimum inside radius?
c. Are sharp outside corners recommended?

6:3:13. *a.* Is draft necessary in die castings?
b. Does the layout man consider draft on his layout for the average die casting? Why?

6:3:14. What tolerances may be expected in die castings?

6:3:15. If a part is to be used where appearance is important, what rules should the designer observe? Explain fully, drawing freehand any illustrations necessary.

6:3:16. Why are beads desirable in thin-walled sections? Explain fully. Draw freehand a section where a bead would be recommended.

6:4. Extrusions. Practically all extrusions used in aircraft are made by a process whereby the material in a plastic state is forced through an orifice under high pressure, the shape of the orifice determining the shape of the extruded section. Since the vast majority of extrusions used at the present time are aluminum alloy, the following applies only to such extrusions, unless otherwise stated.

Before the layout man designs a new extrusion, he should investigate the existing sections; by a slight modification of his design, he may be able to use one of the available shapes. First, look in the standards book and, if nothing there is deemed suitable, check with the standards group, who have not only catalogues from the Aluminum Company of America but also standards books of other aircraft companies. Usually, from one of these sources the layout man can choose a section that may be worked into his design.

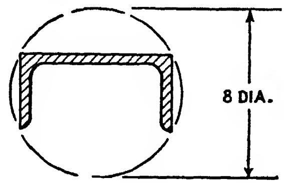


FIG. 6:30.

It is often possible to combine a number of shapes into one extrusion. For example, on a transport windshield, the original design called for seven different shapes. Through ingenious design these seven shapes were combined into one extrusion.

If it is impossible to use any available section and is considered necessary to design a new shape, the layout man should remember that the extrusion process has the following limitations:

The sizes of sections available commercially are those that can be enclosed in an 8-in. diameter circle as shown in Fig. 6:30.

As the thickness of the material decreases, the difficulty of extruding increases; thus a lower limit of thickness is reached beyond which it is impractical to extrude. This limit varies with the size and shape of the section and with the alloy. The limits shown in the following table cover the sizes that can be extruded commercially:

Diameter of Circumscribing Circle, In.	Minimum Thickness of Sections, In.
0 to 2	$\frac{1}{16}$
2 to 4	$\frac{3}{32}$
4 to 6	$\frac{1}{8}$
6 to 8	$\frac{5}{32}$

However, it is sometimes possible to exceed these minimum limits. Under no circumstances should the thickness of any member be less than 0.051 in. without first consulting a member of the production design group.

Shapes that require long and narrow die projections with little support are impractical to manufacture. The limits of the depth and thickness of the die projections depend upon the alloy and

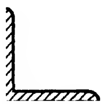


Fig. 6:31.

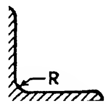


Fig. 6:32.

upon the thickness of section. Extruded shapes of this nature should be avoided where possible; if deemed necessary, they should be discussed with the manufacturer.

Figure 6:31 shows an extruded section with a sharp inside corner; as pointed out in Art. 4:18, this is not recommended because of stress concentrations at this point. The preferred design has a radius as shown in Fig. 6:32; the minimum radius being $\frac{1}{16}$ in. unless the section is large, then $\frac{1}{8}$-in. radius should be considered a minimum.

Reference

"The Technology of Aluminum and Its Light Alloys," by A. von Zeerleder, pp. 153–160, Nordemann Publishing Co., Amsterdam, 1936.

Problems

6:4:1. *a.* Describe the extrusion process.

b. What is the material most frequently used in aircraft for extrusions?

6:4:2. What should the layout man do before designing a new extrusion?

6:4:3. *a.* What are the maximum limiting conditions for an extrusion?

b. What is the recommended minimum thickness of webs based on the size of section?

6:4:4. *a.* What is the absolute minimum thickness of webs?
b. If it is deemed necessary to use thinner webs, what should be done?

6:4:5. Discuss shapes that require die projections.

6:4:6. *a.* Are sharp inside corners recommended? Why?
b. What is considered the minimum inside radius for extrusions?

6:5. Machined Fittings. Fittings are machined from bar stock only when it is considered inadvisable to use forgings. This is true for experimental airplanes where only one or possibly a few airplanes are to be made. In rare cases, it has proved more economical on experimental airplanes to machine the fitting from a rough hand forging than from a large rectangular piece of stock. The material saved by this method together with the decreased machining time more than pay for the cost of the rough forging.

On the first few airplanes of a production order, it is often necessary to use machined fittings, owing to the time element of obtaining forgings. Extreme care should be exercised by the layout man in order to make these machined fittings interchangeable with the forgings that will follow (see Art. 4:16).

If the direction of the fibers or "grain" is important, the layout man should specify by arrows ⟷ the required direction. If the loads are great, the direction of grain should be parallel to the principal loads.

In the event that fittings are to be machined from either rough hand forgings or bar stock, often referred to as "hogging out," the designer should keep in mind the cutters and tools that are available to the shop. The checking group usually has a list of these and every effort should be exercised to use them. Never design a part that will require the purchase of a special cutter or tool, without first consulting a member of the production design group.

Practically all machined fittings are made on a milling machine. Here the work is held stationary on the table while the tool rotates. The table itself may be moved in various directions and may be tilted at the will of the operator; the speed of the cutter also is under his control.

The rotating shaft on which the cutter is fastened is known as the "arbor"; some milling machines have the arbor parallel to the floor and others have it vertical. A great variety of cutters may be obtained for milling machines. Perhaps one of the commonest is the end mill, which derives its name from the fact that teeth are on the end of the rotating tool. There are many types of end mills; however they all have a certain similarity in that a tool something like a drill is used; one end being inserted in the arbor, the other end being shaped and sharpened so that the cutting action occurs on the end as well as the sides.

The layout man need only follow certain general rules in order to obtain a design that is practical and that can be made with a

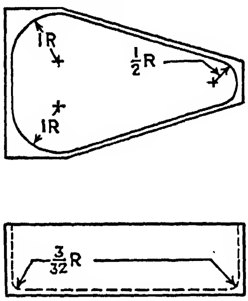


FIG. 6:33.

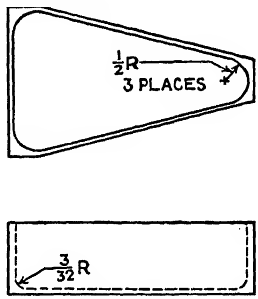


FIG. 6:34.

minimum of difficulty. The design recommendations set forth in Chap. 4 apply to machined fittings as well as to forgings or castings.

As a general rule, the designer should be careful not to specify walls thinner than $3/32$ in. on any machined fitting, owing to excessive deflection during the machining operation.

Endeavor to use the same cutter as much as possible for a given fitting. For example, in Fig. 6:33, a 1-in. diameter end mill will have to be used for one cut, and a 2-in. end mill for the other two. Figure 6:34 shows the same part so designed that the 1-in. end mill can be used at all three places, which saves another setup on the machine. Note the $3/32$-in. radius shown at the bottom of the recess. This is considered the minimum radius and one for which the shop has many cutters. It is a good radius to use when relatively sharp corners are desired.

It is desirable to design any part to reduce the number of machining operations to a minimum. Figure 6:35 shows a

common type of channel fitting in which the center line of the bolt is not parallel to the base of the fitting.

To make the part as shown, the machine operator starts removing the material within the channel, having the axis of the end mill vertical to the base. He removes all the material possible with the axis in this position; then tilts the table and removes the remaining portion of the material, forming the heavy end through which the bolt passes.

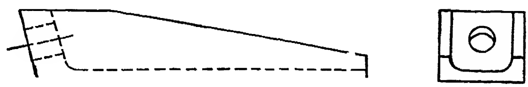


Fig. 6:35.

The part should have been designed as shown in Fig. 6:36, where the end remains 90 deg. to the base and is spot-faced to provide a seat for the bolt. This spot facing should not be considered an extra operation as the end is usually spot-faced to clean up corner radii.

The cut shown in Fig. 6:36 should never be at less than 90 deg. Never specify a closed angle as this type of cut is practically impossible to make.

Figure 6:35 illustrates a conventional type of channel fitting. The machining operations were discussed on the assumption

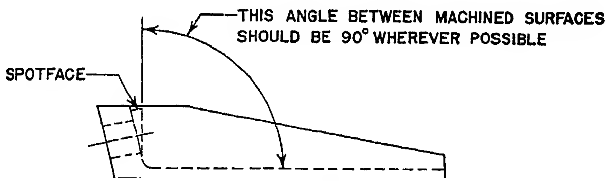


Fig. 6:36.

that the inside of the channel would be removed by an end mill. This assumption was made as it is usually preferable to use that type of cutter on channel fittings. However, there are times when the design may require the use of a side mill to remove the material within the channel. A side mill may be described as being similar to a circular saw, the teeth of this saw being sharpened and shaped so as to cut the metal both on the periphery and on the sides; hence the name "side mill." Since the plane of these cutters is perpendicular to the axis of the arbor, the design must provide suitable clearance between the arbor and the part

being machined. Figure 6:37 shows the minimum recommended radius for arbor clearance, and is a fitting designed so that a side mill will be used to remove the interior of the channel. These side mills may be obtained with various radii on the corners; however the designer should specify a radius for which there are cutters in the shop. As pointed out previously, the checking group usually has a list of standard cutters which should be referred to.

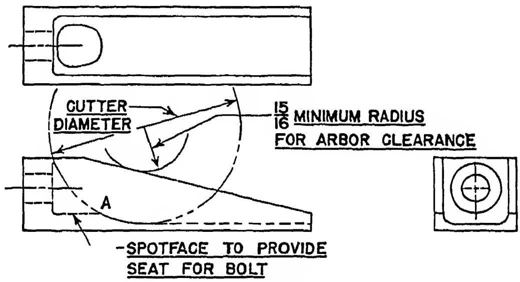


FIG. 6:37.

The spot-facing operation shown in Fig. 6:37 is at times difficult owing to the tool starting to cut on a slope, this point being marked *A*. This slope forces the cutter up, actually bending the shaft in many cases; hence the designer should not use this style of fitting unless absolutely necessary.

At times, it is desirable to mill a step in a fitting as shown in Fig. 6:38. Here a sharp inside corner is shown, which is poor design and should be avoided at all times. Figure 6:39 shows

FIG. 6:38.

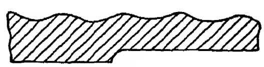


FIG. 6:39.

the same step milled correctly, 0.005 – 0.015 radius being an absolute minimum with 3⁄32 radius as a preferred minimum (see Art. 4:18). If the radius is unimportant, it is wise to note "radius optional," which permits the shop to use a variety of cutters. By showing a radius on the layout such as in Fig. 6:39, with the "radius optional" note, there is little danger of a sharp-cornered cutter being used.

At times, the layout man will have a design that requires either a shim to provide a flat surface for his fitting or a step in the

fitting itself. This may be due to a difference in gages of doublers, extruded angles, etc. In such cases, it is usually better design to provide a step in the base of the fitting, thus eliminating the use of the shim. This step should have a radius in the corner as shown in Fig. 6:39.

It has been frequently pointed out in this text that the designer should avoid abrupt changes in section; hence the fittings illustrated have all been tapered to avoid this condition. The

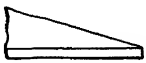


FIG. 6:40.

FIG. 6:41.

question often arises as to whether the taper of the sides should be as shown in Fig. 6:40 or as in Fig. 6:41. Both types are acceptable; however the taper shown in Fig. 6:41 is generally preferred as it facilitates machining to a certain extent and lessens the chances of rejections. When the design calls for a taper as in Fig. 6:40, and the shop takes the tolerances, the cutter may mill into the base at the end. Although it is not serious to cut into the base, it may cause the part to go into salvage which is undesirable from a production standpoint.

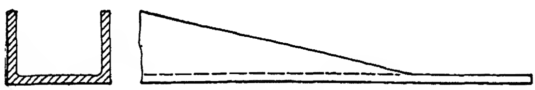


FIG. 6:42.

Figure 6:42 shows a method of avoiding an abrupt change in section—a highly undesirable condition from the manufacturing standpoint. Here the operator has to change the direction of his cutter, which introduces extra operations. This method has advantages from a stress standpoint, but should be used only where necessary. As mentioned previously, Fig. 6:41 shows the preferred method; the point at which this taper stops is usually the point of tangency of the interior fillet as seen in the end view.

Problems

6:5:1. *a.* When is it recommended to machine fittings from bar stock?

b. Is it ever recommended to machine fittings from rough hand forgings? Why?

6:5:2. *a.* If the fitting is to be machined from bar stock, what provision should be made in the design?
b. When the direction of the grain is important, how is this information noted on the layout?

6:5:3. *a.* What is meant by "hogging out?"
b. The fitting should be designed for what cutters and tools?
c. Where is this information usually obtained?

6:5:4. What is considered the minimum practical wall thickness for machined fittings? Why?

6:5:5. The part should be designed so that a maximum (or minimum) number of cutters may be used. Which statement is correct? Explain fully, drawing any necessary freehand sketches.

6:5:6. Should the machining operations be held to a minimum or maximum? Explain with freehand sketches showing incorrect and correct designs for the same part.

6:5:7. Discuss the possibility of making machine cuts at less than 90 deg.

6:5:8. *a.* What is the arbor on a milling machine?
b. What is an end mill?
c. What is a side mill?

6:5:9. *a.* If the design requires the use of a side cutter, what provision should be made?
b. Are sharp inside corners recommended? Why?

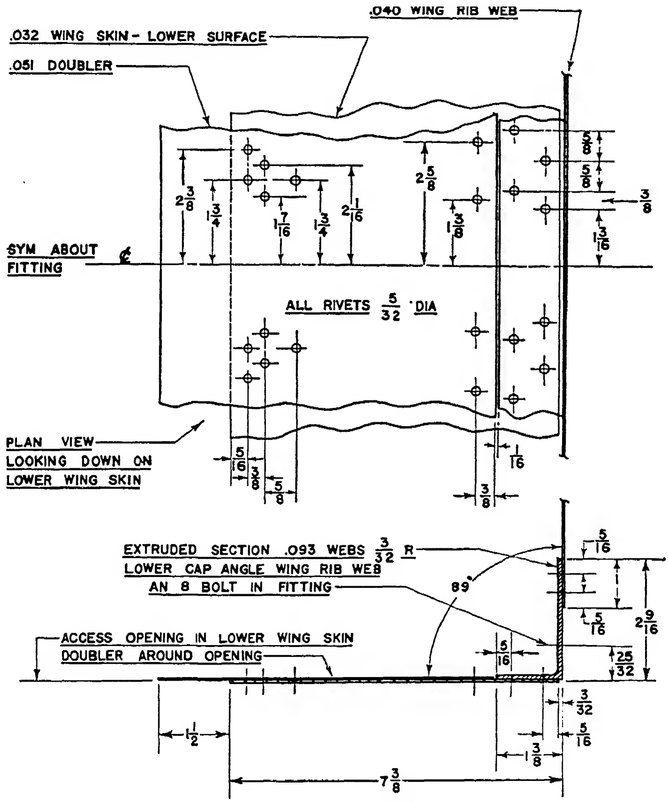
6:5:10. *a.* What is the absolute minimum radius and the preferred minimum radius for inside corners on machined fittings?
b. If the radius is unimportant. what note should appear on the layout?

6:5:11. Sketch three methods of tapering a fitting and point out the preferred method.

6:5:12. An access door was required on the lower surface of a wing near a rib. Owing to high-tension loads that had to be carried across this wing rib, it was deemed advisable to install a machined 24ST aluminum alloy fitting that would distribute this concentrated load from the ½-in. bolt, into the skin and into the doubler around the access hole. The accompanying diagram shows the wing structure between the access opening and the rib. The surrounding rivets are shown as a matter of general inter-

est, though they are not essential in the solution of this problem.

The load from this ½-in. bolt is such that the maximum cross section area required in the fitting is 0.66 sq. in., and nineteen 5⁄32-in. rivets are required to distribute this load into the skin and doubler. Owing to clearances required, the fitting must terminate at the point where the *wing skin* is cut for the access hole. Lay out this machined fitting, full size.



PROB. FIG. 6:5:12.

6:6. Electric Spot Welding.[1] The process of electric spot welding, often referred to as "resistance welding," may be

[1] The general information contained in this article applies not only to aluminum and its alloys, but also to magnesium alloy sheets and stainless steel.

described briefly as the passing of an electrical current of low voltage (6 to 10 volts) and high amperage (20,000 amp., average) through two or more pieces of metal; the current heats the metal sufficiently to fuse them together. In actual practice, the work to be welded is clamped between two copper electrodes. The current is then turned on, and at the point of greatest resistance, which is the contacting faces of the pieces to be welded, the metal is heated sufficiently to become molten. The current is then shut off, and the ensuing solidification of this drop of metal results in a weld. (See Fig. 6:44 for a photograph of a spot welder in use.)

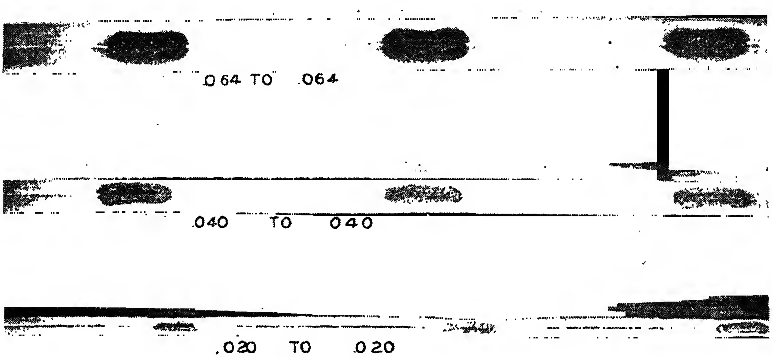


FIG. 6:43.—(*Douglas Aircraft, Inc.*)

In a good weld, this spot of fused metal should extend through the sheet for only about one-half the thickness of the sheet (see Fig. 6:43). If the molten area is too shallow, it is weak; if it is too deep and extends through to the opposite surface, the corrosion resistance of the metal is seriously impaired.

Figure 6:45 is a greatly enlarged view of a section cut through two 0.064 thick 24ST Alclad sheets. The crystalline structure in the center of the weld is the portion of the metal that was actually melted during the spot-welding operation, and is now similar in structure to a casting. The variation in shading and shadows around this crystalline structure is due to the heat effect. The metal adjacent to this central crystalline structure, though not actually reduced to the molten state, was in a plastic form.

The white lines through the center of the weld and on the edges of the sheet are the aluminum coatings. Note how the

aluminum extends into the weld itself; this is characteristic of Alclad because of the lower melting point of aluminum alloy.

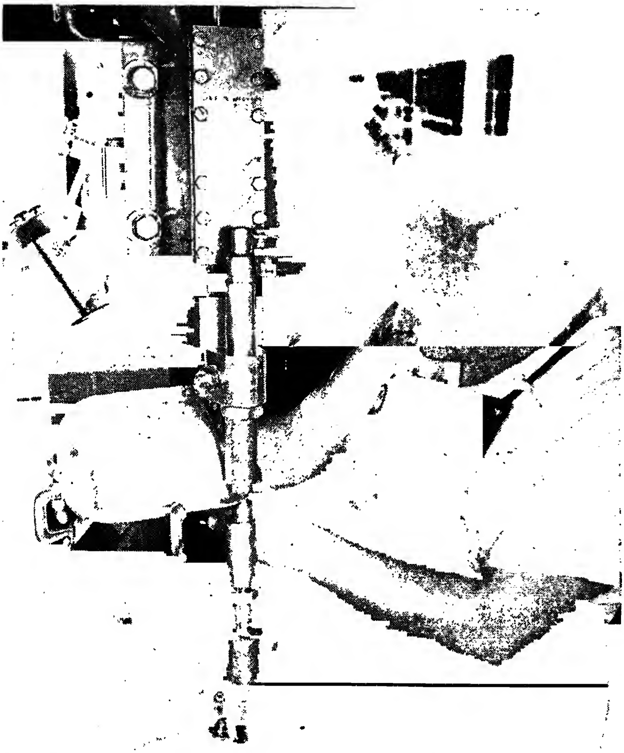


FIG. 6:44.—Spot welder in use. (*Douglas Aircraft, Inc.*)

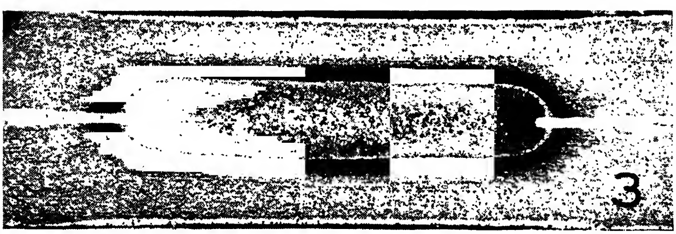


FIG. 6:45.

The melting point of aluminum is 1218°F.; that of aluminum alloy is approximately 1165°F.

This weld is considered a good one, since it extends approximately halfway through the sheet and there is a complete absence

of blowholes and cracks in the weld itself. The Alclad coating on the exterior shows no evidence of having been broken down by surface heating. In other words, the concentration of heat was in the most desirable spot, in the center and not on the surface.

To obtain this condition in the wide variety of materials and gages used in the aircraft industry, the spot-welding machines should be equipped to control the pressure on the electrodes, with a resistance to vary the voltage, and a timing device to regulate the length of time the current flows, usually from $\frac{1}{5}$ to $\frac{1}{25}$ sec. A wide variety of shapes of electrodes should be available which are made of copper or some alloy of copper, since it is highly important to keep the resistance of the electrical circuit to a minimum.

It is considered more difficult to spot-weld aluminum and its alloys and magnesium alloys, than the ferrous metals. The equipment necessary to weld the nonferrous metals is more complex and rugged than that used to weld the ferrous metals which adapt themselves so well to resistance welding. The reason for this difficulty in welding aluminum and its alloys is twofold:

1. The electrical conductivity of aluminum and its alloys is much greater than that of steel: hence more current is required for the same heating effect.

2. The heat conductivity of aluminum and its alloys is also much greater than that of steel, that is, the heat tends to flow away from the weld faster in aluminum and its alloys than in steel. To compensate for this loss, more heat is required, which in turn requires more current. The net result is that the welding of steel requires approximately one-third the current that the welding of aluminum and its alloys requires.

According to the Air Corps, spot welding of aluminum alloy material is prohibited in joining members of the primary structure, except joints between skin to intermediate stiffener combinations. Spot welding of aluminum alloys is permitted in all secondary and nonstructural portions of the airplane, and is considered good design, wherever possible. Consult a member of the production design group if there are questions about the advisability of spot welding in any particular problem.

Spot welding of stainless steel may be used in primary, secondary, and nonstructural members throughout the airplane; with

the exception that main attachment fittings, control surface and control system fittings should not be attached by this method, and should not consist of "built-up" fittings constructed of spotwelded stainless steel parts.

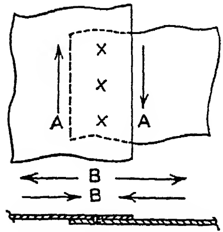


FIG. 6:46.

The simplest form of joint between two flat sheets is the single row of spots through a lap joint as shown in Fig. 6:46. If the load acts as indicated by the arrows *B*, this type of joint is considered only fair; if the principal loads are as in *A*, this is considered a good joint.

If these same sheets are curved sufficiently to give a tubular column effect, as shown in Fig. 6:47, thus reducing the tendency to wave and wrinkle under load, this spot-welded joint is considered good for all types of loads.

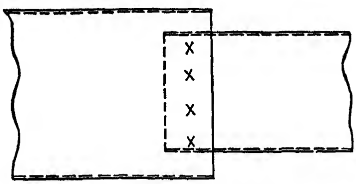


FIG. 6:47.

A good general purpose joint for carrying loads in any direction in the plane of the metal is shown in Fig. 6:48. With proper spacing of welds, the joint efficiency will approach that of a riveted joint. It is often desirable to have a flush joint such as

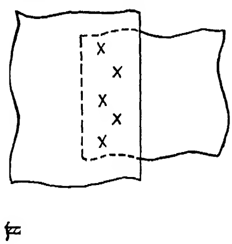


FIG. 6:48.

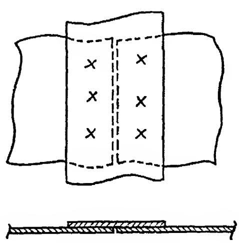


FIG. 6:49.

on the exterior surface. The simplest type is shown in Fig. 6:49; however, the criticisms made of the joint in Fig. 6:46 apply equally here. The same improvement in load-carrying charac-

teristics will be made in this joint by curving the sheet as were made in the joint in Fig. 6:47.

If a flush surface is not essential, a very good type of joint that has no eccentricity is shown in Fig. 6:50. This joint carries loads equally well in any direction in the plane of the metal. The combined thickness of the splice plates should be equal to or slightly greater than the thickness of the heaviest sheet.

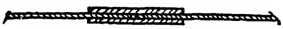


FIG. 6:50.

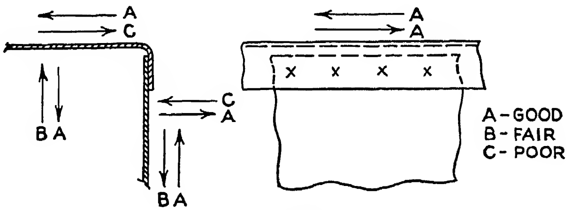


FIG. 6:51.

A typical corner joint used for brackets, boxes, etc., is shown in Fig. 6:51. It has good resistance to distributed loads from the inside, but should not be used where loading from the outside can force the flat sheet away from the flange.

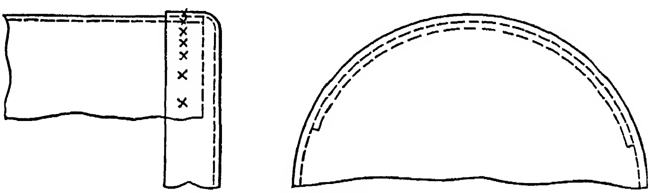


FIG. 6:52.

Figure 6:52 is a commonly used joint for attaching a cap to the end of a tube or cylinder, and is good for any type of loading. The designer must keep in mind the limitations of the equipment, with regard to diameter and length of cylinder that may be welded.

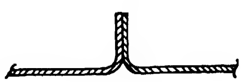


FIG. 6:53.

As far as accessibility for welding is concerned, the type of joint shown in Fig. 6:53 is ideal. It is poor if there is any tendency to pull the joint apart as in tension loads; if there is no tension and only shear, it is considered a good joint.

A common type of flanged corner joint is shown in Fig. 6:54. It is used often where equipment limitations prevent the use of the type shown in Fig. 6:51. If loading conditions are severe, every effort should be made to modify the design to use the joint shown in Fig. 6:51.

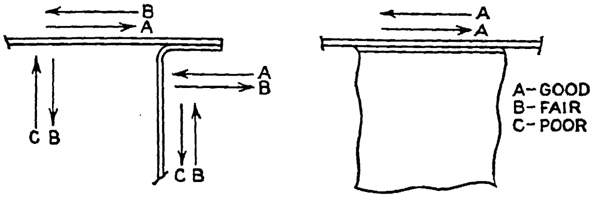


FIG. 6:54.

Where equipment limitations prevent the closure of a tube or cylinder by the method shown in Fig. 6:52, the construction shown in Fig. 6:55 is often used. This is good for both internal and external loading; however, the type shown in Fig. 6:52 is preferable.

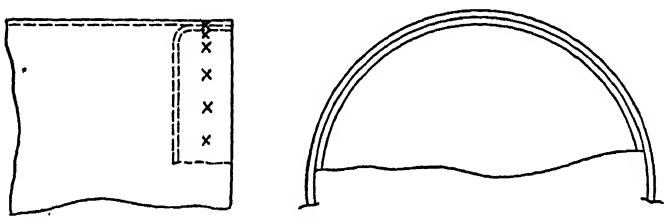


FIG. 6:55.

A design sometimes calls for a filler spot-welded into the end of a flattened tube as shown in Fig. 6:56. This is definitely a mistake for it is impractical to spot-weld a part within a tube; the tube shunts the current around the filler.

FIG. 6:56.

Aluminum and its alloys may be welded together; however for reasons of corrosion resistance, restrictions are placed on the welding of different alloys. It is impossible from a practical standpoint to weld aluminum and its alloy to anything but aluminum or its alloy.

More than two sheets may be welded together; however more than five is not considered satisfactory. The maximum thickness of any spot-welded joint should not be more than 0.20 in.

Care should be exercised by the designer that two sheets of widely varying gage are not specified to be spot-welded together. For example, an 0.016-in. sheet and an 0.072-in. sheet cannot be welded together satisfactorily. A current sufficient to melt the 0.072 will burn the 0.016. A good general rule to follow is that the thickness ratio of members joined should not exceed 3 to 1.

The recommended spacing of spots depends upon the gage. For 0.025 and 0.032, they should be ½ in. apart. If the gage

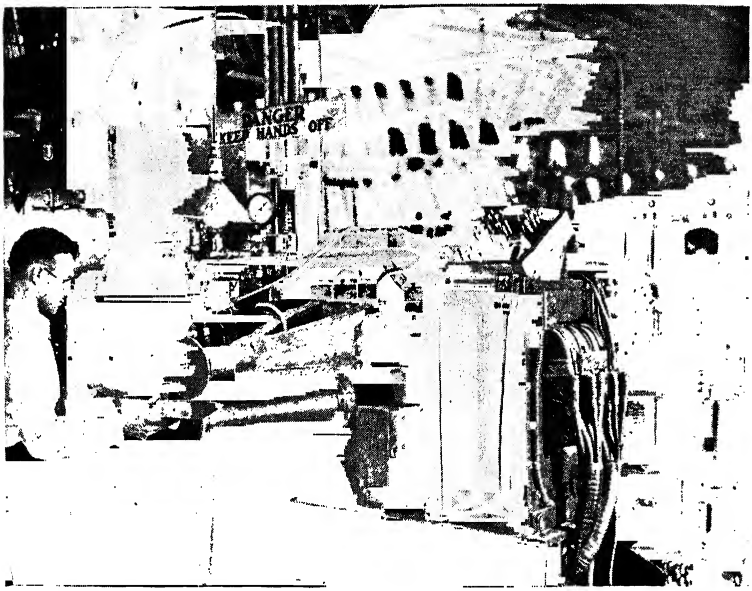


Fig. 6:57.—Seam welder in operation. (*Douglas Aircraft, Inc.*)

increases, the spots become farther apart; conversely, as the gage decreases, the spots are closer together.

Edge distance is as important in spot welding as in riveting. It must be remembered that a spot, to have strength equal to a rivet, must be larger in diameter than the rivet. The metal adjacent to the spot has been heated and has had its strength characteristics altered. The edge distance is a function of the gage welded; for 0.025, 0.032, and 0.040, the edge distance (center of spots to edge of sheet) should not be less than ¼ in. As this dimension is an absolute minimum, the layout man should at all times show more than this on his design.

Another type of spot-welding machine is an adaptation of the conventional type (see Fig. 6:57). In this machine the electrodes have been replaced by two mechanically driven rollers or wheels. The work to be welded is simply rolled between these two wheels, the timing device being so adjusted that the current interruptions will provide the correct spacing. It may be adjusted so there is a series of overlapping spots or a seam with spots spaced far apart. This type of equipment lends itself to high production.

Problems

6:6:1. Describe the process of electric spot welding.

6:6:2. *a.* What is the average voltage used?
b. What is the average amperage?
c. Approximately how long does the current flow?

6:6:3. Sketch freehand two sheets spot-welded together, showing the relation of the weld to the total thickness of sheet. Explain why it should be a good weld.

6:6:4. In spot-welding Alclad sheets, the layer of aluminum extends a short way into the weld. Why?

6:6:5. From what material are the electrodes made? Why?

6:6:6. Why is it more difficult to weld aluminum and its alloys than iron or steel?

6:6:7. *a.* What are the limitations on welding aluminum alloy?
b. What are the limitations on welding stainless steel?

6:6:8. *a.* Sketch a simple lap joint and discuss its load-carrying characteristics.
b. If these sheets are curved as the wall of a cylinder, are those characteristics improved?

6:6:9. Sketch a simple corner joint used for boxes, etc., and discuss its resistance to various types of loading conditions.

6:6:10. Draw a cap on a cylinder, and discuss its characteristics. What limiting conditions should be considered in the design?

6:6:11. Is it recommended to spot-weld fillers in the ends of tubes? Why?

6:6:12. *a.* Is it recommended to weld a 24ST Alclad sheet to a stainless steel sheet?
b. 24ST to 17ST?
c. 24STAL to magnesium sheet?

d. Stainless steel to stainless steel?
e. 24STAL to 24STAL?

6:6:13. How many sheets may be welded together satisfactorily? What is the limiting thickness?

6:6:14. *a.* Is it practical to weld sheets of widely varying gage together?
b. What is a good general rule to follow for this?

6:6:15. *a.* What is the recommended spacing of spots for 0.032 sheet?
b. If the gage is increased, are the spots closer together or farther apart?

6:6:16. What is the minimum edge distance for spots in sheet 0.025 to 0.040?

6:7. Sheet Metal Design and Assembly. There are a number of points the layout man should keep in mind when designing for sheet metal construction. He should be careful to show the correct bend radii and should never call for less than that given in the Appendix. It is common practice for the designer to guess at the bend radii, leaving it to the detailer to determine them accurately. These guesses are often incorrect, resulting in rivets and bolts that ride the radius, or in a general misfit (see Art. 4:7).

Hard stock should be used wherever possible. It is never permissible to specify soft stock in order to keep a rivet or bolt from riding a radius, since all soft stock must be heat-treated before installation on the airplane. If it seems necessary to specify soft stock in order to keep the radius to a minimum, the supervisor should be consulted.

The layout man cannot be too careful about the width of a flange. There are desirable features in keeping the width of a flange to a minimum, mainly the saving of weight. However, minimum flanges are the source of much trouble in the shop and their use should be discouraged wherever possible. The designer should keep in mind that punch countersunk rivets, as shown in Fig. 6:62, require a wider flange than ordinary brazier or round-head rivets. According to the Navy, the minimum width of flange when punch countersunk rivets are used, is $4.8d$ where d is the diameter of the rivet.

It is often necessary to design a sheet metal part that must be attached to two rigid and fixed members, such as a wing rib

extending from the front spar to the center spar. In the attachment of this rib to the spars, the layout man should make provisions for shop variations. Figure 6:58 shows a recommended method of attaching the rib to the spars, which permits adjustments for shop errors and variations.

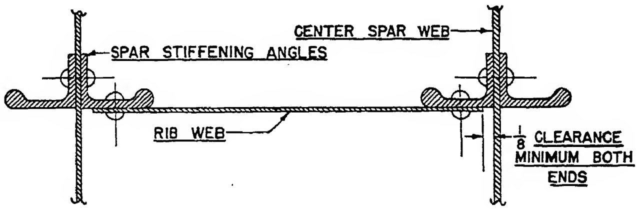


FIG. 6:58.

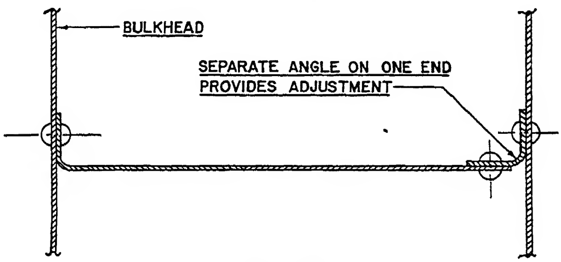


FIG. 6:59.

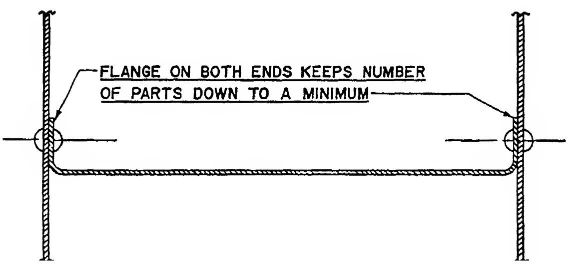


FIG. 6:60.

Quite often for parts smaller than the rib to spar assembly, it is desirable to provide the adjustment at one end only, as shown in Fig. 6:59. This is particularly true in cases where the bulkheads are rigid with little or no deflection possible, or where they cannot be adjusted to take up the variations and shop tolerances.

When the bulkheads are comparatively free to deflect, or if the design is of such nature that they can be moved slightly on assembly to allow for variations, it is desirable to use the type of

construction shown in Fig. 6:60. This cuts the number of parts down to a minimum, which is highly desirable from a production standpoint.

Aluminum alloy rivets are used in most cases for joining sheet metal parts (see Art. 5:2). Electric spot welding is used occasionally; however in this article we shall consider only riveted joints. Spot-welded joints were discussed in Art. 6:6.

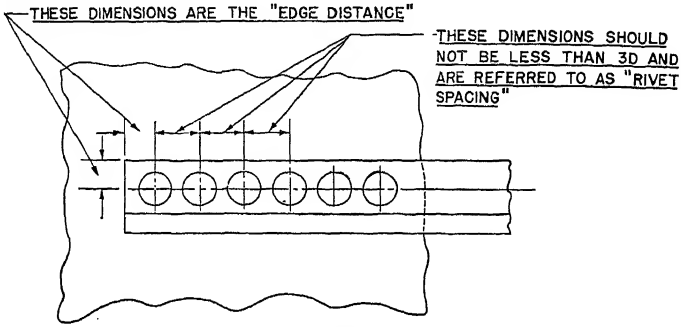


FIG. 6:61.

Rivets are usually spaced no closer than three times their diameter, often spoken of as $3d$ (see Fig. 6:61). Some designers feel that $4d$ is an ideal spacing and is considered preferable in many cases, but in no case call for rivets closer than $3d$ on centers.

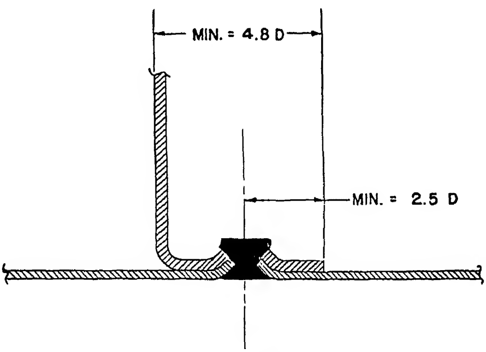


FIG. 6:62.

The edge distance for rivets may usually be determined accurately from the Drafting Room Manual. In general, a good rule to remember is that the edge distance should not be less than twice the rivet diameter except in very thin sheet where it should be greater than $2d$.

For flush riveting, edge distance greater than normal is required. According to the Navy, the minimum edge distance should be $2.5d$ (see Fig. 6:62).

The layout man should keep in mind that accessibility is highly important for all riveted and bolted joints. Too often the designer overlooks the fact that the rivet will be driven from one side and bucked from the other. Provision should be made not only for the gun and buck, but also for the operator's hands and arms. In some cases, manual or pneumatic squeezers can be

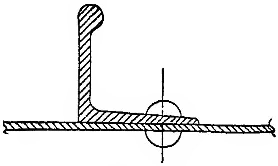


Fig. 6:63.

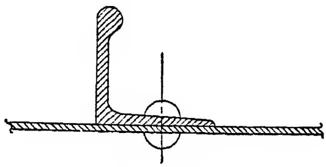


Fig. 6:64.

used; they also require ample clearance for their operation. if ever in doubt about the space required for riveting, consult a member of the production design group.

The choice of rivets is governed by two factors: strength and practicability. The strength considerations are discussed in Chap. 8; the practical side of rivet selection was discussed in Art. 5:2.

When two sheets are to be spliced together, the splice should be so located as to fall on a longeron, stringer, spar cap, or some member that will provide a backing for the splice. When a

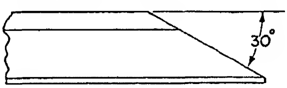


Fig. 6:65.

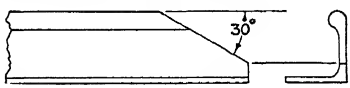


Fig. 6:66.

splice has a good solid backing, larger rivets may be used than when no backing is provided.

At times, it will be necessary to rivet a wide angle to some member. These rivets should not be placed at the edge of the angle as shown in Fig. 6:63, even though the edge distance may be $2d$ or more; they should be placed in the approximate center of the angle as shown in Fig. 6:64.

It is conventional design to scarf the ends of a bulb angle stiffener as shown in Fig. 6:66. In Fig. 6:65 is shown a type of scarfing that in some respects is superior to Fig. 6:66; however,

its use is discouraged as it limits the shop in the methods used to scarf the angle. In some cases, cutoff dies are used; then the scarf should be designed as shown in Fig. 6:66 to provide clearance for the dies. At times a circular saw is used to make the scarf; then the method shown in Fig. 6:65 would be satisfactory. However, all scarfs should be shown as in Fig. 6:66.

Frequently, the designer must attach parts that are tapered as shown in Figs. 6:67 and 6:68. If the attachment is by means of bolts, the part must be spot-faced to provide a flat spot normal to the axis of the bolt, as shown in Fig. 6:67. This spot facing for bolts is necessary even for very small angles.

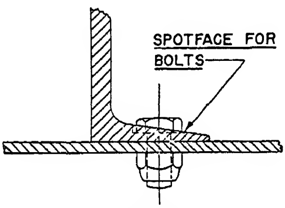


FIG. 6:67.

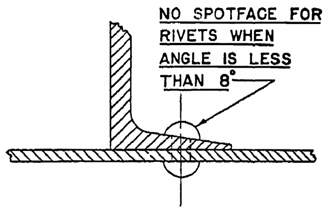


FIG. 6:68.

Figure 6:68 shows the tapered part being attached by rivets. Here it is unnecessary to spot-face when the angle of taper is 8 deg. or less. For angles over 8 deg., the part should be spot-faced to provide a flat spot for the head. It has been proved by tests that, up to 8 deg., the strength of the rivet is not affected by letting the manufactured or bucked head conform to the slope of the part.

Problems

6:7:1. *a.* Should the design call for hard or soft stock? Why?

b. Why is it poor practice for the designer to guess at bend radii?

6:7:2. *a.* Why is the use of minimum flange widths discouraged?

b. What is the recommended width of flange when punch countersunk rivets are used?

6:7:3. *a.* Sketch a recommended method of attaching a wing rib to the spars.

b. If adjustment is to be on one end only sketch a method of attachment.

c. Sketch the style of connection that keeps the number of parts to a minimum. When is this type of connection recommended?

6:7:4. *a.* What is the minimum recommended spacing for rivets?

b. What is considered by many designers as an ideal average spacing?

6:7:5. *a.* What is a good rule to follow for edge distance of rivets?

b. Does this apply to very thin sheets?

c. Make a sketch illustrating what is meant by "edge distance" and "rivet spacing."

6:7:6. *a.* For flush rivets, what is the minimum recommended edge distance?

b. In general, where should splices in flat sheets be located?

6:7:7. *a.* When necessary to rivet a wide angle to any member, should the rivet be placed with $2d$ edge distance?

b. Sketch the recommended location of the rivets.

6:7:8. Discuss methods of scarfing the ends of bulb angle stiffeners, stating what is preferable and why.

6:7:9. *a.* Is it necessary to spot-face for boltheads and nuts for small angles?

b. Is it necessary to spot-face for rivets for small angles? Explain fully.

6:8. Pulley Brackets. It is not the part of this article to determine the geometry of the bracket, that is, the relation of the plane of the mounting surface to the plane formed by the path of the cables,[1] or the angle of bend (wrap angle) of the cable around the pulley, as this subject was discussed in Art. 2:9.

After the geometry of the problem has been determined, there are certain facts about pulley bracket design that should be considered.

Left-hand and right-hand brackets should be avoided wherever possible. Frequently the base may be designed so that identical parts can be used on both sides of the airplane. Pulley brackets

[1] See TM 1-435, pp. 74-80, for a discussion of cables and the methods used for wrapping and splicing.

must be rigid; deflections must be kept down to an absolute minimum since there is no such thing as a too rigid pulley bracket, especially in flight control systems. It is considered better practice to use cast aluminum alloy or magnesium alloy pulley brackets instead of sheet metal brackets because of the possible deflection of sheet metal brackets. Sheet metal brackets may be used on lightly loaded relatively unimportant systems; usually, on main flight controls it is better to use castings.

All pulley brackets used on the flight control system should be designed for pulleys having antifriction bearings, as shown on the standards sheet AN210. Generally speaking, the diameter of the pulley should be at least twenty times the diameter of the cable. This rule of twenty times the diameter need not be adhered to in brake controls and other systems subjected to intermittent use.

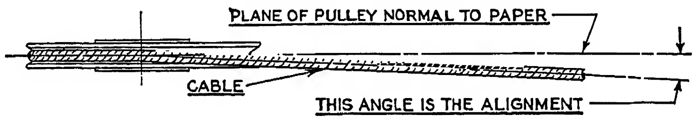


Fig. 6:69.

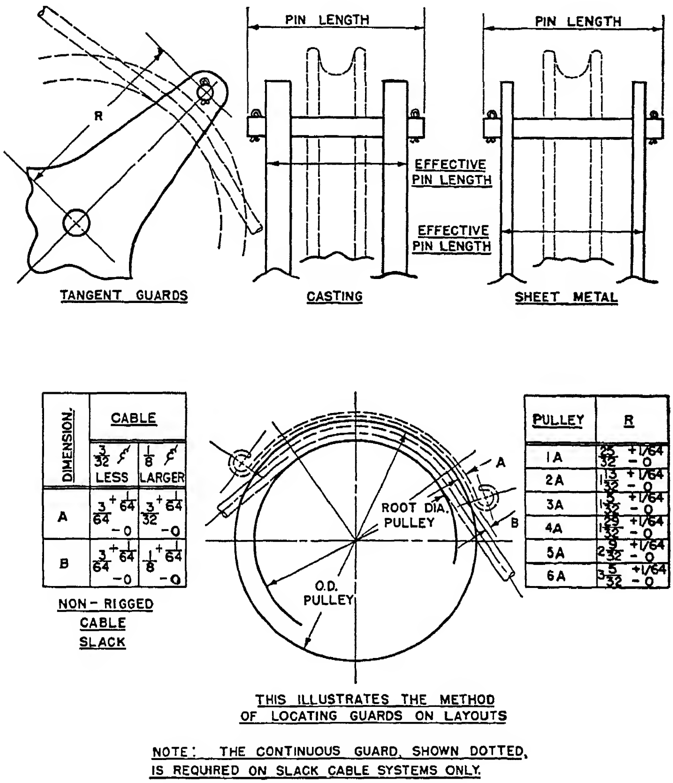
These larger diameter pulleys are used in flight control systems to keep the friction down to a minimum. The word "friction" as used here should probably be termed "energy loss," as over 85 per cent of the so-called friction of the system is actually an energy loss. This loss exists when any cable is bent around a pulley and then moved. The loss occurring is due to the work of bending and unbending the cable around the pulley. The energy is dissipated in heat and is never recovered; hence it appears as "friction" at the controls. The stiffness of the cable and the radius of bend are the principal factors determining the magnitude of this loss. As the radius of bend increases, the loss decreases, that is, the loss varies inversely as the radius.

In the design of a pulley bracket, the alignment of the cable, as it approaches the pulley, should be given careful consideration. The alignment of a cable with its pulley is defined as the angle through which the cable is bent as measured perpendicular to the plane of the pulley. This is shown graphically in Fig. 6:69.

Where a control cable has an angular motion with respect to the plane of the pulley, the maximum misalignment resulting

from this motion should not exceed 1 deg. for neutral position of controls and 2 deg. for any position between half and full movement of the controls.

Cable guards are very important in all pulley bracket design. In a system that permits slacking of the cable, such as the automatic pilot follow-up, a continuous guard should be provided



DIMENSION	CABLE	
	$\frac{3}{32}$" LESS	$\frac{1}{8}$" LARGER
A	$\frac{3}{64}{}^{+\frac{1}{64}}_{-0}$	$\frac{3}{32}{}^{+\frac{1}{64}}_{-0}$
B	$\frac{3}{64}{}^{+\frac{1}{64}}_{-0}$	$\frac{1}{8}{}^{+\frac{1}{64}}_{-0}$

PULLEY	R
1A	$\frac{25}{32}{}^{+1/64}_{-0}$
2A	$1\frac{13}{32}{}^{+1/64}_{-0}$
3A	$1\frac{5}{32}{}^{+1/64}_{-0}$
4A	$1\frac{29}{32}{}^{+1/64}_{-0}$
5A	$2\frac{9}{32}{}^{+1/64}_{-0}$
6A	$3\frac{5}{32}{}^{+1/64}_{-0}$

FIG. 6:70.

between the points of tangency of pulley and cable. Since there is no slacking in the majority of cable systems, pins at or close to the point of tangency of cable and pulley are provided. For the design of guards and the location of pins, see Fig. 6:70.

The use of cast guards or rivets and spacers is discouraged. On all multiple pulley brackets (brackets having two or more pulleys) the guards should be removable.

The importance of properly locating the pulley bracket should be realized. Even though the bracket itself is exceedingly rigid, unless the mounting surface is equally rigid, deflections will be introduced that are very undesirable. A pulley bracket mounted on the web of a spar or bulkhead is not satisfactory unless the web is reinforced by suitable stiffeners.

Problems

6:8:1. *a.* What is meant by the "wrap angle" of a cable?
b. Is it desirable to have left- and right-hand pulley brackets?

6:8:2. *a.* Are sheet metal pulley brackets recommended for flight control systems? Why?
b. Where is the use of sheet metal brackets recommended?

6:8:3. *a.* What series of pulleys are recommended for flight controls?
b. In general, what is the ratio of the diameter of the pulley to the diameter of the cable?

6:8:4. Discuss the reasons for the desirability of large-diameter pulleys in flight controls.

6:8:5. *a.* Sketch a cable and pulley, showing just what is meant by "alignment."
b. What is the maximum permissible misalignment?

6:8:6. *a.* In systems that permit a slacking of the cable, what type of guard is recommended?
b. In other systems where there is no slacking of the cable, what type of guard is recommended?
c. Where are they located?

6:8:7. *a.* In cast pulley brackets, are guards that are cast integral with the bracket recommended?
b. Are rivets and spacers recommended as guards?

6:8:8. Discuss the location of pulley brackets with respect to rigidity.

6:9. Drop Hammer. The drop hammer is used chiefly for low production, where there are too many parts to be formed by hand and not enough to warrant expensive steel dies. Figure 6:71 is an illustration of a drop hammer. When the operator pulls on the control rope, there is a snubbing action on the

rotating drums on top of the machine which results in raising the hammer. When the operator releases the control rope, the snubbing action ceases and the hammer falls. A skilled operator can vary the blow of the hammer from practically nothing up to the maximum, by the pull on his control rope.

Figure 6:72 is a photograph of a typical drop hammer part showing the piece being lifted from the female die. The lower or

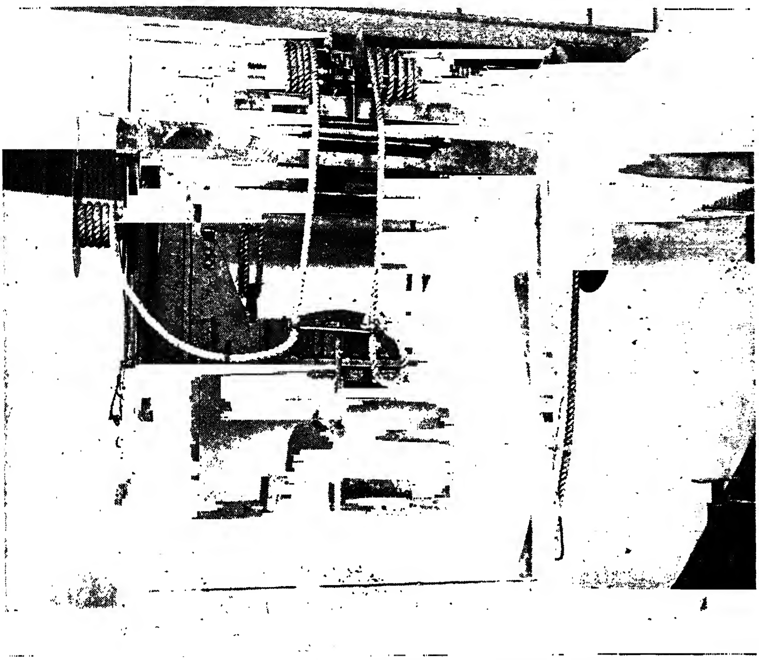


FIG. 6:71.—Drop hammer in operation. (*Douglas Aircraft, Inc.*)

female half of the die is usually made from cast zinc or Kirksite (special high-grade zinc alloyed with aluminum, copper, and magnesium) and is securely fastened to the "anvil" or base of the machine. The upper or male half of the die is usually of lead and is attached to the hammer.

The operator of the drop hammer must be a good sheet metal worker, for there is often some hand forming during the process. The basic principles of operation are those of stretching and shrinking the metal. The designer must understand these prin-

ciples before the drop hammer can be used to its maximum efficiency.

ST aluminum alloy should never be used for drop hammer work. The following materials are listed in the order of their

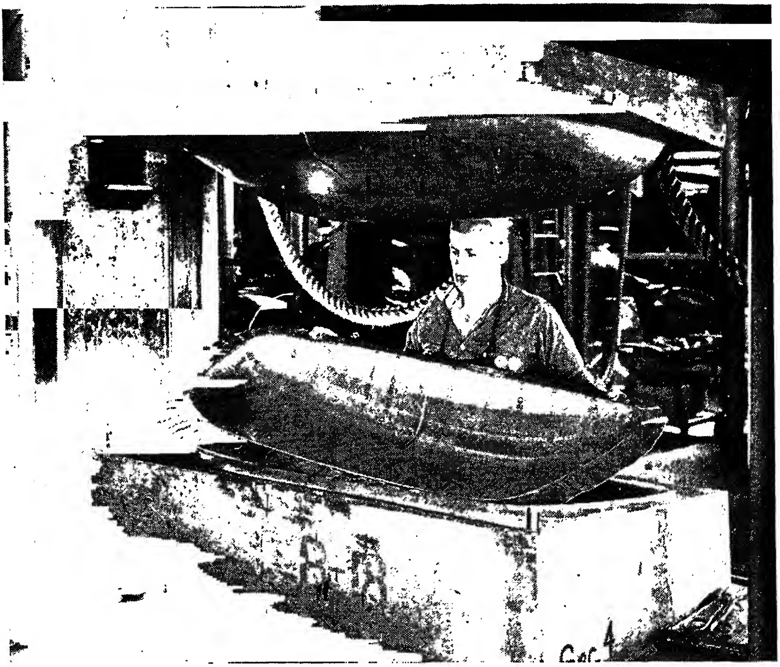


FIG. 6:72.—Drop-hammer part being removed from the die. (*Douglas Aircraft, Inc.*)

forming adaptability: 3SO, 24SO, 52SO; the 3SO being most ductile. Stainless steel may be formed on the drop hammer in the soft state; however, if the forming is severe, the part often

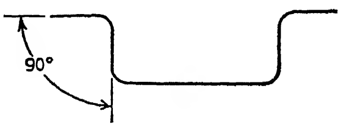


FIG. 6:73.

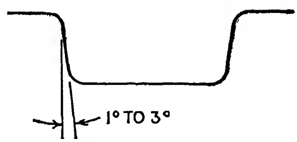


FIG. 6:74.

requires annealing during the forming process. Magnesium alloy sheet can be formed; however, it is difficult and expensive since it

must be worked while hot (500 to 750°F.), and should be used only after consulting a member of the production design group.

Parts to be made on the drop hammer should have not less than 1 to 3 deg. draft. Figure 6:73 shows an incorrectly designed part; Fig. 6:74 shows the same part with the correct draft. Draft is necessary in drop hammer work in order to remove the parts from the dies.

Problems

6:9:1. *a.* In general, what is the use of the drop hammer?
b. Describe briefly its operation.

6:9:2. *a.* From what material is the lower half of the die generally made?
b. Is the upper half of the die made from the same material?

6:9:3. *a.* Is 24ST recommended for drop hammer work?
b. List the alloys of aluminum used, the most ductile given first.

6:9:4. *a.* Can stainless steel be formed in the drop hammer?
b. Is magnesium alloy sheet recommended for drop hammer work? Why?

6:9:5. *a.* Is draft necessary for drop hammer work? Why?
b. Sketch two parts, one correctly designed, the other incorrectly as regarding draft, giving the amount of draft necessary.

6:10. Power Brake.[1] "Brake" is the name applied to the machine that "breaks" up sheet metal. Small machines are usually hand operated, but machines of large capacity are necessarily power operated, hence the name "power brake." The correct spelling of "brake" is at times discussed, some insisting it should be "break" since it actually breaks the metal. In this text it will be spelled "brake" as that is the form used by a prominent manufacturer of these machines.

Figure 6:77 is a close-up view of a large power brake showing how a corrugated sheet may be made. Corrugated sheets are often rolled; however this photograph clearly shows the dies and the method of their attachment to the ram and to the bed.

[1] See TM 1-435, p. 5, for folding machine; page 10, for cornice brake.

There are certain limitations to the power brake. Parts must necessarily be broken up along a straight line. The length

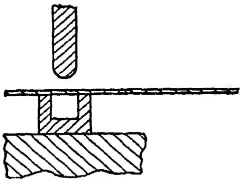


FIG. 6:75.

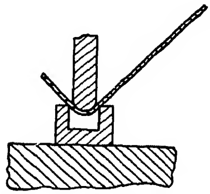


FIG. 6:76.

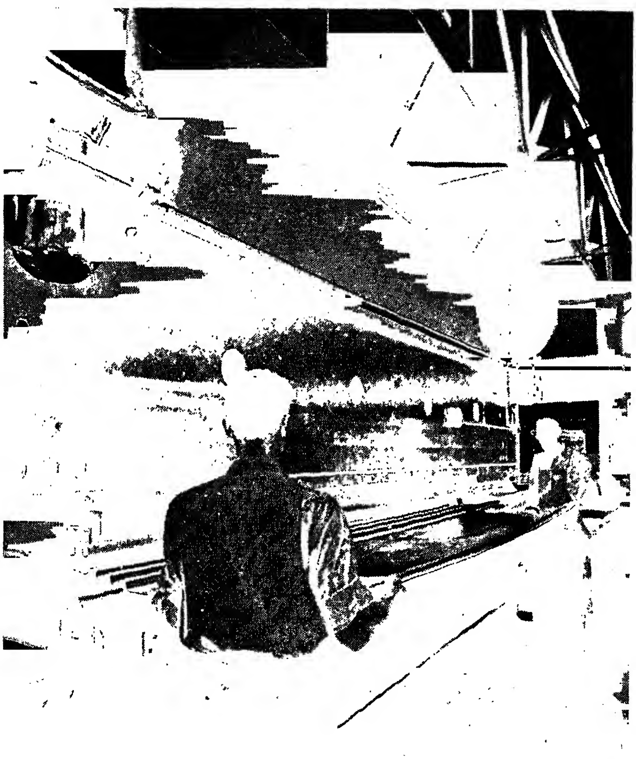


FIG. 6:77.—Forming corrugations on a power brake. (*Douglas Aircraft, Inc.*)

of the part, the width of the flange, and the gage of the material are also limited by the capacity of the machine.

In Fig. 6:75, a schematic diagram shows the upper male die and the lower female die, with a flat sheet of metal in place ready

to be broken up. Figure 6:76 shows the part broken up 90 deg.

Figure 6:78 shows clearly why there are limitations to the height of flange that can be formed. If the operator of the machine tries to form a flange of insufficient height, the condition

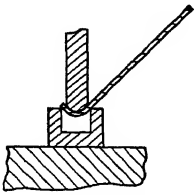


Fig. 6:78.

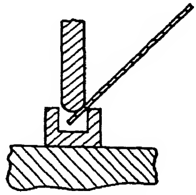


Fig. 6:79.

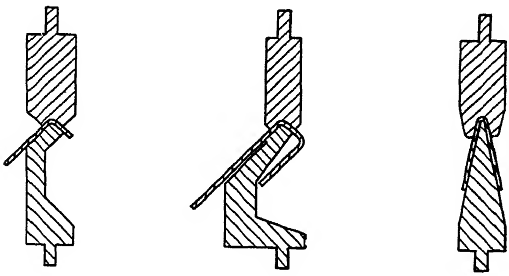


Fig. 6:80.

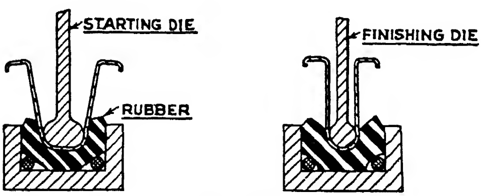


Fig. 6:81.

shown in Fig. 6:79 results. Figure 6:80 shows several common shapes of power brake dies and their uses.

At times rubber is utilized in forming complicated shapes on the power brake. Figure 6:81 shows how a hat section may be broken up using rubber as a female die.

This is not a complete treatise on the power brake, but shows only some of the common operations in order to acquaint the layout man with its operation and uses.

Problems

6:10:1. *a.* What is a "brake"?

b. Why are some brakes referred to as "power brakes"?

6:10:2. Discuss the limitations of a power brake.

6:10:3. Make two freehand sketches showing how a flat sheet may be bent up 90 deg. in a power brake.

6:10:4. Make two freehand sketches showing why there are limitations to the height of a flange that may be broken up.

6:10:5. Illustrate with sketches how rubber is sometimes used to facilitate the forming of complicated shapes in the power brake.

CHAPTER 7

FUNDAMENTAL PRINCIPLES OF FITTING ANALYSIS

It is best to begin the study of fitting analysis by considering some of the fundamental types of loading and defining certain terms that will be used frequently in this part of the text. The nomenclature and symbols used will be the Standard Structural Symbols in Par. 1.1 of ANC-5.

7:1. Tension.[1] Tension may be defined as "pull." When a man hangs on a rope, the rope is in tension. An elevator cable is in tension when the pilot moves the control column.

The designer is mainly interested in the tension load per square inch, and this load per unit area is known as the *tensile stress.*

The term "stress" always implies *a force per unit area* and is a measure of the intensity of the force acting on a definite plane passing through a given point. The stress distribution may or may not be uniform; it depends upon the nature of the loading. Uniform tensile stresses have the load distributed equally across the section. In airplane stress analysis, the unit area is always the square inch.

To determine the tension stress in a member

$$f_t = \frac{P}{A} \qquad (7:1)$$

where f_t = tensile stress in pounds per square inch

P = total load on section in pounds

A = area of cross section in square inches

This equation assumes the stress distribution to be uniform and, in practically all cases, this assumption may be made.

Illustrative Example. A lift strut of 0.65 sq. in. cross-sectional area carries a design load of 25,000 lb. What is the tensile stress in the strut?

$$f_t = \frac{P}{A}$$

[1] See C.A.B. 27, pp. 1–2, for a general discussion of stresses on aircraft structures and for tensile stresses.

where $P = 25{,}000$ lb.
$A = 0.65$ sq. in.

$$f_t = \frac{25{,}000}{0.65} = 38{,}470 \text{ psi}$$

7:2. Compression.[1] This is just the opposite of tension. Where tension was a "pull," compression might be defined as a "push," or a "squeezing together." The legs of a chair are in compression when it is occupied. A man climbs a flagpole; the pole supports his weight and is thus in compression. As in tensile stresses, compressive stresses may be considered to be distributed uniformly over the area. The compressive stress is determined similarly to the tensile stress, and is written

$$f_c = \frac{P}{A} \tag{7:2}$$

where f_c = compressive stress in pounds per square inch
P = total load on section in pounds
A = area of cross section in square inches

Illustrative Example. A link in the landing gear mechanism carries 18,500 lb. in compression. The cross-sectional area of this link is 0.39 sq. in. What is the compressive stress in the link?

$$f_c = \frac{P}{A}$$

where $P = 18{,}500$ lb. and $A = 0.39$ sq. in.

$$f_c = \frac{18{,}500}{0.39} = 47{,}450 \text{ psi}$$

7:3. Shear.[2] A thorough discussion of shear would become too complex for this text; however, it is important that the layout man understand simple shear and what is meant by single and double shear. One of the simplest examples of shearing action is that of a pair of scissors cutting a sheet of paper. As the blades of the scissors close, the paper is cut by a shearing action between the blades.

[1] See C.A.B. 27, p. 2, for a discussion of compressive stresses.
[2] See C.A.B. 27, p. 3, for a discussion of shearing stresses.

A simple example of a shearing action as commonly used in airplanes is that of a riveted joint as shown in Fig. 7:1, where the load P is applied as shown by the arrows. When the load becomes sufficiently great, the rivet will begin to shear, as indicated by Fig. 7:2. As the load is increased, the rivet will eventually fail in shear as in Fig. 7:3. These cases are examples of *single shear* since the rivet is sheared in only one place.

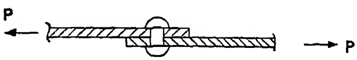


FIG. 7:1.

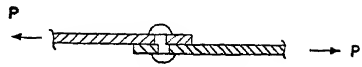


FIG. 7:2.

However, the construction may be similar to that shown in Fig. 7:4 with the load acting as indicated by arrows P. As the load is increased it will become great enough to begin to shear the rivet as shown in Fig. 7:5. The important thing to note is that the rivet is being sheared in two places, whereas in Fig. 7:2 it was sheared in one place. This is known as "double shear."

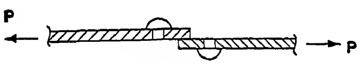


FIG. 7:3.

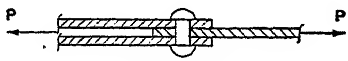


FIG. 7:4.

Now continue adding to the load P, and eventually the joint will fail in double shear as shown in Fig. 7:6. The joint shown in double shear will carry twice as much load as the joint in single shear, since in double shear the rivet is sheared at *two* sections simultaneously.

Figures 7:1 to 7:6 might be altered slightly to show a bolted joint; the last three illustrations could conveniently be made

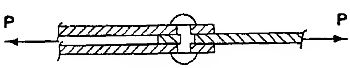


FIG. 7:5.

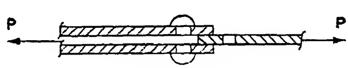


FIG. 7:6.

into a male and female fitting with a bolt and nut joining them. The foregoing statements about single and double shear apply equally well to a bolted joint and to a riveted joint.

The layout man should design his fittings and joints for double shear wherever possible, owing to the greater load-carrying capacity. The methods used to calculate rivets and bolts in shear are discussed in Chap. 8.

Another shearing action frequently encountered is that shown in Fig. 7:7, where a bolt in the end of a tension link tends to shear out the end of the link. Although the shearing action takes place along the lines *A-B* and *C-D* in the figure, the calculated area in shear is double the area along the line *E-F*. This method is conservative as the student may readily see; however it is the generally accepted one and should be followed.

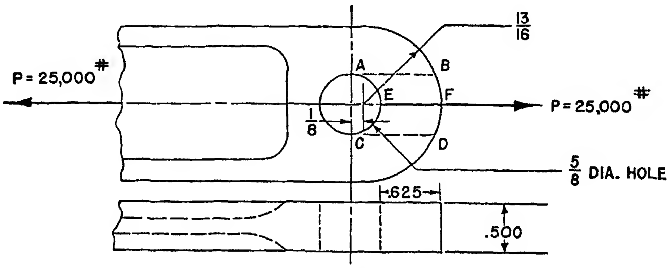


Fig. 7:7.

The average direct shear stress is given by the equation

$$f_s = \frac{S}{A} \qquad (7:3)$$

where f_s = shearing stress in pounds per square inch
S = shearing force or shearing load in pounds
A = area of cross section in square inches

Applying this to the example in Fig. 7:7, first find the area along the line *E-F* and double it.

$$A = 2(0.625 \times 0.5) = 0.625 \text{ sq. in.}$$

The load *P* is also the shearing force *S* in this example; therefore,

$$S = 25{,}000 \text{ lb.}$$

$$f_s = \frac{S}{A} = \frac{25{,}000}{0.625} = 40{,}000 \text{ psi.}$$

Now consider a type of construction where a bushing is used in the fitting, the bolt transmitting the load through the bushing into the fitting itself as shown in Fig. 7:8. The area in shear is calculated as in Fig. 7:7; the calculated area in shear is double the area along the line *E-F* which is twice the area figured from the hole in the fitting to the end. As in Fig. 7:7, the shearing

area is along the lines A-B and C-D; however, this conservative method of determining the area in shear is recommended.

7:4. Bearing. Another simple loading condition with which the layout man should be familiar is known as "bearing." Consider Fig. 7:7 again where the bolt is being forced against the end of the link by the load P. This bolt is bearing against the hole, that is, it is held in position by the metal surrounding the hole. Assuming the bolt is 5/8 in. in diameter, the projected area of this

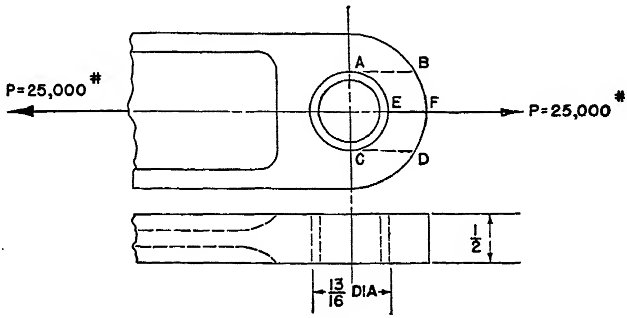


FIG. 7:8.

material bearing against the bolt is $\frac{5}{8} \times \frac{1}{2} = 0.313$ sq. in., since the thickness of the link is 1/2 in.

The bearing stress is given by the equation

$$f_{br} = \frac{P}{A} \tag{7:4}$$

where f_{br} = bearing stress in pounds per square inch
P = total load on section in pounds
A = projected area of cross section in bearing in inches

Illustrative Examples.

1. What is the bearing stress of the bolt on the end of the link shown in Fig. 7:7?

$$f_{br} = \frac{P}{A} = \frac{25{,}000}{0.313} = 79{,}900 \text{ psi}$$

2. What is the bearing stress of the bushing on the fitting given in Fig. 7:8?

$$f_{br} = \frac{P}{A} = \frac{25{,}000}{0.813 \times 0.5} = 61{,}500 \text{ psi}$$

It is important to note that the bearing stress was considerably reduced by the addition of a bushing. Frequently, the layout man should resort to this method of reducing bearing stresses; it is considered good design practice to do so (see Art. 5:5).

7:5. Ultimate Stress. The layout man should be able not only to calculate the actual stress within a part, but also to find correctly the ultimate stress. The ultimate stress is found in ANC-5 in the tables entitled "Mechanical Properties of Materials."

The following are the standard structural symbols as used in ANC-5:

$$
\begin{aligned}
F_{tu} &= \text{ultimate tensile stress} \\
F_{cu} &= \text{ultimate compressive stress} \\
F_{su} &= \text{ultimate shear stress} \\
F_{br} &= \text{ultimate bearing stress}
\end{aligned}
$$

For example, to find the ultimate tensile stress of 24ST Alclad sheet, look in table 5-5, page 5-8 of ANC-5. Line 1 reads "F_{tu} Ultimate Stress." Following across under column 2 marked "24ST Alclad Sheet," 56,000 is found, which is the figure desired. This would be written

$$F_{tu} = 56{,}000 \text{ psi}$$

In the same manner, it may be found that for 24ST Alclad sheet

$$
\begin{aligned}
F_{cu} &= 56{,}000 \text{ psi} \\
F_{su} &= 34{,}000 \text{ psi} \\
F_{br} &= 82{,}000 \text{ psi}
\end{aligned}
$$

The ultimate stress is the stress at which failure may be expected. If a simple tension load on a strip of 24ST Alclad sheet were gradually increased, the strip would break when the stress reached 56,000 psi.

7:6. Margin of Safety.[1] This is usually abbreviated M.S. and is given by the following equations:

[1] See C.A.B. 27, p. 5, for a discussion of factor of safety. The student should note the difference between "margin of safety" and "factor of safety."

$$\text{M.S.*} = \frac{\text{ultimate load}}{\text{design load}} - 1 \tag{7:5}$$

or

$$\text{M.S.*} = \frac{\text{ultimate unit stress}}{\text{design load unit stress}} - 1 \tag{7:6}$$

Applying this to the example in Fig. 7:7, what is the margin of safety in bearing of the bolt on the fitting? Assume that the link is a 14ST forging.

$$\text{M.S.} = \frac{F_{br}}{f_{br}} - 1 \qquad \text{[from Eq. (7:6)]}$$

From column 1, line 15, Table 5-11, page 5-14 of ANC-5, is found $F_{br} = 93{,}000$ psi. In Art. 7:4, f_{br} was determined and found to be 79,900 psi. Substituting these values in Eq. (7:6) gives

$$\text{M.S.} = \frac{93{,}000}{79{,}900} - 1 = 1.164 - 1.000 = 0.164$$

In speaking of this, it is said that the margin of safety of the bolt in bearing on the link is 16.4 per cent.

In actual practice, the layout man obtains the load from the stress department, and *at that time* finds out from the stress man what margin of safety should be held using the given load. The student should realize that in many cases the ultimate stress as obtained from ANC-5 cannot be used for his margin of safety equation (7:6).

There are many simple loading conditions, such as pure tension, where the ultimate stress from ANC-5 may be used; however, for more complex conditions such as columns, the ultimate must be reduced. If it is understood that the ultimate stress in Eq. (7:6) actually means in many cases the *allowable stress*, many errors will be avoided. The layout man should consult the stress group when in doubt about using the ultimate stress as given in ANC-5, for any particular problem.

Also, if it is remembered that the ultimate load actually means the allowable load, the correct usage of Eq. (7:5) will be simplified.

It is obvious that, if the margin of safety comes out a negative quantity, the design is not satisfactory; in all cases, a positive margin of safety is required.

* Niles and Newell, Vol. I, p. 33.

Illustrative Example.

To apply the foregoing information to a practical problem, strength-check the end of the fitting shown in Fig. 7:9. This is a 14ST aluminum alloy forging. The design loads are

+53,000 lb. tension
−59,200 lb. compression

There is an AN12 bolt through the end into a mating female fitting. In this example, only the end of the male fitting given in Fig. 7:9 and the AN12 bolt will be strength-checked.

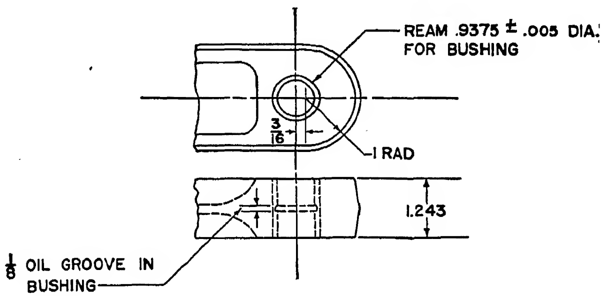


FIG. 7:9.

Strength of AN12 *Bolt.* From the AN book, the strength of this bolt in single shear is found to be 33,135 lb. As the mating part is a female fitting, the bolt is in double shear. Therefore, the allowable load is

$$2 \times 33{,}135 = 66{,}270 \text{ lb.}$$

$$\text{M.S.} = \frac{66{,}270}{59{,}200} - 1 = 1.120 - 1 = 0.12$$

Bearing of Bolt on Steel Bushing. Allowance must be made for the oil groove in the steel bushing.

$$\begin{aligned} \text{Projected area of bolt} &= 0.750 \times 1.243 = 0.933 \text{ sq. in.} \\ \text{Area removed by oil groove} &= 0.125 \times 0.750 = \underline{-0.093} \text{ sq. in.} \\ \text{Net area in bearing} &= 0.840 \text{ sq. in.} \end{aligned}$$

$$f_{br} = \frac{59{,}200}{0.840} = 70{,}500 \text{ psi}$$

$$F_{br} = 175{,}000 \text{ psi}$$

since both the bolt and the bushing are heat-treated to 125,000 psi (ANC-5, page 4-12).

$$\text{M.S.} = \frac{175{,}000}{70{,}500} - 1 = 2.48 - 1 = 1.48$$

Bearing of Bushing on Link.

Area of bearing $= 0.937 \times 1.243 = 1.165$ sq. in.

$$f_{br} = \frac{59{,}200}{1.165} = 50{,}800 \text{ psi}$$

$$F_{br} = 93{,}000 \text{ psi}$$

since link is a 14ST forging.

$$\text{M.S.} = \frac{93{,}000}{50{,}800} - 1 = 1.83 - 1 = 0.83$$

Caution: If there is a reversal of load, there are special factors to be applied to the bearing between the bolt and the fitting. Check with the stress department for these factors.

Tension across Link at Bolt Hole.

$$\begin{aligned} \text{Area in tension} &= (2.000 - 0.938)1.243 \\ &= 1.062 \times 1.243 \\ &= 1.32 \text{ sq. in.} \\ f_t &= \frac{53{,}000}{1.32} = 40{,}150 \text{ psi} \\ F_{tu} &= 65{,}000 \text{ psi} \\ \text{M.S.} &= \frac{65{,}000}{40{,}150} - 1 = 1.62 - 1 = 0.62 \end{aligned}$$

Shear in End of Link.

$$\begin{aligned} \text{Area in shear} &= 2\left(1.187 - \frac{0.938}{2}\right) \times 1.243 \\ &= 2(1.187 - 0.469) \times 1.243 \\ &= 2 \times 0.718 \times 1.243 = 1.785 \text{ sq. in.} \\ f_s &= \frac{53{,}000}{1.785} = 29{,}700 \text{ psi} \\ F_{su} &= 39{,}000 \text{ psi} \\ \text{M.S.} &= \frac{39{,}000}{29{,}700} - 1 = 1.313 - 1 = 0.313 \end{aligned}$$

7:7. Stress and Strain. The student should never confuse the terms "stress" and "strain," as they are not synonyms. It was pointed out in Art. 7:1 that stress always implies a force per unit area, and is a measure of the intensity of the force. Strain refers to the elongation per unit length of a member in stressed condition. "Strain" should never be used in place of the terms "elongation" and "deflection." There are several kinds of strains: axial strain, lateral strain, and shearing strain. However, it is beyond the scope of this text to discuss them.

7:8. Manufacturing Tolerances Applied to Stress Analysis. In the calculation of stresses, nominal dimensions are generally used. The problem shown in Fig. 7:9 uses an AN12 bolt whose nominal diameter is ¾ in., although by referring to the AN book it may be determined that the actual diameter will vary from 0.7445 to 0.7490 in.

In the bearing of the bolt on the bushing, the *nominal* diameter (0.750) was used, and not the minimum diameter as figured from the tolerances (0.7445).

In shear, the nominal diameter is also used, or nominal dimensions of the part. There are always certain tolerances allowed the shop; however, it is standard practice to use the nominal dimensions in calculations for shear. In Art. 7:6, these nominal dimensions were used when the calculations for the shear in the end of the link were made.

In ordinary calculations for tension and compression, nominal dimensions are used, *with one notable exception:* When tensile stresses are calculated for bolts, use the *minimum root diameter* which is the diameter at the bottom of the threads.

In the AN series of bolts, the single shear strength is calculated for the nominal diameter; however, the tensile strength at root diameter is calculated for the *minimum* root diameter.

Problems

7:1. *a*. Define "stress."

b. Define "strain."

7:2. Write the equation for (*a*) tensile stress; (*b*) compressive stress; (*c*) shear stress; (*d*) bearing stress.

7:3. Sketch a fitting or link having a single large bolt for attachment and show just what area would be used in the strength

check for (*a*) tension through the end of link; (*b*) bolt shearing out end of link; (*c*) bearing of bolt on fitting.

7:4. Write the equation for margin of safety both in terms of loads and in terms of stresses.

7:5. A 24ST aluminum alloy fitting machined from bar stock is subjected to a tension load of 11,250 lb. The area in tension is 0.22 sq. in. $F_{tu} = 62{,}000$ psi. Find f_t and M.S.

7:6. The design compressive load in this same fitting is 10,730 lb. $F_{cu} = 62{,}000$ psi. Find f_c and M.S.

7:7. A ⅜-in.-diameter Ni steel pin (heat treated to 125,000 psi) in double shear carries a load of 14,300 lb. $F_{su} = 75{,}000$ psi. Find f_s and M.S.

7:8. This same pin passes through a steel bushing H.T. to 125,000 psi pressed into the fitting. $F_{br} = 175{,}000$ psi. A 100 per cent M.S. must be held between bolt and bushing. What is the required length of bushing?

7:9. How is the required margin of safety determined for any design problem?

7:10. *a.* The tension load in a member having 0.58 sq. in. cross-sectional area is 19,800 lb. What is the tensile stress in that member?

b. What is the stress in an AN12 bolt carrying 30,000 lb. tension?

7:11. The compressive force in a strut is 32,300 lb. The area is 0.92 sq. in. What is the compressive stress?

7:12. *a.* The shearing force on an AN5 bolt is 7210 lb. Should the fitting be designed so the bolt is in single or double shear? Why?

b. What is the shearing stress in either case?

7:13. If the AN5 bolt of Prob. 7:12 is bearing on a fitting ⅞ in. long, what is the bearing stress?

7:14. What is the bearing stress of an AN8 bolt on a fitting 1⅛ in. long when the load is 46,200 lb.?

7:15. What is the ultimate tensile stress of:

(*a*) 24SRT Alclad sheet?

(*b*) 24ST extruded shapes?

(*c*) 24ST bar (up to 3 in. thick)?

(*d*) SAE 4130 C.M. (chrome-moly) steel normalized sheet 0.125 in. thick?

(*e*) SAE 4130 C.M. steel H.T. 125,000 psi?

7:16. What is the ultimate compressive stress of:
(*a*) SAE 2330 Ni steel bar H.T. 125,000 psi?
(*b*) 24ST bar (up to 3 in. thick)?
(*c*) 14ST forging?
(*d*) SAE 4130 C.M. steel normalized bar $1\frac{1}{4}$ in. thick?

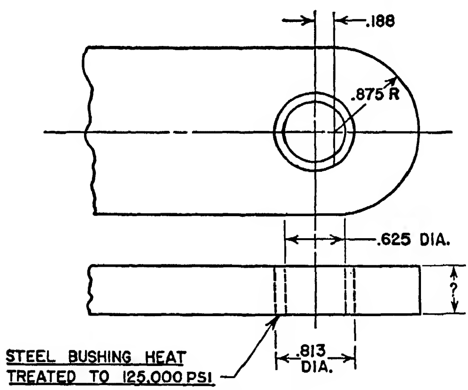
7:17. What is the ultimate shear stress of:
(*a*) SAE 4130 C.M. steel rod H.T. 180,000 psi?
(*b*) SAE 2330 Ni steel rod H.T. 125,000 psi?
(*c*) 24ST sheet?
(*d*) 24ST Alclad sheet?

7:18. What is the ultimate bearing stress of:
(*a*) 24ST extruded shapes?
(*b*) $52S\frac{1}{2}H$ sheet?
(*c*) 24ST bar (up to 3 in. thick)?
(*d*) 14ST forging?

7:19. Determine the thickness and strength check the end, including the bolt, of the 24ST aluminum alloy link shown in the accompanying diagram. The design loads are as follows:

+30,000 lb. tension
−38,000 lb. compression

Using these loads and the ultimates from ANC-5 for bar stock, a 20 per cent margin of safety must be kept for all conditions



PROB. FIG. 7:19.

except for the bearing of the bolt on the bushing where a 100 per cent margin should be maintained. This link attaches to a female fitting with an AN10 bolt.

CHAPTER 8

STRENGTH CALCULATIONS FOR RIVETED AND BOLTED JOINTS

8:1. Introduction. Much of the knowledge gained in Chap. 7 has a direct application to the design of riveted or bolted joints, in particular, that part pertaining to shear and bearing. Since riveted and bolted joints are calculated similarly, little mention will be made of bolted joints in this chapter. If the student understands the underlying principles used in the calculations of riveted joints, there will be no difficulty in applying these principles to bolted joints.

8:2. Conditions to Be Investigated. All riveted joints should be checked for the shear of the rivet and for the bearing of the sheet on the rivet. The values for the allowable shear and bearing strengths for aluminum alloy rivets and sheet are obtained from the table on page 5-20 of ANC-5. In this text, only aluminum alloy rivets and sheet will be considered, as the majority of the problems encountered by the layout man will be for that material, though the principles involved apply equally well to steel.

Knowing the gage of the material to be joined and the alloy of the rivets or bolts to be used, it is customary to obtain from the tables in ANC-5 the allowable bearing and shear values; and then in the design of the joint to use the lower of these two values.

For example, a $\frac{5}{32}$-in.-diameter A17ST rivet has a single shear strength of 518 lb. (from ANC-5, page 5-20). The allowable bearing strength of this rivet on an 0.040-in.-thick 24ST Alclad sheet is 512 lb. Therefore, this joint is weakest in bearing; often referred to as being "critical in bearing."

8:3. Joints of Maximum Efficiency. In the design of riveted joints, it is desirable to choose rivets that have approximately the same strength in shear and in bearing. The ideal joint, from a strength standpoint, would be to have the shear and bearing values the same, which would give the maximum joint efficiency.

As pointed out in Art. 5:2, there are practical considerations in the selection of rivet sizes which often determine the size to be used. Unless impractical, the designer should keep the shear and bearing values as near the same as possible.

To illustrate: Consider a simple lap joint between two 0.032 24ST Alclad sheets. The load across this joint is 500 lb. per running inch. How many and what size of A17ST rivets are required per running inch?

From the ANC-5 rivet tables on page 5-20 for $\frac{1}{8}$-in. AD rivets, the allowable load in single shear is 331 lb., and the allowable load in bearing on 0.032 24ST Alclad sheet is 328 lb. The corresponding values for the $\frac{5}{32}$-in.-diameter rivets are: shear = 518 lb.; bearing = 409 lb. Since the shearing and bearing values of the $\frac{1}{8}$-in.-diameter rivets are nearly the same and since it is practical to drive $\frac{1}{8}$-in.-diameter rivets in 0.032 sheet, this would be the most efficient rivet to specify.

8:4. Calculations for Spacing of Rivets. Having determined the diameter of rivet, next calculate the number of rivets per running inch to carry the given load of 500 lb. per inch.

The $\frac{1}{8}$-in.-diameter rivet is critical in bearing with an allowable load of 328 lb. The number of rivets per inch may be calculated as follows:

$$\text{Number of rivets per inch} = \frac{\text{load per inch}}{\text{critical strength of rivet}} \qquad (8:1)$$

Substituting these values, gives

$$\text{Number of rivets per inch} = \frac{500}{328} = 1.524$$

In order to have 1.524 rivets per inch, it is obvious that they must be less than 1 in. apart. To obtain the required spacing we write,

$$\text{Rivet spacing} = \frac{1.000}{\text{number of rivets per inch}} \qquad (8:2)$$

Substituting the above values gives

$$\text{Rivet spacing} = \frac{1.000}{1.524} = 0.656 \text{ in. on centers}$$

That is, a row of $\frac{1}{8}$-in.-diameter A17ST rivets spaced $\frac{5}{8}$ in. on centers, will carry 500 lb. per running inch.

Example: A simple riveted lap joint between an 0.081 and an 0.091 24ST Alclad sheet carries a shearing load of 1890 lb. per running inch. What size and spacing of A17ST rivets are required to carry this load?

For $\frac{3}{16}$-in. rivets:

$$\begin{aligned}\text{Allowable in shear} &= 745 \text{ lb.}\\ \text{Allowable in bearing} &= 1245 \text{ lb.}\end{aligned}$$

For $\frac{1}{4}$-in. rivets:

$$\begin{aligned}\text{Allowable in shear} &= 1325 \text{ lb.}\\ \text{Allowable in bearing} &= 1660 \text{ lb.}\end{aligned}$$

Notice that the allowable in bearing is taken for the lighter of the two sheets. From the foregoing, it is apparent that $\frac{1}{4}$-in. rivets would be the more efficient and that they are critical in shear. From Eq. (8:1):

$$\text{Number of rivets per inch} = {}^{1890}\!/_{1325} = 1.43$$

From Eq. (8:2):

$$\text{Rivet spacing} = \frac{1.00}{1.43} = 0.70 \text{ in. or } {}^{11}\!/_{16} \text{ in. on centers}$$

As this spacing is less than $3d$, it is apparent there should be two rows staggered so that the requirements in Fig. 8:1 may be met.

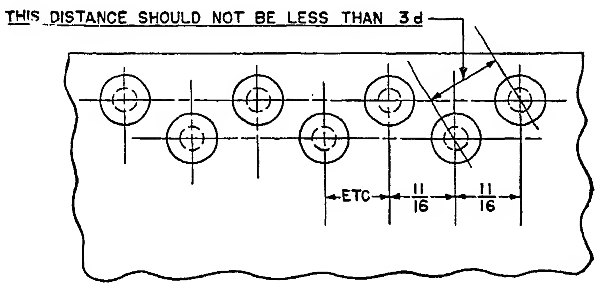


FIG. 8:1.

This problem made no mention of whether or not there was a backing for the splice. If the splice occurred on a spar where there was a heavy cap member, or on a large extruded longeron, or similar member, $\frac{1}{4}$-in. rivets would probably be used. However, if this splice occurred where there was no backing, such as

between wing stringers, $\frac{1}{4}$-in. rivets would not be recommended owing to the tendency to stretch the metal on driving such heavy rivets. In this case $\frac{3}{16}$-in. rivets should be used to lessen this tendency of stretching the metal, although the efficiency of the joint would be decreased. As pointed out in Art. 6:7, it is good design to lap skin splices where there is a backing for the rivets; the laps are purposely located to fall on longerons, frames, etc. There is some stretching even when smaller rivets are used; however, it is usually not bad enough to show objectionably. But when the seam has a good solid backing, this reinforcement prevents any stretching of the skin from showing; so the more efficient rivet may be used.

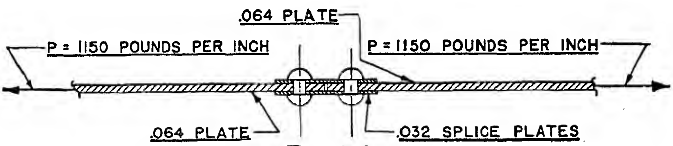


FIG. 8:2.

The allowable strength of rivets, as given in ANC-5, applies to all types of heads: flat, brazier, round, countersunk, or counterpunched.

Thus far, only riveted joints in single shear have been considered. Now a type of joint where the rivets are in double shear, as shown in Fig. 8:2, will be discussed. In a joint of this nature, the combined thickness of the splice plates must be equal to or greater than the thickness of the plate being spliced. If the splice occurs between plates of different thickness, the combined thickness of the splice plates should be equal to or greater than the thickness of the thinner of the two plates being spliced.

In Fig. 8:2, the rivets are A17ST and the sheet is 24ST Alclad. First, determine the diameter of rivet that will give maximum efficiency.

For $\frac{1}{8}$-in. rivets:

Allowable in shear $= 2 \times 331 = 662$ lb.
Allowable in bearing on 0.064 sheet $= 656$ lb.

For $\frac{5}{32}$-in. rivets:

Allowable in shear $= 2 \times 518 = 1036$ lb.
Allowable in bearing on 0.064 sheet $= 819$ lb.

From the foregoing it is obvious that the maximum joint efficiency will be maintained by the use of $\frac{1}{8}$-in.-diameter rivets. It should be noticed that the allowable shear was doubled, since the rivet is in double shear. For bearing, the allowable for 0.064 sheet was used, as the load P is on the 0.064 sheet. At the splice, the load divides, half going through the upper splice plate and half through the lower plate.

The calculations for the number of rivets per inch and the spacing of the rivets are made similarly to those for joints in single shear. Since the load P is 1150 lb. per running inch,

From Eq. (8:1):

$$\text{Number of rivets per inch} = {}^{1150}\!/_{656} = 1.753$$

From Eq. (8:2):

$$\text{Rivet spacing} = \frac{1.000}{1.753} = 0.571 \text{ in.}$$

A convenient short cut to determine the spacing of the rivets is as follows:

$$\text{Rivet spacing} = \frac{\text{allowable strength of rivet}}{\text{load per running inch}} \qquad (8:3)$$

Substituting our values,

$$\text{Rivet spacing} = {}^{656}\!/_{1150} = 0.571 \text{ in.}$$

which is identical to that obtained above. In this case, a spacing of $\frac{9}{16}$ in. O.C. (on centers) would be specified, as it is usually undesirable to give rivet spacing in thirty-seconds.

8:5. Eccentrically Loaded Joints—Symmetrical Rivet Pattern. Consideration has thus far been given to joints having no eccentric loading. Frequently, the layout man encounters a condition as shown in Fig. 8:3.

In this joint, there is a load P acting at the end of an arm, resisted by the four rivets numbered 2, 3, 4, and 5. There is a combination of loads on these four rivets; the simplest load is the opposite reaction to the load P which is $P/4$ on each rivet.

To maintain equilibrium in this joint, there are forces on the rivets that provide a turning moment, resisting the moment from the load P. These forces are denoted P_2, P_3, P_4, and P_5.

The total force on any rivet is the sum of the forces on that rivet added vectorially. To determine these forces P_2, P_3, P_4, and P_5, certain calculations must be made.

There are definite limitations to the method presented in this and the following article. These equations are valid only for joints with rivets of the same area and same material, applied throughout to the same sheet thickness. A combination of rivets, or rivets and bolts, in the same joint is not considered in this discussion, as that condition is not often encountered by the layout man. In the event such a joint is necessary, the stress department should be consulted, for the computations are more

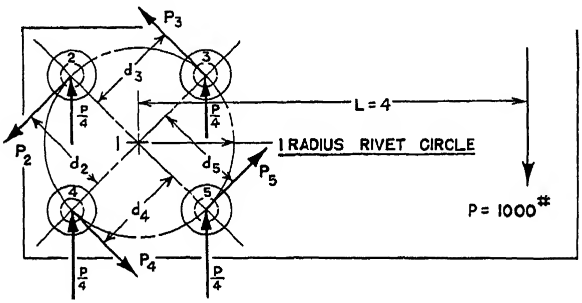


FIG. 8:3.

complex than any given here. When these limitations are understood, proceed as follows:

Determine the centroid of the group of rivets. In this example, it is obvious that the centroid is at point 1, since the rivet pattern is symmetrical.

The moment M from load P is

$$M = PL \tag{8:4}$$

where P = applied load

L = distance from applied load to centroid of group of rivets.

Now find the inertia of the group of rivets.

$$I = d_2^2 + d_3^2 + d_4^2 + d_5^2 \tag{8:5}$$

where d_2, d_3, etc., is the distance from each rivet to the centroid of the rivet group.

In the case shown in Fig. 8:3, $d_2 = d_3 = d_4 = d_5$, therefore, it may be written

$$I = 4d_2^2 \tag{8:5a}$$

The force on each rivet providing the turning moment to maintain equilibrium is obtained as follows:

$$\begin{aligned} P_2 &= \frac{Md_2}{I} \qquad P_4 = \frac{Md_4}{I} \\ P_3 &= \frac{Md_3}{I} \qquad P_5 = \frac{Md_5}{I} \end{aligned} \tag{8:6}$$

Having the magnitude and direction of each load, solve for the direction and magnitude of the resultant load. It is usually possible to determine the critical rivet by inspection. In this problem, it will be rivet 3 or 5. The magnitude of the resultant will be the same for either rivet; hence arbitrarily solve for rivet 3. Before attempting to determine this resultant load, work through the example using the dimensions and loads given in Fig. 8:3.

Solving for Eq. (8:4),

$$M = PL = 1000 \times 4 = 4000 \text{ in.-lb.}$$

Solving for Eq. (8:5*a*),

$$I = 4d_2^2 = 4 \times 1^2 = 4$$

Substituting these values in Eq. (8:6) gives

$$P_3 = \frac{Md_3}{I} = \frac{4000 \times 1}{4} = 1000 \text{ lb.}$$

The direct reaction per rivet from load P is $P/4 = 250$ lb. The directions and magnitudes of the reactions on rivet 3 have now been determined. To solve for the resultant vectorially, plot the 250-lb. force to some convenient scale parallel to the direction in Fig. 8:3. This is shown in Fig. 8:4 as line A-B. Now plot line B-C to the same scale, parallel to the line of force in Fig. 8:3. To close the force polygon, draw the line C-A, which is the magnitude and direction of the resultant force on rivet 3. By scaling this line, it is found to be 1188 lb.

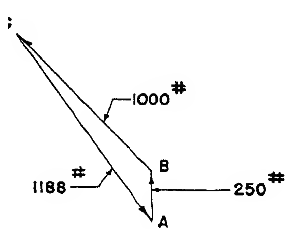


FIG. 8:4.

The magnitude of the resultant force having been found, the selection of rivets to carry this load may now be made. Assume the plate is 0.064 24ST Alclad. Then from the rivet tables on page 5-20 of ANC-5, it may be found that ¼-in.-diameter A17ST rivets are good for 1325 lb. in single shear and 1312 lb. in bearing. Therefore, rivets 3 and 5 of Fig. 8:3 must be ¼ in. in diameter. As pointed out previously in this article, all rivets in the joint should be the same diameter, therefore ¼-in.-diameter rivets should be specified.

8:6. Eccentrically Loaded Joints—Unsymmetrical Rivet Pattern. Usually, the pattern is symmetrical; however, at times it will be necessary to have an unsymmetrical pattern as shown in Fig. 8:5. In this case, the problem becomes more involved.

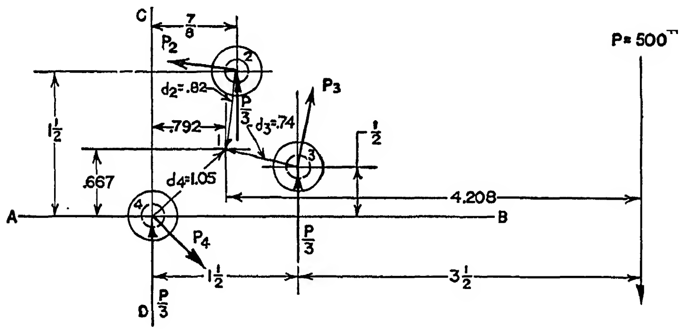


FIG. 8:5.

The first step is to determine the centroid of the group of rivets. To do this, construct two base lines at right angles to each other, such as lines A-B and C-D, through some convenient rivet. Now take moments about the horizontal base line A-B.

Rivet 2,	1×1.5	$= 1.5$
Rivet 3,	1×0.5	$= 0.5$
Rivet 4,	1×0	$= 0$
	3	$= 2.0$

The first column is the number of rivets; second column is the distance from base line A-B; and the third column is the moment.

Now having the total moment of these three rivets, in order to obtain the center of gravity (c.g.) from the base line A-B, write

$$\text{Distance of c.g. from } A\text{-}B = \frac{2}{3} = 0.667$$

where 2 = total moment

3 = total number of rivets.

In like manner, solving for the center of gravity from line *C-D*,

Rivet 2,	1×0.875	$= 0.875$
Rivet 3,	1×1.5	$= 1.50$
Rivet 4,	1×0	$= 0$
	3	$= 2.375$

$$\text{Distance from } C\text{-}D = \frac{2.375}{3} = 0.792$$

The intersection of these neutral axes is the centroid of the group of rivets, and is point 1 in Fig. 8:5.

By simple arithmetic, the moment arm for load P is calculated, and found to be 4.208 in.; therefore, from Eq. (8:4),

$$M = 500 \times 4.208 = 2104 \text{ in.-lb.}$$

Determine the distance from each rivet to the centroid of the group. By scaling,

$$d_2 = 0.82$$
$$d_3 = 0.74$$
$$d_4 = 1.05$$

Therefore,

$$\begin{aligned} I &= 0.82^2 + 0.74^2 + 1.05^2 \quad \text{[From Eq. (8:5)]} \\ &= 0.672 + 0.548 + 1.103 \\ &= 2.323 \end{aligned}$$

Now, plot the direct reaction on each rivet due to the load P; each rivet carries a load of $P/3 = 500/3 = 167$ lb.

By inspection, it may be seen that rivet 3 will have the highest resultant load. Therefore, solving for P_3, using Eq. (8:6),

$$P_3 = \frac{Md_3}{I} = \frac{2104 \times 0.74}{2.323} = 670 \text{ lb.}$$

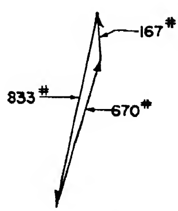


FIG. 8:6.

Plotting the vector diagram shown in Fig. 8:6, the resultant load is found to be 833 lb.

If the plate is 0.072 24ST Alclad, 1/4-in.-diameter A17ST rivets will be required critical in single shear.

In the foregoing discussion, only riveted joints have been considered; however this information applies equally well to bolted joints. The same limitations apply to eccentrically loaded bolted joints as apply to riveted joints, that is, the same size of bolt must be used throughout to the same sheet thickness.

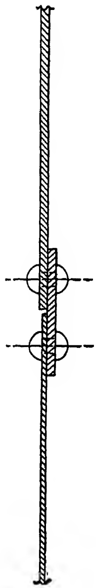


FIG. 8:7.

8:7. Connections between Sheets of Different Gages. It frequently happens that two sheets of different gages are but-joined by a single splice plate as shown in Fig. 8:7. Of course, the thickness of the splice plate should be equal to or greater than the thickness of the thinner of the two plates. In the selection of rivets, the designer may find he should have a different size rivet through the thin plate than through the thick plate, to have the most efficient joint. In other words, he would have two sizes of rivets in the same joint. This is not considered good practice as it leads to confusion and possible errors in the shop; use the same size of rivets in the joint, preferably the larger of the two.

For example, in Fig. 8:7, say that one sheet is 0.032 and the other is 0.040 24ST Alclad, and that the splice plate is to be 0.040 24ST Alclad. What is the most efficient size of rivet, and what rivets would be actually called for?

From ANC-5, page 5-20, for the 0.032 sheet use $\frac{1}{8}$-in.-diameter AD rivets critical in bearing at 328 lb.; for the 0.040 sheets, use $\frac{5}{32}$-in. rivets critical in bearing at 512 lb. Since it is considered poor design to mix sizes of rivets, specify the $\frac{5}{32}$-in. rivets for the entire joint.

8:8. Spacing Called for on a Layout vs. Calculated Spacing. In the majority of cases, the load obtained from the stress department is the one on which the stress man in his report bases his margins of safety. If the rivet spacing on the layout is greater than the calculated required spacing, it means that a negative margin would be shown in the stress report, which is not permissible; hence the *layout man should never specify rivet spacing greater than the calculated spacing.* It is usually customary to choose the nearest sixteenth under the calculated dimension. Suppose the

calculated spacing is 0.704 in.; on the layout the spacing should be $^{11}/_{16}$ in. on centers; if the figures show 0.99 in., specify $^{15}/_{16}$ in., not 1 in., on centers.

8:9. Actual Strength of Riveted Joints Compared to Calculated Strength. By referring to page 5-20 of ANC-5, it may be seen that the shearing strength of the rivet is based on the ultimate shear stress F_{su}. That is, the allowable single shear strength of a $^{3}/_{16}$-in.-diameter A17ST rivet is given as 745 lb., and is based on the ultimate shear stress $F_{su} = 27{,}000$ psi. Also, the allowable bearing strength of the 24ST Alclad sheet is based on the ultimate bearing strength $F_{br} = 82{,}000$.

Since it was pointed out in Art. 7:5 that the ultimate stress is the stress at which the part will fail, the student may wonder why riveted joints are designed right up to the ultimate stress with very small margins of safety. Actually, a riveted joint will carry considerably more load than the calculations indicate. This is due to three things:

1. Work hardening of material.
2. Swelling of the rivet.
3. Friction.

When an A17ST rivet is driven, there is a metallurgical change within the rivet, which is known as "work hardening." This change slightly increases the strength properties of the rivet; hence the values given in ANC-5 for F_{su} are actually lower than F_{su} of the driven rivet.

When a hole is drilled for a rivet, the hole must be *larger* than the rivet. When the rivet is driven, it swells and fills the hole; hence the diameter of the driven rivet is greater than the diameter of the original rivet; which means there is more area in shear and in bearing than had been shown in the calculations.

After a joint is riveted, the parts are held securely together. Any tendency for the parts to move relative to each other is resisted by a force, owing to the friction between these parts. This frictional force may be considerable, yet it should be neglected in the design calculations. This explains the apparent anomaly, that riveted joints are actually much stronger than their calculated strength.

8:10. Allowable Bearing Values for Materials Not Given in ANC-5 Rivet Tables. The question often arises as to the allowable bearing value in a riveted joint when the parts are 24SRT

sheet, 14ST forging, or other material that has F_{br} values not covered in the tables in ANC-5.

In any joint, the allowable bearing stress of the softest material is the one to be considered in the calculations. However, an allowable bearing value of 90,000 psi for *any* aluminum alloy rivet is permitted by ANC-5 because of the three previous reasons. Hence, a joint between 24SRT sheets attached with A17ST rivets (or *any* aluminum alloy rivet) would have F_{br} for the 24SRT sheet = 93,000 psi, F_{br} for the driven A17ST rivet = 90,000 psi. Therefore, this joint would be designed to F_{br} = 90,000, and the values would be obtained from the tables on page 5-20 of ANC-5.

Likewise, 14ST forgings whose value for F_{br} = 93,000 psi, would be critical in bearing on the driven rivet at F_{br} = 90,000.

If a steel bolt passed through the forging, obviously the bearing of the 14ST dural would be critical, and the bearing strength would be calculated on F_{br} = 93,000 psi. When the foregoing principles are thoroughly understood, the student should be able to calculate *any* riveted or bolted joint.

8:11. Limitations on Rivet Diameter with Respect to Sheet Gage. It will be noted that certain bearing values are not given in tables in ANC-5. This is because the ratio of rivet diameter to thickness of sheet is greater than 5.5; when this ratio is greater than 5.5, the joint is not recommended. Always design riveted joints so that the values for allowable bearing may be taken from the table in ANC-5.

Reference

"Airplane Structures," by Niles and Newell, Vol. I, Art. 11:6, p. 346; Art. 11:7, p. 355; Art. 11:10, p. 363.

Problems

In the following problems, any positive margin of safety is considered satisfactory unless otherwise stated.

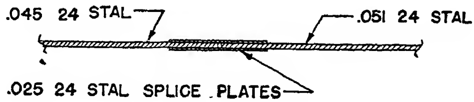
8:1. *a.* All riveted joints should be checked for what two conditions?

b. What would give maximum rivet efficiency?

8:2. A simple lap joint between 0.032 and 0.040 24ST Alclad sheets carries 480 lb. per running inch. Design the joint using A17ST rivets.

8:3. At times the layout man finds that, to obtain maximum joint efficiency, he should use very large rivets. Is it good design to do so in all cases? Why?

8:4. Design the splice in the accompanying figure, using A17ST rivets. The load is 900 lb. per running inch.



PROB. FIG. 8:4.

8:5. In eccentrically loaded joints, the load on the critical rivet is calculated. What determines the sizes of the balance of the rivets in that joint? Why?

8:6. Why is it important that on a layout the specified spacing of rivets should never exceed the calculated spacing?

8:7. What is the allowable load in shear on a $\frac{5}{32}$-in. A17ST rivet? On a 24ST rivet?

8:8. What is the allowable load in bearing on an 0.051 24ST Alclad sheet of a $\frac{3}{16}$-in. rivet? On 0.051 24ST sheet?

8:9. A simple lap joint between an 0.040 and an 0.051 24ST Alclad sheet carries 675 lb. per running inch. What size and spacing of A17ST rivets should be specified?

8:10. An 0.072 24ST Alclad wing skin attaches to the fuselage with an 0.081 24ST Alclad angle. The shear load is 1630 lb. per running inch. Design the rivet pattern, assuming the attach angle may have any desired width of flange to rivet.

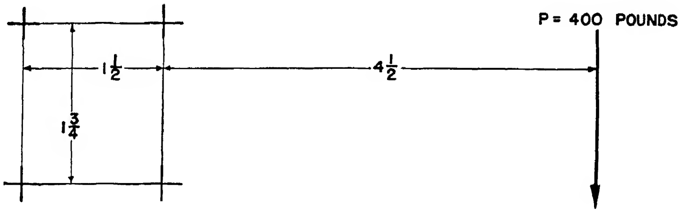
8:11. A 14ST forged fitting carrying 14,950 lb. attaches to a bulkhead. The design is such that the rivets are in single shear. The thickness of the forging is $\frac{3}{16}$ in., whereas the thickness of the bulkhead 24ST extruded angle, to which it attaches, is 0.21 in.

a. How many and what size of AD rivets are recommended?

b. If the same size DD rivets are to be used, how many are necessary?

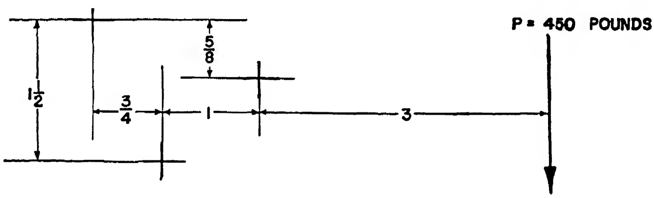
8:12. An 0.064 24ST Alclad spar web splices to an 0.072 24ST Alclad web by means of 0.040 24ST Alclad plates on each side of the web. This splice carries 1240 lb. per inch. Design the rivet pattern using A17ST rivets.

8:13. What size of A17ST rivets is required in the joint of the accompanying figure, assuming the sheet is 0.051 24ST Alclad? Show all the calculations.



PROB. FIG. 8:13.

8:14. What size of A17ST rivets is required in the joint shown in the accompanying figure? The plate is 0.081 24ST Alclad. Show all the calculations.



PROB. FIG. 8:14.

CHAPTER 9

BENDING AND TORSIONAL STRESSES

9:1. Simple Bending.[1] A simple example of bending is that of a beam supported at each end with a load concentrated at the center, as shown in Fig. 9:1. The load will cause the beam to deflect at the center, as shown in Fig. 9:2; the deflection stretches

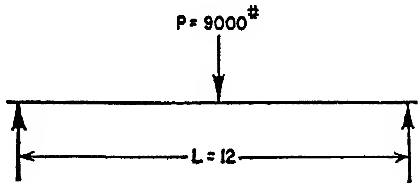


FIG. 9:1.

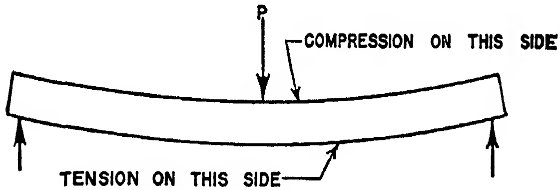


FIG. 9:2.

the fibers on the lower surface and compresses those on the upper surface. Somewhere between these two extremes will be a place where there is neither stretching nor compression; this place is known as the "neutral axis," the location of which depends on the cross section of the beam.

The stress due to this bending is not a uniform stress, but varies in intensity from a maximum at the extreme fiber to zero at the neutral axis.[2] The maximum stress is given by the equation

$$f_b = \frac{My}{I} \tag{9:1}$$

[1] See C.A.B. 27, p. 3 and 7, for a brief discussion of bending stresses.

[2] See C.A.B. 27, p. 10, for a discussion of stress distribution.

where f_b = bending stress
M = applied bending moment
y = distance from neutral axis to extreme fiber
I = moment of inertia

In the example illustrated by Fig. 9:1, the maximum bending moment is $PL/4$. No time will be spent in this text on the determination of bending moment equations; any standard engineering reference book has tables of bending moments for various beams and types of loading conditions. However, it is necessary to learn how to determine the moment of inertia I and the distance from the neutral axis to the extreme fiber y. No attempt will be made to define the complicated term "moment of inertia," but its use will be studied.

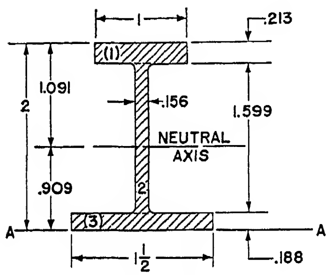


FIG. 9:3.

9:2. Determination of Neutral Axis. Assume that the beam of Fig. 9:1 has a constant cross section as shown in Fig. 9:3. First, determine the location of the neutral axis in relation to the base line A-A. This is done by dividing the section into a convenient number of rectangles and taking moments about this base line. The rectangles are numbered (1), (2), and (3) for convenience in identifying the parts.

By calculating the area of each rectangle and multiplying that area by the distance of its c.g. from the base line A-A, a moment is obtained. The sum of these moments divided by the sum of the individual areas gives the distance of the c.g. above the base line. This is shown in the accompanying calculations. To be mathematically correct, it should be written:

Breadth × height = area. Area × arm = moment

However, instead of repeating the column for area, it is convenient to write it in the following form:

Rectangle	Breadth × height	= area × arm	= moment
(1)	1.000 × 0.213	= 0.213 × 1.894 =	0.403
(2)	0.156 × 1.599	= 0.249 × 0.988 =	0.246
(3)	1.500 × 0.188	= 0.282 × 0.094 =	0.027
		0.744 × 0.909 =	0.676

0.909 = neutral axis of entire section

9:3. Determination of Moment of Inertia. The inertia I is calculated from

$$I = \Sigma I_0 + \Sigma A d^2 \tag{9:2}$$

where Σ = symbol denoting "summation"
I_0 = inertia of each rectangle about its own neutral axis
A = area of each rectangle
d = distance of neutral axis of rectangle from neutral axis of entire section

For a rectangle,[1]

$$I_0 = \frac{bh^3}{12}$$

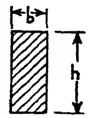


FIG. 9:4.

where b = breadth
h = height (see Fig. 9:4)

To calculate $I = \Sigma I_0 + \Sigma A d^2$, prepare tables as shown below. It is customary in work of this nature to omit the area of the fillets.

Rectangle

(1) $$I_0 = \frac{1.000 \times 0.213^3}{12} = 0.0008$$

(2) $$I_0 = \frac{0.156 \times 1.599^3}{12} = 0.0532$$

(3) $$I_0 = \frac{1.500 \times 0.188^3}{12} = 0.0008$$

$$\Sigma I_0 = 0.0548$$

Rectangle	Area $\times$ d^2	= Ad^2
(1)	0.213×0.985^2	= 0.2065
(2)	0.249×0.079^2	= 0.0016
(3)	0.282×0.815^2	= 0.1872
	$\Sigma A d^2$	= 0.3953
	ΣI_0	= 0.0548
	I	= 0.4501

The student should notice in the above calculations for I_0 that the values for rectangles (1) and (3) are practically negligible. Areas such as these are usually neglected in calculations.

[1] From "Machinery's Handbook," 10th ed., p. 344, or many other engineering reference books.

As mentioned in Art. 9:1, the bending moment at the center of the beam is $PL/4$. Substituting the values given in Fig. 9:1, we have

$$M = \frac{PL}{4} = \frac{9000 \times 12}{4} = 27,000 \text{ in.-lb.}$$

The maximum tensile stress due to bending may now be found. As was pointed out in Art. 9:1, the maximum stress is at the extreme fiber; the lower edge of the beam is stretched, owing to the deflection from load P; hence it is in tension.

Substituting the values in Eq. (9:1), we have

$$f_b = \frac{My}{I} = \frac{27,000 \times 0.909}{0.4501} = 54,500 \text{ psi tension}$$

To find the maximum compressive stress, use for y the distance from the neutral axis to the extreme fiber on the upper surface, since the upper surface is in compression.

Again substituting in Eq. (9:1).

$$f_b = \frac{My}{I} = \frac{27,000 \times 1.091}{0.4501}$$
$$= 65,400 \text{ psi compression}$$

Another examination of the section in Fig. 9:3, shows that there is considerably less metal on the upper than on the lower surface; hence, it is reasonable to expect that the stresses would be higher on the upper surface where there is less area. In solving for f_b, the compression stresses were found to be higher than the tension stresses.

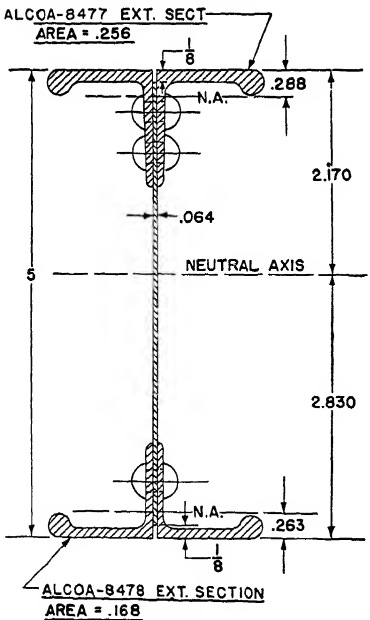


FIG. 9:5.

Now apply this to a typical form of aircraft construction as shown in Fig. 9:5. Here aluminum alloy extruded sections form the cap material, and an Alclad sheet is the web. Actually, the area removed by the rivet holes on the tension

side of the beam should be deducted, but it is common practice to disregard these rivet holes, as they are such a small per cent of the total area. Hence, no deductions will be made for them in this example. The areas and the location of the centers of gravity for the extruded sections may usually be found in the Standards Book or the Extrusion Catalog of the Aluminum Company of America. In some cases, it may be necessary for the layout man to calculate the area and location of the center of gravity himself. In this problem, the area and location of the c.g. of each extruded section are given in order not to complicate the work (see Appendix).

First, find the neutral axis of the entire section.

		Area $\times$ arm	= moment
Upper caps:	2×0.256	$= 0.512 \times 4.712 =$	2.412
Web:	0.064×4.750	$= 0.304 \times 2.500 =$	0.760
Lower caps:	2×0.168	$= 0.336 \times 0.263 =$	0.088
		$1.152 \times 2.830 =$	3.260

2.830 = neutral axis of entire section

Now calculate

$$I = \Sigma I_0 + \Sigma A d^2$$

$$I_0 \text{ upper caps} = 2 \times 0.0262 = 0.0524$$

$$I_0 \text{ web} = \frac{0.064 \times 4.75^3}{12} = 0.5714$$

$$I_0 \text{ lower caps} = 2 \times 0.0171 = 0.0342$$

$$\Sigma I_0 = 0.6580$$

$$Ad^2 \text{ upper caps} = 0.512 \times 1.882^2 = 1.814$$

$$Ad^2 \text{ web} = 0.304 \times 0.330^2 = 0.036$$

$$Ad^2 \text{ lower caps} = 0.336 \times 2.567^2 = 2.212$$

$$\Sigma Ad^2 = 4.062$$

$$\Sigma I_0 = 0.658$$

$$I = 4.720$$

Assume that this beam has a bending moment of 85,000 in.-lb., which causes compression in the upper cap and tension in the lower cap. Find f_b at the extreme fibers.

$$f_b = \frac{My}{I} = \frac{85{,}000 \times 2.83}{4.72} = 51{,}000 \text{ psi tension}$$

$$f_b = \frac{My}{I} = \frac{85{,}000 \times 2.17}{4.72} = 39{,}000 \text{ psi compression}$$

At times, it is necessary to determine the neutral axis about a vertical line, such as *C-D* in Fig. 9:6, and to determine the moment of inertia about the vertical neutral axis. This is done using the same system as was used to determine the horizontal neutral axis and the moment of inertia about it.

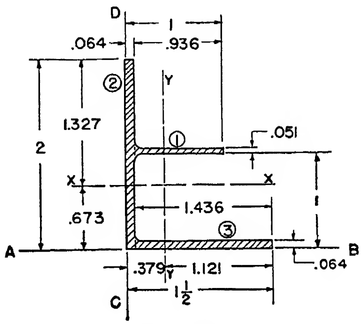


FIG. 9:6.

First, determine the horizontal neutral axis and the moment of inertia.

Rectangle		Area	× arm	= moment
(1)	0.936 × 0.051	= 0.0478	× 1.026	= 0.0491
(2)	0.064 × 2.000	= 0.1280	× 1.000	= 0.1280
(3)	1.436 × 0.064	= 0.0919	× 0.032	= 0.0029
		0.2677	× 0.673	= 0.1800

0.673 = neutral axis of entire section

I_0 rectangle (1) = neglect

$$I_0 \text{ rectangle (2)} = \frac{0.064 \times 2^3}{12} = 0.043$$

I_0 rectangle (3) = neglect

$$\Sigma I_0 = 0.043$$

Rectangle	Area × d^2	= Ad^2
(1)	0.0478 × 0.353^2	= 0.006
(2)	0.1280 × 0.327^2	= 0.014
(3)	0.0919 × 0.641^2	= 0.038

$$\Sigma Ad^2 = 0.058$$
$$\Sigma I_0 = 0.043$$
$$I_{x\text{-}x} = 0.101$$

Note: $I_{x\text{-}x}$ means that this is the value of the moment of inertia taken about the x-x axis. The horizontal axis is usually referred to as the x axis, whereas the vertical axis is referred to as the y axis.

Now, determine the vertical neutral axis and the moment of inertia about that axis which will be referred to as $I_{y\text{-}y}$

Rectangle		Area	× arm	= moment
(1)	$0.051 \times 0.936 =$	0.0478	$\times 0.532 =$	0.0254
(2)	$2.000 \times 0.064 =$	0.1280	$\times 0.032 =$	0.0041
(3)	$0.064 \times 1.436 =$	0.0919	$\times 0.782 =$	0.0719
		0.2677	$\times 0.379 =$	0.1014

$0.379 =$ neutral axis of entire section

$$I_0 \text{ rectangle (1)} = \frac{0.051 \times 0.936^3}{12} = 0.0035$$

$$I_0 \text{ rectangle (2)} = \text{neglect}$$

$$I_0 \text{ rectangle (3)} = \frac{0.064 \times 1.436^3}{12} = 0.0158$$

$$\Sigma I_0 = 0.0193$$

Rectangle	Area × d^2	= Ad^2
(1)	$0.0478 \times 0.153^2 =$	0.0011
(2)	$0.1280 \times 0.347^2 =$	0.0154
(3)	$0.0919 \times 0.403^2 =$	0.0149
	$\Sigma Ad^2 =$	0.0314
	$\Sigma I_0 =$	0.0193
	$I_{y\text{-}y} =$	0.0507

Comparison of the values of $I_{x\text{-}x}$ and $I_{y\text{-}y}$ shows that the moment of inertia about the y-y axis is less than about the x-x axis. This means that the section is stronger about the x-x axis, which is apparent from examining the section in Fig. 9:6.

9:4. Allowable Bending Stresses. In figuring the margin of safety on a beam in bending, the layout man should not use the ultimate stresses (F_{cu} and F_{tu}) as his allowable without first consulting the stress department. Usually, the "modulus of rupture in bending" is determined experimentally; and this value is used as the allowable stress in the calculations for the margin of safety. In the absence of more specific information or for a rough check, the layout man may use the ultimate stress in his margin of safety calculations.

9:5. Deductions for Holes. In Fig. 9:5, the rivet holes were neglected in the calculations because, on the compression or upper side of the beam, the rivets could be considered as completely filling the hole, owing to their swelling on being driven. But on the lower or tension side, the rivet holes should be deducted from the total area. As explained previously, this reduction in area and inertia is often neglected when the holes are small as the error introduced is negligible.

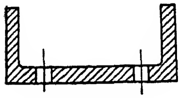


Fig. 9:7.

However, there are many times when the layout man should be careful to deduct for rivet or bolt holes, in his calculations. A simple case would be a channel fitting in tension attached by means of bolts or rivets, as shown in Fig. 9:7.

In the strength-check calculations, the net area should be used; the net area being the area cross-sectioned in Fig. 9:7.

Another instance where the layout man should be careful to make proper deductions for rivet or bolt holes is in heavy spar caps where the area removed for such holes is great. When in doubt as to the area to use, either check with the stress department or use the net area; the net area is the total area less the area of any holes such as for bolts, rivets, or lightening.

9:6. Calculations for Minor Changes in a Section. When the properties of a section have been calculated and it is found

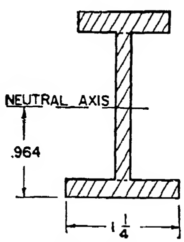


Fig. 9:8.

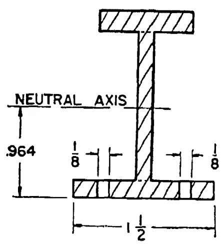


Fig. 9:9.

necessary to make changes for any reason, it is unnecessary to recalculate the section completely. By using the existing figures and making proper corrections, the new neutral axis and the revised moment of inertia may be readily found, and much time saved in many cases.

For example, it was found necessary to reduce the width of the bottom flange of Fig. 9:3 from $1\frac{1}{2}$ to $1\frac{1}{4}$ in., to provide clearance; this new section is shown in Fig. 9:8. The calculations for

this new section will be the same as if it had been decided to deduct from the original section two ⅛-in.-diameter rivet holes, as shown in Fig. 9:9. This is because the net width of the bottom flange in both figures is the same. The dimensions not shown in these figures are identical to the dimensions shown in Fig. 9:3.

		Area × arm = moment
From calculations in Art. 9:2,		$0.744 \times 0.909 = 0.676$
deduct ¼ in. from lower flange:	$-0.188 \times 0.25 =$	$-0.047 \times 0.094 = -0.004$
		$0.697 \times 0.964 = 0.672$

0.964 = revised neutral axis

To calculate the revised Ad^2, using the corrected neutral axis,

$$\text{Area} \times d^2 = ad^2$$

$$0.213 \times 0.930^2 = 0.1842$$
$$0.249 \times 0.024^2 = 0.0001$$
$$(0.282 - 0.047) = 0.235 \times 0.870^2 = 0.1779$$
$$\Sigma Ad^2 = 0.3622$$
$$\Sigma I_0 = 0.0548 \text{ (not enough change to read on slide rule)}$$
$$I = 0.4170$$

$$f_b = \frac{27{,}000 \times 0.964}{0.417} = 62{,}450 \text{ psi tension}$$

$$f_b = \frac{27{,}000 \times 1.036}{0.417} = 67{,}100 \text{ psi compression}$$

The student should notice that the stresses on the upper or compression side changed but little; however, the stresses on the lower or tension side changed considerably. This is logical as the material was removed from the lower flange; hence the remaining material would have to carry more load. The stresses on the upper side are increased slightly, owing to the shifting of the neutral axis.

As mentioned earlier in this article, the calculations for the section shown in Figs. 9:8 and 9:9 will be identical since the net width of the lower flange is the same for both sections. That is, when flange material is removed as is indicated by the rivet holes in Fig. 9:9, the calculations are the same as if the flange width itself had been cut down.

Now consider a case where a hole is added to the web of Fig. 9:3, as shown in Fig. 9:10. When material is removed from a vertical web, the net height cannot be used in the calculations. *Net widths* may be considered but *never net heights*. This is because of the fact that in the equation

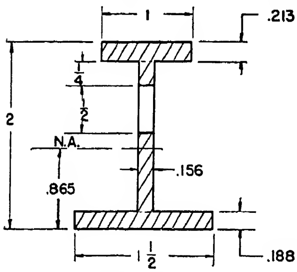


FIG. 9:10.

$$I_0 = \frac{bh^3}{12} \text{ (see Art. 9:3)}$$

the inertia varies directly to the breadth but as to the cube of the height. The basic dimensions for Fig. 9:10 are the same as those for Fig. 9:3; however, a hole ½ in. in diameter has been added to the web. In the calculations for Fig. 9:10, a net height of 1½ in. cannot be used but the depth of 2 in. must be held, making the necessary deductions to compensate for this hole.

		Area × arm = moment
From the calculations in Art. 9:2		$0.744 \times 0.909 = 0.676$
deduct for hole in web:	$-0.156 \times 0.5 =$	$-0.078 \times 1.287 = -0.100$
		$0.666 \times 0.865 = 0.576$

$0.865 =$ revised neutral axis

From calculations in Art. 9:3 $\quad \Sigma I_0 = 0.0548$

deduct I_0 of hole: $\quad -\dfrac{0.156 \times 0.50^3}{12} = -0.0016$

Revised $\Sigma I_0 = 0.0532$

Now calculate the revised Ad^2 using the corrected neutral axis. Figure 9:10 is divided into the same rectangles as Fig. 9:3; this is necessary in order to use some of the original calculations made for Fig. 9:3.

$$\begin{aligned}
\text{Area} \times d^2 &= Ad^2 \\
0.213 \times 1.029^2 &= 0.225 \\
0.249 \times 0.123^2 &= 0.004 \\
0.282 \times 0.771^2 &= 0.168 \\
-0.078 \times 0.422^2 &= -0.014 \text{ (deduct for hole)} \\
\Sigma Ad^2 &= 0.383 \\
\Sigma I_0 &= 0.053 \\
I &= 0.436
\end{aligned}$$

Comparing this value with the moment of inertia of the original section, it may be seen that it has decreased but little; this is logical, as material removed from the web changes the inertia slightly, owing to its proximity to the neutral axis. By placing the hole directly on the neutral axis, the change in the moment of inertia would be negligible since the distance between the center of the hole and the neutral axis of the entire section would be zero; hence Ad^2 would also be zero. The only change would be the I_0 of the hole itself which may safely be neglected in the majority of cases.

Whenever metal is to be removed, it should be taken from the web if possible. When holes are to be added, it is desirable, from a strength standpoint, to have the center of the hole coincide with the neutral axis of the entire section.

9:7. Combined Stresses.[1] Quite often bending stresses are accompanied by other stresses such as tension or torsion, or both. Combined stresses may become exceedingly complicated; hence they are omitted. The average layout man will seldom have such complex problems to solve. In the event that he is confronted by such a condition, he should refer to ANC-5, Niles and Newell, or the stress department, preferably the last.

9:8. Centroid of a Trapezoid. It often happens that the layout man must find the centroid of a trapezoid, as shown in Fig. 9:11. To do this, he may use the following equation:[2]

$$y = \frac{d(a + 2b)}{3(a + b)} \qquad (9:3)$$

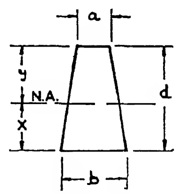


FIG. 9:11.

where the letters refer to the dimensions in Fig. 9:11.

To determine x, we may say that

$$x = d - y$$

Substituting the value for y from Eq. (9:3),

$$x = d - \frac{d(a + 2b)}{3(a + b)}$$

[1] See C.A.B. 27, pp. 8 and 9, for a discussion of combined stresses.

[2] "Machinery's Handbook," 10th ed., p. 346.

In order to obtain a common denominator, multiply the first term in the right-hand side of the equation by

$$\frac{3(a+b)}{3(a+b)}$$

which gives

$$x = \frac{3d(a+b)}{3(a+b)} - \frac{d(a+2b)}{3(a+b)}$$

which may be written

$$x = \frac{3d(a+b) - d(a+2b)}{3(a+b)}$$

Removing the parentheses in the numerator, we have

$$x = \frac{3ad + 3bd - ad - 2bd}{3(a+b)}$$

Combining,

$$x = \frac{3ad - ad + 3bd - 2bd}{3(a+b)} = \frac{2ad + bd}{3(a+b)}$$

Factoring out d in the numerator,

$$x = \frac{d(2a+b)}{3(a+b)} \tag{9:4}$$

To determine the neutral axis of the trapezoid shown in Fig. 9:12, substitute the dimensions shown in Eq. (9:3) or (9:4). Equation (9:4) will arbitrarily be chosen. Substituting values,

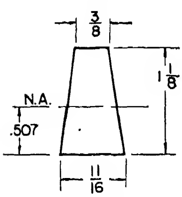


FIG. 9:12.

$$x = \frac{1.125(2 \times 0.375 + 0.6875)}{3(0.375 + 0.6875)}$$

$$\frac{1.125(1.4375)}{3(1.0625)} = 0.507$$

Another convenient way of determining the centroid of a trapezoid is to use the table given on page 57 of Niles and Newell, Vol. I. In this example, use the ratio of $^{11}\!/_{16}$ to $^{3}\!/_{8}$, thus,

$$\text{Ratio} = \frac{0.6875}{0.375} = 1.833$$

From the table in Niles and Newell, it may be found that when the ratio is 1.80, distance $x = 0.4523$. Interpolation of values for x between 1.80 and 1.85 gives $x = 0.4509$, when the ratio is 1.833.
Substituting this in the example,

$$x = 0.4509 \times 1.125 = 0.507$$

The moment of inertia of a trapezoid may be found in many standard engineering reference books; also see Art. 10:9, Eqs. (10:3) and (10:4).

9:9. Torsional Stresses—Round Solid Sections.[1] Torsion is a twisting or rotational force. A shaft driven by a pulley is in torsion. In aircraft, many members in the flight controls system are in torsion; hence the layout man should have a basic understanding of the principles of torsion and of the methods used to determine torsional stresses and deflections. Figure 9:13 is the end view of a solid shaft having an arm attached to it. There is a load P acting on the arm at a distance L from the center of the shaft. The twisting moment or "torque" is given as

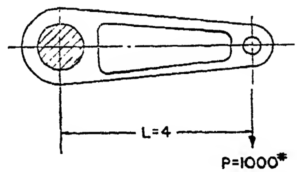


FIG. 9:13.

$$T = PL \tag{9:5}$$

where T = torsional moment in inch-pounds
P = applied load in pounds
L = distance of applied load from center of shaft in inches

If the other end of the shaft has an opposite torsional moment (which it will have if the shaft is in equilibrium), each cross section of this solid shaft is subjected to a shearing stress given by the equation

$$f_s = \frac{Tr}{I_p} \tag{9:6}$$

where T = torsional moment
r = radius of shaft
I_p = polar moment of inertia

In torsion, as in bending, the intensity of the shearing stress is not uniform throughout the cross-sectional area. The maximum

[1] See C.A.B. 27, p. 4, for a discussion of torsional stresses.

stress occurs at the extreme fiber, and it is the maximum stress in which the designer is interested. In Eq. (9:6), the radius of the shaft is used which gives the stress at the extreme fiber.

In[1] ANC-5, this equation is given in more general terms, as

$$f_s = \frac{Ty}{I_p} \tag{9:6a}$$

where y is the distance from the neutral axis (center of the circular shaft) to the given fiber.

In practically all cases, the layout man will not be concerned with the stresses at any point except at the extreme fiber; hence Eq. (9:6) will be the one most commonly used.

The polar moment of inertia of any section is the sum of the moments of inertia about its principal axes. Tables of polar moments of inertia for various sections are not given in this text for they may be obtained from any standard engineering reference book. For a solid circular shaft[2]

$$I_p = \frac{\pi D^4}{32} \tag{9:7}$$

where D = outside diameter of shaft.

In Fig. 9:13, the shaft is 1 in. in diameter. Substituting this value in Eq. (9:7) gives

$$I_p = \frac{\pi \times 1^4}{32} = \frac{\pi}{32} = 0.0982$$

Solving for the torsional moment, substitute the values given in Fig. 9:13 into Eq. (9:5).

$$\begin{aligned} T &= PL \\ &= 1000 \times 4 = 4000 \text{ in.-lb.} \end{aligned}$$

Solving for the shearing stress due to this torque, substitute the values in Eq. (9:6)

$$f_s = \frac{Tr}{I_p} = \frac{4000 \times 0.5}{0.0982} = 20{,}350 \text{ psi}$$

To determine the margin of safety, the allowable stress for a solid circular shaft is the torsional modulus of rupture F_{st}.

[1] Eq. 1:6, p. 1-7.

[2] "Machinery's Handbook," p. 490, 10th ed.

Assume this bar to be 24ST aluminum alloy. From page 5-9 of ANC-5, F_{st} = 45,000 psi is found. This is one of the few errors in ANC-5, and it is suggested the student immediately correct it to read 56,000 psi (instead of 45,000) as the former figure is believed to be nearer the correct value. However, this value should be verified by the stress department of the company in which the student is employed.

$$F_{st} = 56{,}000$$
$$\text{M.S.} = \frac{56{,}000}{20{,}350} \quad 1 = 2.75 - 1 = 1.75$$

9:10. Torsional Stresses—Round Aluminum Alloy Tubes. The calculation of the shearing stress due to torsion in a round tube is more complicated. Figure 9:14 shows a round tube where D is the outside diameter and t is the wall thickness. To illustrate: Consider the problem shown in Fig. 9:13 again. This time use a 1-in.-diameter 24ST aluminum alloy tube having a wall thickness of 0.065. The torque on this shaft remains the same no matter what is used for the shaft.

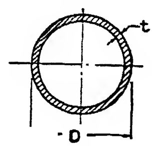


Fig. 9:14.

As might be expected, the polar moment of inertia will be different for the tube. A convenient way of determining the polar moment of inertia is from the table on page 312 of Niles and Newell, Vol. I.[1] In it is given the moment of inertia for a large range of diameters and wall thicknesses, with a footnote explaining that the polar moment of inertia will be twice the value given in the table. Referring to this table for a 1-in.—0.065 tube, we find

$$I = 0.02097$$
$$I_p = 2 \times 0.02097 = 0.04194$$

Now substituting in Eq. (9:6) gives

$$f_s = \frac{4000 \times 0.5}{0.04194} = 47{,}700 \text{ psi}$$

In the determination of the margin of safety, F_{st} cannot be used without a correction factor being applied to compensate for the

[1] Also see p. 8-1 of ANC-5.

tendency of the wall to collapse. In actual practice, use tables from ANC-5 to determine the allowable torsional modulus of rupture.

Returning to the problem,

$$D = 1$$
$$t = 0.065$$

The ratio of D/t gives

$$\frac{D}{t} \qquad 0.065 = 15.38$$

Since the tube is 24ST aluminum alloy, use the graph on page 5-19 of ANC-5. Find the value of D/t (which was 15.38) along the lower scale. Following up until crossing the curve for 24ST tube and carrying across to the left margin, the allowable stress is found to be 36,200 psi.

Now the margin of safety may be determined.

$$\text{M.S.} = \frac{36{,}200}{47{,}700} - 1 = 0.759 - 1.000 = -0.241$$

As this is a negative margin of safety, it is obvious that it is unsatisfactory.

Now try a 1 in.—0.095 aluminum alloy tube (24ST). Using the same procedure,

$$I_p = 2 \times 0.02796 = 0.05592$$
$$f_s = \frac{4000 \times 0.5}{0.05592} = 35{,}780 \text{ psi}$$
$$\frac{D}{t} \qquad 0.095 = 10.52$$

From page 5-19 of ANC-5,

$$\text{Allowable } F_{st} = 42{,}000 \text{ psi}$$

therefore,

$$\text{M.S.} = \frac{42{,}000}{35{,}780} - 1 = 1.174 - 1 = 0.174$$

As this is a positive margin of safety, it will be considered satisfactory.

9:11. Torsional Stresses—Round Steel Tubes. In the design of steel tubes, there is still another factor to consider in the

determination of the allowable stress. This is the ratio of the free length to the outside diameter and is expressed as L/D.

By free length is meant the distance between the points where the torque is applied and where it is resisted by an equal and opposite torque.

Now return to Fig. 9:13 and work through the problem again using a 1 in.—0.049 C.M. steel tube. Assume the length to be 20 in.

$$I_p = 2 \times 0.016594 = 0.033188$$
$$f_s = \frac{4000 \times 0.5}{0.033188} = 60{,}300 \text{ psi}$$
$$\frac{D}{t} = \frac{1}{0.049} = 20.41$$
$$\frac{L}{D} = \frac{20}{1} = 20$$

Refer to the graph on page 4-37 of ANC-5. Find the value of D/t on the lower border. Follow up until crossing the line $L/D = 20$. Read across to the left and find a constant 0.57.

The allowable stress (torsional modulus of rupture) is the ultimate tensile stress times this constant. In this example, F_{tu} for C.M. steel is 95,000 (from ANC-5, page 4-11, column 2). Therefore,

$$\text{Allowable stress} = 95{,}000 \times 0.57 = 54{,}200 \text{ psi}$$
$$\text{M.S.} = \frac{54{,}200}{60{,}300} - 1 = 0.898 - 1.000 = -0.102$$

As this is a negative margin of safety, it is obviously unsatisfactory.

Now try a 1 in.—.058 C.M. steel tube, the free length remaining the same.

$$I_p = 2 \times 0.019111 = 0.038222$$
$$f_s = \frac{4000 \times 0.5}{0.038222} = 52{,}300 \text{ psi}$$
$$\frac{D}{t} = \frac{1}{0.058} = 17.24$$
$$\frac{L}{D} = \frac{20}{1} = 20$$

From page 4-37 of ANC-5, the constant is found to be 0.59.

Therefore,

$$\text{Allowable stress} = 0.59 \times 95{,}000 = 56{,}050 \text{ psi.}$$

$$\text{M.S.} = \frac{56{,}050}{52{,}300} - 1 = 1.072 - 1 = 0.072$$

As this is a positive margin of safety, it will be considered satisfactory for the example; however, the layout man should always remember to obtain his required margins of safety from the stress department.

In the design of steel tubes in torsion, it is preferable to keep the L/D ratio to 20 or less. For high L/D ratios the deflections become excessive.

9:12. Torsional Deflection—Round Sections. Another fact the designer must bear in mind is that *torsional deflections should be kept to a minimum*, which is particularly true in flight controls. Deflection of circular sections is given by the equation

$$\phi = \frac{TL}{GI_p} \tag{9:8}$$

where ϕ = angle of twist in radians
T = torsional moment in inch-pounds
L = free length in inches
G = modulus of elasticity in shear
I_p = polar moment of inertia

In ANC-5, this equation is given in another form:

$$\phi = \frac{TL}{GJ} \qquad 9{:}8a)$$

where J = torsional constant.

For circular sections, J becomes I_p. It is recommended that students use the form given in Eq. (9:8), which is exceedingly convenient.

It should be noted that the angle of twist ϕ is given in radians. Of course, for practical work, this angle must be converted to degrees.

$$1 \text{ radian} = 57.2958 \text{ deg.}$$

The modulus of elasticity in shear G is found in ANC-5.

Return again to Fig. 9:13 with the 1-in. diameter solid 24ST aluminum alloy shaft. The torsional moment was 4000 in.-lb.

The free length was 20 in. The modulus of elasticity in shear (same as the modulus of rigidity) G is found on page 5-9 of ANC-5, and is 3,800,000 psi. I_p was found to be 0.0982 (see Art. 9:9). Substituting these values in Eq. (9:8) gives

$$\begin{aligned} \phi &= \frac{TL}{GI_p} \\ &= \frac{4000 \times 20}{3{,}800{,}000 \times 0.0982} = 0.2141 \text{ radian} \\ &= 0.2141 \times 57.2958 = 12.26 \text{ deg.} \end{aligned}$$

Now consider the 1 in.—0.065 24ST aluminum alloy tube. Substituting the values in Eq. (9:8),

$$\begin{aligned} \phi &= \frac{4000 \times 20}{3{,}800{,}000 \times 0.04194} = 0.502 \text{ radian} \\ &= 0.502 \times 57.2958 = 28.78 \text{ deg.} \end{aligned}$$

Investigating the 1 in.—0.049 C.M. steel tube,

$$\begin{aligned} \phi &= \frac{4000 \times 20}{11{,}000{,}000 \times 0.033188} = 0.219 \text{ radian} \\ &\quad 0.219 \times 57.2958 = 12.56 \text{ deg.} \end{aligned}$$

In this case, the value for G is found on page 4-11 of ANC-5 in column 2.

In control systems, the torsional deflection should never be more than 20 per cent of the normal rotation of the tube. It is preferable to keep the torsional deflection down to 5 per cent of the normal rotation, but this often necessitates such large tubes that it is impractical from the standpoint of weight as well as lack of space to permit the installation of such a large tube.

9:13. Open Sections Subjected to Torsional Loads. At times, the designer may be forced to use some other shape in torsion; if possible, he should use a closed section. On page 188 of Vol. I, Niles and Newell, are a number of formulas for torsion on symmetrical sections. The stress f_s is the actual stress in the member, due to the torque T. For allowable stresses, the designer should use the torsional modulus rupture F_{st} from the tables in ANC-5 for his particular material. However, *this applies only to the solid sections.* For the open sections, the ellipse

and the rectangle, the layout man should consult the stress department for the allowable stress. Niles and Newell also have equations for the determination of angular twist for all sections except the open rectangle. This was omitted as the formula is too complex to be practical.

For thin-shelled sections such as a wing or fuselage, refer to page 189 of Niles and Newell, in particular to Eqs. (7:5) and (7:6) on that page. Here again the actual stress is given, and the designer should consult the stress department for the allowable stress.

Open sections are particularly undesirable in torsion, owing to high stresses and excessive torsional deflections. Niles and Newell, page 192, have information pertaining to the stresses and torsional deflection of open sections. The portion of Art. 7:7 on page 192, containing Eqs. (7:8) and (7:9) with the explanation and use of these formulas, should be studied along with this article.

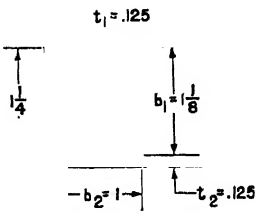


FIG. 9:15.

To illustrate the poor torsional characteristics of an open section, compare the stress and the angular deflection of an extruded angle with that of a tube of the same material and cross-sectional area. Choose the 1 × 1¼-in. 24ST extruded angle Alcoa-734H-H, shown in Fig. 9:15, having an area of 0.267 sq. in.

From Eq. (7:8) on page 192 of Niles and Newell,

$$f_s = \frac{3T}{b_1 t_1^2 + b_2 t_2^2}$$

Substituting,

$$f_s = \frac{3T}{1.125 \times 0.125^2 + 1 \times 0.125^2}$$

Factoring the denominator gives

$$\begin{aligned} f_s &= \frac{3T}{0.125^2(1.125 + 1)} \\ &= \frac{3T}{0.01562 \times 2.125} = \frac{T}{0.0332} \\ &= 90.3T \end{aligned}$$

From Eq. (7:9) on page 192 of Niles and Newell,

$$\theta = \frac{3TL}{(b_1t_1^3 + b_2t_2^3)G}$$

Substituting the values for this example,

$$\theta = \frac{3TL}{(1.125 \times 0.125^3 \times 1 \times 0.125^3)G}$$

Factoring the denominator gives

$$\begin{aligned}\theta &= \frac{3TL}{0.125^3(1.125 \quad 1)G} \\ &= \frac{3TL}{0.001953 \times 2.125G} - \frac{3TL}{0.00415G} \\ &= 723\,\frac{TL}{G}\end{aligned}$$

Niles and Newell use the Greek letter theta (θ) to represent the angular deflection in radians; ANC-5 uses phi (ϕ).

Having determined the maximum stress and the angular deflection of the extruded angle, now determine the same characteristics of a tube having the same area as the angle, which was 0.267 sq. in.

Consider a 1½—0.058 tube which has an area of 0.263 sq. in., and $I_p = 0.0684 \times 2 = 0.1368$. From Eq. (9:6),

$$f_s = \frac{Tr}{I} = \frac{0.75T}{0.1368} = 5.48T$$

From Eq. (9:8),

$$\phi = \frac{TL}{GI_p} \quad \frac{TL}{0.1368G} = 7.31\,\frac{TL}{G}$$

Comparing the angle and the tube of equal cross section,

	Angle	Tube
F_s.	$90.3T$	$5.48T$
	$723\,\frac{TL}{G}$	$7.31\,\frac{TL}{G}$

That is, the stress in the angle is 16 times greater than in the tube; the angle deflects 99 times more than the tube.

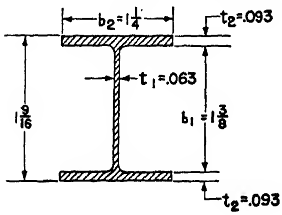


FIG. 9:16.

Now compare an H or I section in torsion with a tube having the same area. Figure 9:16 shows such a section. Its area is

$$
\begin{aligned}
2 \times 0.093 \times 1.25 &= 0.232 \\
1.375 \times 0.063 &= \underline{0.087} \\
& 0.319 \text{ sq. in.}
\end{aligned}
$$

From Niles and Newell, Vol. I, page 192,

$$
\begin{aligned}
f_s(\text{max.}) &= \frac{3T}{b_1t_1^2 + 2b_2t_2^2} = \frac{3T}{1.375 \times 0.063^2 + 2 \times 1.25 \times 0.093^2} \\
&= \frac{3T}{0.0054 + 0.0216} = \frac{3T}{0.027} = 111.1T \\
\theta &= \frac{3TL}{(b_1t_1^3 + 2b_2t_2^3)G} = \frac{3TL}{(1.375 \times 0.063^3 + 2 \times 1.25 \times 0.093^3)G} \\
&= \frac{3TL}{(0.000344 + 0.002012)G} = \frac{3TL}{0.002356G} = 1274\frac{TL}{G}
\end{aligned}
$$

From ANC-5, page 8-1, it is found that a 1¾—0.058 tube has an area of 0.3083 sq. in. From the same table, we find

$$I_p = 0.11046 \times 2 = 0.22092$$

Therefore,

$$f_s = \frac{Tr}{I_p} = \frac{0.875T}{0.2209} = 3.96T$$

and

$$\phi = \frac{TL}{GI_p} = \frac{TL}{0.2209G} = 4.53\frac{TL}{G}$$

Comparing the I section with the tube,

	I section	Tube
f_s	$111.1T$	$3.96T$
ϕ	$1274\frac{TL}{G}$	$4.53\frac{TL}{G}$

Therefore, the stress in the I section is 28 times greater than the stress in the tube; the I section deflects 281 times more than the tube.

The reason why open sections are so poor in torsion is that the internal resisting forces tend to form a circle. In a tube, the metal is so placed that these resisting forces naturally fall within the walls of the tube. In open sections, there is no metal through which these internal forces can pass; hence they are very inefficient in resisting a torsional moment.

To summarize, it may be said, *never use an open section to carry torsion;* always use a tube if possible. The designer should choose as large a diameter as possible, keeping the wall thickness to a minimum. This enables him to maintain a high weight-strength ratio.

When it is said that the wall thickness should be kept to a minimum, the designer should not specify a tube where the D/t ratio is greater than 40, without first consulting the stress group. For high D/t ratios, the tube may fail in local buckling, which is undesirable.

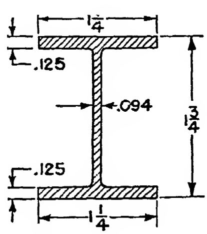
9:14. Selection of Tube Sizes. Tubing used in primary structures or in control systems should have no less than 0.035 wall, although lighter gages are permitted elsewhere. The designer should in no case specify a size and gage of tube not listed as a standard purchased size; these sizes may usually be obtained from the material release group. If the design is for a military plane, the designer should use only those sizes of tubes listed as AN standard; these standard sizes may be obtained from page 8-1 of ANC-5.

Reference

Niles and Newell, Vol. I, pp. 94–107, for tables giving bending moments for various types of beams.

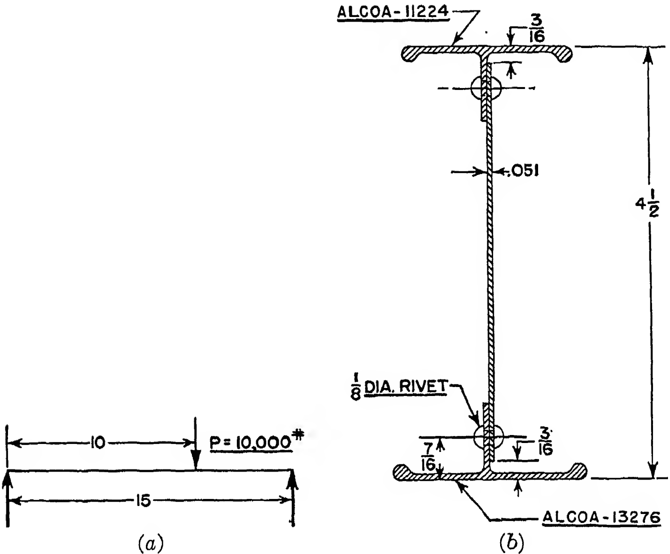
Problems

9:1. Determine I for the accompanying diagram.



PROB. FIG. 9:1.

9:10. Calculate the maximum f_b for the beam of constant cross section given in the accompanying diagrams, and note whether tension or compression exists. Neglect all rivet holes. The dimensions for the cap sections may be obtained from the Appendix.



PROB. FIG. 9:10.

9:11. Recalculate Prob. 9:10 deducting for the 1/8-in.-diameter rivet hole in the tension flange.

9:12. The torsional moment on a 24ST aluminum alloy shaft 1 1/8 in. in diameter is 12,500 in.-lb. Find f_s and the margin of safety.

9:13. What is the margin of safety of a 5/16-in.-diameter solid chromemoly steel rod subjected to 420 in.-lb. torque?

9:14. The twisting moment of a 1 1/4—0.095 24ST aluminum alloy tube is 8000 in.-lb. Find f_s and M.S.

9:15. What wall thickness should be specified for a 1 1/8-in.-diameter 24ST tube carrying 2400 in.-lb. torque? Any reasonable positive margin of safety is satisfactory. Determine ϕ when the length is 20 in.

9:16. A 1 1/2—.065 C.M. steel tube 18 in. long has an applied torsional moment of 11,000 in.-lb. What is the margin of safety? Determine ϕ.

9:17. What torque must be applied to a 1—0.035 C.M. steel tube 40 in. long to have a margin of safety equal to zero? Determine the angular deflection.

9:18. A 2-in.-diameter C.M. steel tube is subjected to a torsional moment of 11,000 in.-lb.

a. What wall thickness would be recommended in order to keep the torsional deflection down to 4 deg. or less when the free length is 11 in.?

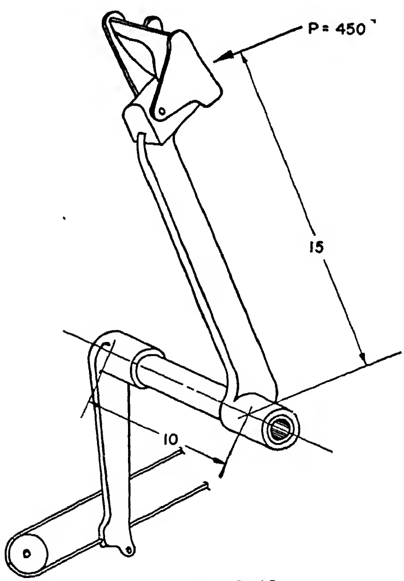
b. What is the torsional stress on the tube?

c. What will the actual torsional deflection be?

d. What is the torsional modulus of rupture?

e. What is the margin of safety? (Any reasonable positive margin will be satisfactory.)

9:19. The accompanying sketch shows a rudder pedal and its attachment to the cable system. The necessary dimensions and loads are given.



PROB. FIG. 9:19.

The torque tube connecting the pedal to the cable attaching arm is limited to 2 in. outside diameter. The torsional deflection of this tube must not exceed 4 deg.

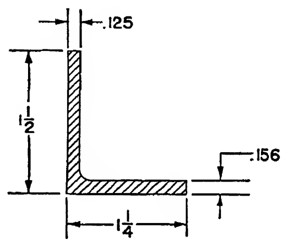
a. What is the lightest 24ST aluminum alloy tube that would be satisfactory?

b. What is the actual torsional deflection?

c. What is the torsional stress?

d. What is the margin of safety?

9:20. Determine f_s and ϕ for the 24ST extruded angle, shown in the accompanying diagram, when $T = 920$ in.-lb. and $L = 35$ in.



PROB. FIG. 9:20.

9:21. *a*. What wall thickness for a 1¾-in. O.D. 24ST tube should be specified, to have an area approximately equal to the area of the angle shown in Prob. 9:20? (Use only standard tube sizes as listed in Niles and Newell or in ANC-5, and use the tube having an area just smaller than the area of the angle.) Find A, I_p, and D/t for this tube.

b. Determine M.S. and ϕ for this tube when $T = 920$ in.-lb. and $L = 35$ in.

c. Prepare a table comparing the stress, the torsional deflection, and the area of the angle given in Prob. 9:20 and the tube chosen in Prob. 9:21*a*.

CHAPTER 10

STRENGTH OF STRUTS AND COLUMNS

10:1. Introduction.[1] "Column action" is something with which every designer should be familiar, for design problems will be often encountered where a strut or some member carries a compression load, and this member will have to be investigated for column action. It is customary to refer to members carrying compression as "struts" or "columns"; those carrying tension as "ties." This chapter will deal exclusively with members under a compressive force.

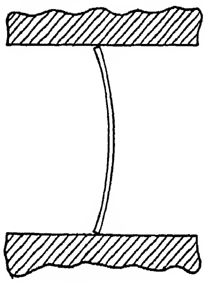


FIG. 10:1

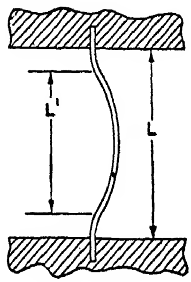


FIG. 10:2.

10:2. Coefficient of Fixity. Since the method of end attachment affects the problem considerably, the first step is for the designer to determine the coefficient of fixity, referred to as c. In Fig. 10:1, a column with no end restraint is shown, commonly referred to as "pin ended," although in many cases there is no actual pin, but some type of attachment that offers little or no restraint. A yardstick could be used as a very good example of this type of column. By resting one end on the floor and exerting a downward force on the other end with the palm of the hand, it will deflect under load very much as shown.

Figure 10:2 shows a beam restrained at the ends and its characteristic deflection. In aircraft structures, there are a number of

[1] See C.A.B. 27, pp. 5 and 6, for a discussion of column action.

types of connections that offer restraint to the extent that this restraint must be considered. For example, steel tubes welded at both ends as in a steel tube fuselage, or columns supported in a number of places between the points where the load is applied, are not treated as pin ended because of the restraint offered by the attachment. Although other examples could be cited, these two will be sufficient.

In other cases, although there is some restraint offered by the end attachment, the restraint is neglected and the strut is considered as pin ended. For example, the strut may be attached to the supporting members by heavy angles riveted to both strut and supporting member. These heavy attachment angles offer some restraint; however, it is customary to neglect it in the strength calculations. This is a very common type of attachment and one that the layout man will frequently encounter.

The coefficient of fixity c is 1 for pin-ended columns and 2 for restrained columns. This is usually expressed as

$c = 1$ for pin-ended columns
$c = 2$ for restrained columns

In theory, the maximum coefficient of fixity for a restrained column is 4; however, in aircraft design the maximum permissible value is $c = 2$. This is because it is practically impossible to have a beam restrained sufficiently to use any higher value. Although the ends may be welded, deflection is still possible under load; that is, the welded cluster of tubes at the ends will rotate under load.

The designer should seldom use a coefficient of fixity greater than 1 without first checking with the stress department.

10:3. Long and Short Columns. The length of the column is extremely important, and the ratio L/ρ is constantly used where L = length in inches and ρ (Greek letter rho) denotes the radius of gyration.

The radius of gyration is determined as follows:

$$\rho = \sqrt{\frac{I}{A}} \tag{10:1}$$

where I = moment of inertia

A = cross-sectional area of the column in square inches.

If the column is not symmetrical about both axes, such as an *H* column, there will be two values for the moment of inertia, one about the x-x axis and the other about the y-y axis; hence there will be two values for the radius of gyration. It is customary to use the least radius of gyration in all calculations; in fact, it is often referred to as the least radius of gyration.

As mentioned previously, the ratio L/ρ plays a very important part in the calculation for columns, and is commonly known as the "slenderness ratio." If the value for L/ρ is above a certain figure, the column is known as a "long column." Below this value, it is a "short column." This dividing line between short and long columns is known as the "critical L/ρ." Niles and Newell refer to this as the "transitional L/ρ." In this text, the term "critical L/ρ" will be used, as that is the term used in ANC-5; however, the student is cautioned to remember that "transitional" and "critical" L/ρ are one and the same thing.

Values for critical L/ρ may be obtained from pages 4-2 and 5-2 of ANC-5, or from the table on page 297 of Niles and Newell, Vol. I. As pointed out in the previous paragraph, when the actual slenderness ratio is above the critical values, it is a long column and falls within the Euler (pronounced "oiler") range. On the contrary, when the actual L/ρ is less than the critical values, it is a short column and is in the Johnson range. If the actual slenderness ratio falls exactly on the critical L/ρ, it is considered a long column. Euler and Johnson were two famous mathematicians who did a vast amount of original research on columns; hence, they are honored by having the formulas they developed named for them.

The student should notice that ANC-5, pages 4-2 and 5-2, refers to L'/ρ with a footnote explaining that

$$\frac{L'}{\rho} = \frac{L}{\rho\sqrt{c}} \tag{10:2}$$

This is presented in a slightly different manner in Niles and Newell; however, since ANC-5 is considered the standard reference for the stress department, L'/ρ and the various tables in ANC-5 will be used in the major part of this work. However, there are several nomograms in Niles and Newell that are very useful; hence the student should thoroughly understand the difference between L'/ρ and the L/ρ. He should be careful to

note whether the charts, curves, etc., are based on L'/ρ or L/ρ and calculate values accordingly. Fortunately, the majority of strut and column problems encountered will have $c = 1$, in which case the radical drops out of Eq. (10:2) and gives $L'/\rho = L/\rho$. That is, when $c = 1$,

$$\frac{L'}{\rho} = \frac{L}{\rho\sqrt{1}} = \frac{L}{\rho}$$

The difference between L and L' is graphically shown in Fig. 10:2. When the $\sqrt{c}$ is introduced into the L/ρ factor, the effective column length as shown in Fig. 10:2 is actually being determined. That is, a restrained column of length L has the same strength as a pin-ended column of length L'.

It is desirable to keep L'/ρ as low as possible; in no case should it exceed 150.

10:4. Types of Column Failure. There are two different ways in which a member may fail under compression:

1. As a column.
2. By crushing.

A yardstick under compression will bow out in the center and will collapse if the load is not removed; that is, the yardstick fails as a column. Figure 10:1 shows this type of failure. The other type of failure occurs in very short columns where L'/ρ is small and is known as crushing. Here the member may collapse somewhat like an accordion, the column being literally crushed. This crushing action should be investigated in many cases when L'/ρ is less than 40. When the strut length is such that the designer should check the strength for crushing action, he should check also the strength for failure as a column and use the smaller of these two values as an allowable.

10:5. Round Steel Tubes as Columns. There is more than one way to strength-check a tube acting as a column. First, use equations on page 4-2 of ANC-5, as every designer should understand these equations and their uses. Then the use of certain nomograms and charts will be explained that simplify the work in many cases. Fortunately, most problems encountered by the layout man may be solved by these nomograms and graphs.

Example. A chrome-moly (X-4130) steel tube 35 in. long carries a compressive load of 5000 lb., $c = 1$. What size of tube will be required to maintain a positive margin of safety?

This will be solved using two methods: (*a*) use of equations on page 4-2 of ANC-5; (*b*) use of nomogram on page 304 of Vol. I, Niles and Newell.

Method a. The first step is for the designer to assume a tube size and then work through the problem. If there is a negative margin of safety, he should work through again, using a stronger tube. Conversely, if he has an excessive margin of safety, he should cut down the size—thus saving weight—and repeat his calculations.

For this example, try a 1-in.-diameter tube having a wall thickness of 0.065 in., conveniently written 1 diameter—0.065, or 1—0.065.

$$\rho = 0.3314 \text{ (ANC-5, page 8-1)}$$

$$\frac{L'}{\rho} = \frac{35}{0.3314\sqrt{1}} = \frac{35}{0.3314} = 105.6$$

$$\text{Critical* } \frac{L'}{\rho} = 91.5 \text{ (ANC-5, page 4-2)}$$

Therefore, we have a long column.

The area of the 1—0.065 tube is 0.1909 sq. in. (ANC-5, page 8-1).

$$f_c = \frac{P}{A} = \frac{5000}{0.1909} = 26{,}200 \text{ psi}$$

The allowable stress is given by the equation on page 4-2 of ANC-5.

$$F_c = \frac{286 \times 10^6}{(L'/\rho)^2} = \frac{286 \times 1{,}000{,}000}{(105.6)^2} = \frac{286{,}000{,}000}{11{,}150} = 25{,}640 \text{ psi}$$

As the allowable is less than the actual stress, the selected tube size is unsatisfactory and a stronger tube must be chosen.

Now try a 1⅛—0.049 tube

$$\left.\begin{aligned}\rho &= 0.3808\\ A &= 0.16564\end{aligned}\right\} \text{(ANC-5, page 8-1)}$$

$$\frac{L'}{\rho} = \frac{35}{0.3808\sqrt{1}} = \frac{35}{0.3808} = 91.9$$

* It should be noted that in the table on p. 4-2 of ANC-5, lines 2 and 3 are both for X-4130 steel, one having a value of $F_{ty} = 75{,}000$ and the other $F_{ty} = 85{,}000$. By referring to p. 4-11 of ANC-5, columns 2 and 4, it may be found that the steel where $F_{ty} = 85{,}000$ is a special tubing. In this text only the X-4130 steel, $F_{ty} = 75{,}000$ psi will be used.

This is again over the critical slenderness ratio and is a long column.

$$f_c = \frac{5000}{0.1656} = 30{,}200 \text{ psi}$$

$$F_c = \frac{286 \times 1{,}000{,}000}{91.9^2} = \frac{286{,}000{,}000}{8445} = 33{,}850 \text{ psi}$$

As the allowable is greater than the actual stress, this tube would be satisfactory. The student has probably noticed that the area of the 1⅛—0.049 tube was less than the area of the 1—0.065 tube, yet the 1⅛—0.049 tube was the stronger. This illustrates clearly how the designer, by choosing tubes of larger diameter and cutting down on the wall thickness, may often save weight and gain strength at the same time.

Method b. The nomogram on page 304 of Vol. I, Niles and Newell, will be used but no attempt will be made to explain it in this text, as it is explained fully beginning with the last paragraph on page 303 and on pages 305, 307, and 308. The information contained therein should be thoroughly understood by the student.

Drawing a line on the nomogram from the proper scale on the right-hand edge where length = 35, to the 5000-lb. load on the scale on the left-hand side, it may be found at a glance that 1—0.065 is not strong enough and that the smallest satisfactory tube would be 1⅛—0.049. Niles and Newell do not have nomograms prepared for all materials and alloys; however, the student should familiarize himself with the available nomograms for they have a distinct advantage and save time.

Example. Find F_c for a 1¼—0.035 C.M. steel tube 21 in. long. $c = 1$.

$$\rho = 0.4297 \text{ (ANC-5, page 8-1)}$$

$$\frac{L'}{\rho} = \frac{21}{0.4297} = 48.9$$

From page 4-17 of ANC-5,

$$F_c = 61{,}800 \text{ psi}$$

when $L'/\rho = 48.9$ and $c = 1$.

Example. Find F_c for the same tube as used in the previous example; however, this time $c = 2$.

$$\frac{L'}{\rho} = \frac{21}{0.4297\sqrt{2}} = \frac{21}{0.4297 \times 1.414} = 34.6$$

$$F_c = 67{,}500 \text{ psi}$$

when $L'/\rho = 34.6$ and $c = 2$.

The graph on page 4-20 of ANC-5 for "Allowable Column Stress for Heat Treated Steel Round Tubing" clearly shows some interesting facts.

Notice the general shape of any of the curves; note how the curvature reverses. Each of these curves is plotted using two

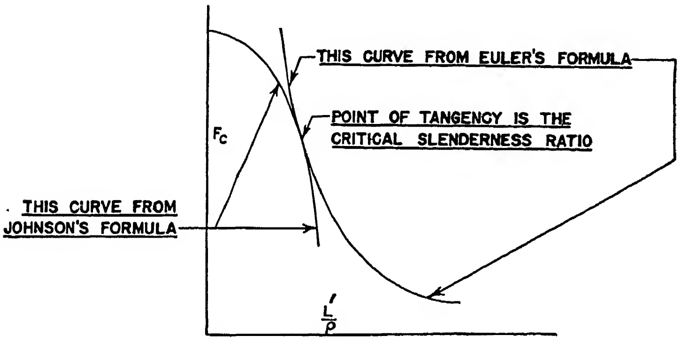


FIG. 10:3.

different equations; the curve on the left side of the graph is obtained from Johnson's parabolic formula, and the one on the right-hand side from Euler's formula for long columns. When these curves are plotted, they actually look somewhat like Fig. 10:3.

As pointed out in Fig. 10:3, the point of tangency of the two curves is the critical value of L'/ρ. Consider the upper curve on page 4-20 of ANC-5 for $F_{tu} = 180{,}000$ psi. By inspection of that curve, the point of tangency seems to be around the value for $L'/\rho = 60$. Turning to page 4-2 of ANC-5, the critical L'/ρ is given as 58.9. Likewise the student may check the critical points for the other two curves and find that they agree with the table on page 4-2 of ANC-5.

Notice, in this graph on page 4-20 of ANC-5, how the curves diverge widely for the various heat-treats in the short-column

range, then in the long-column range they converge into one line. This means that *there is no advantage in heat-treating a tube used as a long column.* This is a very important fact which is often overlooked, a fact which the student will do well to remember.

Often the designer would like to know the column strength of a C.M. steel tube of a certain size and length. On pages 4-21 to 4-27 of ANC-5 are graphs giving column strengths for various tubes. The graph on page 4-22 for tubes of 0.035-in. wall thickness will be discussed where curves are drawn for 10 different diameters. The column strength is read along the left-hand edge, and the column length is read at the bottom of the page. Care should be exercised to read along the correct value of c.

Example. What is the strength of a 5/8—0.035 C.M. steel tube 15 in. long? $c = 2$.

From the graph on page 4-22 of ANC-5, find the point along the lower border where length = 15 when $c = 2$. Following up until intersecting the curve for 5/8-in.-diameter tube, then reading across in the left-hand margin, the allowable load of 3900 lb. is found.

Example. What is the strength of the same tube used in the previous example, if $c = 1$?

Again referring to the same graph in ANC-5, read the length in the scale for $c = 1$. Then following up until crossing the curve, the allowable load of 3050 lb. is determined.

The student should notice that from these curves the allowable *loads* in pounds are obtained; not the allowable stress in pounds per square inch.

Example. What is the strength of 1¾—0.058 C.M. steel tube 20 in. long? $c = 2$.

From page 4-24 of ANC-5, find the line for length = 20 in. when $c = 2$. Following up until intersecting the curve for 1¾-in.-diameter tube, the allowable load of 20,800 lb. is determined.

Example. What is the strength of the column in the foregoing example, when $c = 1$?

Again use the same graph and the allowable load will be 20,800, the same as in the previous example. Notice that the allowable load is the same in this problem when $c = 1$ and $c = 2$.

When the column is very short, there is no gain in strength by restraining the ends. Also, there would be no increase in strength by decreasing the length. This is a fact that should not be overlooked by the designer.

On the graphs, pages 4-21 to 4-27, ANC-5, there are pertinent data in the upper right-hand corner, which is frequently useful to the designer, such as: ultimate tensile strength, allowable bending moment, and allowable torsional moment for the tubes given on the graph.

10:6. Round Aluminum Alloy Tubes as Columns. In the design of aluminum alloy round tubes as columns and struts, the designer may use the equations given on page 5-2 of ANC-5 to calculate allowable stresses, as the allowable stresses for steel tubes were calculated under method *a* in Art. 10:5. However, in most cases, it is more convenient to use the graph in Fig. 5-1, page 5-15 of ANC-5 to determine the allowable stress. Here there are four curves: one for 24SRT, one for 24ST, and two for 17ST tubing. By referring to page 5-6 of ANC-5, it may be seen that the two curves for 17ST are for tubes "before stretching" and "after stretching" (or "stretched" as referred to by ANC-5). When an aluminum alloy tube is stretched after it is made, the strength properties are slightly improved. The tubing usually called for on details is the stretched tubing; hence the designer should use that curve. However, 24ST tubing is used more frequently than 17ST; here the difference is not enough to have two curves, so there is only one on the graph for 24ST tubing.

Notice in the long-column range that the three curves come together, as the curves for heat-treated steel tubing converged into one line in the Euler range.

For small L'/ρ values, the tube may be critical in crushing. At the left-hand edge of this graph, page 5-15 of ANC-5, notice the dotted lines for various values of D/t. This particular portion of the graph has been replotted to a large scale in the upper right-hand corner. When the L'/ρ value is such that it is read from this extreme left-hand portion of the graph, the D/t value must be found, and the enlarged view in the upper right-hand corner used to find the allowable in crushing.

Example. What is the allowable stress for a 1-0.049 24ST tube, 7 in. long? $c = 1$.

$$\left.\begin{aligned} A &= 0.1464 \\ \rho &= 0.3367 \end{aligned}\right\} \text{(ANC-5, page 8-1)}$$

$$\frac{L'}{\rho} = \frac{7}{0.3367} = 20.8$$

Therefore,

$$F_c = 41{,}000 \text{ psi}$$

From the graph for aluminum alloy tubes, it may be found that, when L'/ρ is 20.8, the design is in that region where the column may be critical in crushing; hence the D/t value must be calculated.

$$\frac{D}{t} = \frac{1}{0.049} = 20.4$$

From the upper right-hand corner of this graph, it may be seen that, when $D/t = 20.4$,

$$F_{cc} = 46{,}800 \text{ psi}$$

Therefore, this strut is critical under column action.

Example. What is the allowable stress of a 1½—0.065 24ST tube, 33 in. long? $c = 2$.

$$\left.\begin{aligned} A &= 0.2930 \\ \rho &= 0.5079 \end{aligned}\right\} \text{(ANC-5, page 8-1)}$$

$$\frac{L'}{\rho} = \frac{33}{0.5079\sqrt{2}} = \frac{33}{0.5079 \times 1.414} = 46.0$$

From Fig. 5-1, page 5-15 of ANC-5, the value of $F_c = 30{,}200$ psi is determined.

10:7. Recommended Sizes of Round Tubing. The data on recommended tube sizes, as given in Art. 9:14, applies equally well to tubes subjected to compressive forces; in fact, that information applies to any tubes used in aircraft.

10:8. Streamlined Tubes as Columns. Streamlined tubes are used so seldom in present-day aircraft design that little time will be devoted to them. On page 8-3 of ANC-5 are data for calculating the properties of these tubes; on page 4-18 are curves for allowable stresses in C.M. steel streamlined tubes; Fig. 5-2, page 5-15 of ANC-5, gives the allowable stresses in aluminum alloy tubes. The use of these curves is the same as for round tubes.

It is interesting to note that streamlined tubes are so designed that the D/t ratio is always 25.

10:9. Tees, Angles, Etc., as Columns. The calculations for odd-shaped sections such as angles, tees, and hat sections as columns may become quite complicated because of the introduction at times of what is known as a form factor. There can be no definite figures given for these form factors since they vary with the shape and the proportions of the sections used. Consult the stress department if such a problem is encountered.

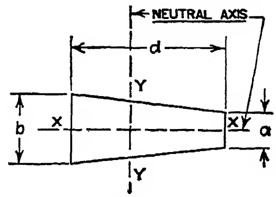


Fig. 10:4.

There is one section frequently used in aircraft that should be considered, and that is the H section which lends itself so readily to forgings. The layout man often encounters a design in which a forged link has to be checked for column action; hence such a link will be investigated.

The following equations[1] will be useful in the calculation for stresses in a forged link. The student is not expected to memorize these formulas; they are stated for reference only. Since draft plays an important part in the design of forgings, the moment of inertia of a trapezoid will be necessary in order to perform the required calculations. Figure 10:4 shows a typical trapezoid, and the notations used in the equations refer to the dimensions in this figure.

$$I_{x\text{-}x} = \frac{d(b + a)(b^2 + a^2)}{48} \qquad (10:3)$$

$$I_{y\text{-}y} = \frac{d^3(a^2 + 4ab + b^2)}{36(a + b)} \qquad (10:4)$$

The determination of the location of the neutral axis will not be discussed as it was considered in Art. 9:8.

Example. A forged 14ST aluminum alloy link is shown in Fig. 10:5. Using a design load of 65,000 lb. compression, what is the margin of safety? Figure 10:6 gives the dimensions of a section through the link. Notice that both ends of the link are designed to fit into a female attachment; the restraint offered by this type of attachment is sufficient to warrant the use of $c = 2$. However, about the other axis, there is a truly pin-ended

[1] Eq. (10:4) is form "Machinery's Handbook," 10th ed., p. 346.

column; hence the link will be investigated for column action about both axes. In Fig. 10:6, $c = 1$ about the x-x axis; about

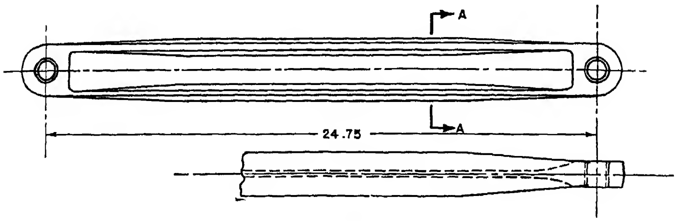


FIG. 10:5.

the y-y axis, $c = 2$. To do this, calculate the radius of gyration about *both* axes, using Eq. (10:1).

$$\rho = \sqrt{\frac{I}{A}}$$

First calculate the area of the section shown in Fig. 10:6.

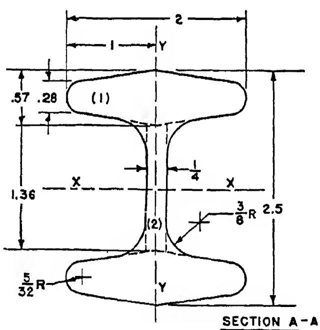


FIG. 10:6.

$$(1)\quad \frac{(0.28 + 0.57)}{2} \times 1 \times 4 = 1.70$$

$$(2)\quad 0.25 \times 1.36 = \underline{0.34}$$

$$\text{sq. in. area } 2.04$$

It is customary to neglect the area of the fillets in calculations such as this, unless the radius is quite large.

For the calculation of the moment of inertia about the x-x axis, use Eq. (9:2).

$$I = \Sigma I_0 + \Sigma A d^2$$

I_0 of the trapezoid marked (1) in Fig. 10:6, is calculated from Eq. (10:3). Substituting the values in this equation gives

$$a = 0.28$$
$$b = 0.57$$
$$d = 1$$

Therefore

$$I_0 = 4\left[\frac{(0.57 + 0.28)(0.57^2 + 0.28^2)}{48}\right]$$
$$= 4\left(\frac{0.85 \times 0.4033}{48}\right) = 4 \times 0.007142 = 0.02857$$

I_0 of the rectangle marked (2) in Fig. 10:6 is calculated from the equation[1]

$$I_0 = \frac{bh^3}{12}$$
$$= \frac{0.25 \times 1.36^3}{12} = \frac{0.25 \times 2.515}{12} = 0.0524$$

Therefore,

$$\Sigma I_0 = 0.0286 + 0.0524 = 0.0810$$

$$Ad^2 = 4\left[\frac{(0.28 + 0.57)}{2} \times \left(1.25 - \frac{0.57}{2}\right)^2\right]$$
$$= 4(0.425 \times 0.965^2) = 4 \times 0.396 = 1.584$$

Substituting these values in the equation $I = \Sigma I_0 + \Sigma Ad^2$ gives

$$I_{x\text{-}x} = 0.0810 + 1.584 = 1.665$$

Notice that ΣI_0 is less than 5 per cent of the total I; hence in this case no appreciable error will be introduced if the designer neglects I_0 entirely in his calculations.

$$\rho_{x\text{-}x} = \sqrt{\frac{I_{x\text{-}x}}{A}} = \sqrt{\frac{1.665}{2.04}} = \sqrt{0.816} = 0.903$$

For the calculations of the moment of inertia about the y-y axis, again use

$$I = \Sigma I_0 + \Sigma Ad^2$$

I_0 of the trapezoid marked (1) in Fig. 10:6 will be calculated from Eq. (10:4).

[1] From p. 344 of "Machinery's Handbook," 10th ed., or many other standard engineering reference books.

Substituting the values for this example, gives

$$
\begin{aligned}
I_0 &= 4\left[\frac{0.28^2 + 4 \times 0.28 \times 0.57 + 0.57^2}{36(0.28 + 0.57)}\right] \\
&= 4\left(\frac{0.0784 + 0.6384 + 0.3249}{36 \times 0.85}\right) = 4\left(\frac{1.0417}{30.6}\right) \\
&= 4 \times 0.03405 = 0.1362
\end{aligned}
$$

I_0 of the rectangle is

$$
\begin{aligned}
I_0 &= \frac{bh^3}{12} \\
&= \frac{1.36 \times 0.25^3}{12} = \frac{1.36 \times 0.01563}{12} = 0.0018
\end{aligned}
$$

Therefore,

$$\Sigma I_0 = 0.1362 + 0.0018 = 0.138$$

To find Ad^2 of the trapezoid, its centroid must first be found. From Niles and Newell, page 57, Vol. I, $x = 0.4431$ when the ratio is $0.57/0.28 = 2.035$. Since the height of this trapezoid is 1, the centroid is 0.4431 from the y-y axis. Therefore,

$$
\begin{aligned}
Ad^2 &= 4\left[\frac{(0.28 + 0.57)}{2} \times 0.4431^2\right] \\
&= 4(0.425 \times 0.1965) = 4 \times 0.0835 = 0.3340 \\
I_{y\text{-}y} &= 0.138 + 0.334 = 0.472
\end{aligned}
$$

In this case ΣI_0 is nearly 30 per cent of the total I and so should not be neglected.

$$
\begin{aligned}
\rho_{y\text{-}y} &= \sqrt{\frac{I_{y\text{-}y}}{A}} = \sqrt{\frac{0.472}{2.04}} \\
&= \sqrt{0.231} = 0.4802
\end{aligned}
$$

Earlier in this article, it was stated that $c = 1$ about the x-x axis; about the y-y axis, $c = 2$. Therefore,

$$
\begin{aligned}
L'_{x\text{-}x} &= L_{x\text{-}x} = 24.75 \\
L'_{y\text{-}y} &= \frac{L_{y\text{-}y}}{\sqrt{2}} = \frac{24.75}{1.414} = 17.5 \\
\frac{L'_{x\text{-}x}}{\rho_{x\text{-}x}} &= \frac{24.75}{0.903} = 27.41 \\
\frac{L'_{y\text{-}y}}{\rho_{y\text{-}y}} &= \frac{17.5}{0.4802} = 36.42
\end{aligned}
$$

For all practical purposes, it is accurate enough to determine the critical L'/ρ from page 5-2 of ANC-5 for round aluminum alloy tubes. As the worst condition has a value for L'/ρ of 36.42, there is no doubt that this is a short column. If ever in doubt whether a design is a long or short column, consult the stress group.

The allowable stress F_c is calculated from Eq. (1:26) on page 1-9 of ANC-5.

$$F_c = F_{co}\left[1 - \frac{0.385(L'/\rho)}{\pi\sqrt{E/F_{co}}}\right] \qquad (10:5)$$

There is no value given in ANC-5 for F_{co} for 14ST aluminum alloy forgings; 60,000 psi is a reasonable figure and is obtained from data prepared by the Aluminum Company of America.

$$E = 10{,}300{,}000 \text{ (ANC-5, page 5-14)}$$

Substituting values in Eq. (10:5),

$$\begin{aligned} F_c &= 60{,}000\left(1 - \frac{0.385 \times 36.42}{\pi\sqrt{10{,}300{,}000/60{,}000}}\right) \\ &= 60{,}000\left(1 - \frac{14.02}{\pi\sqrt{171.7}}\right) = 60{,}000\left(1 - \frac{14.02}{41.18}\right) \\ &= 60{,}000(1 - 0.3405) = 60{,}000 \times 0.6595 \\ &= 39{,}570 \text{ psi} \end{aligned}$$

The actual stress is

$$f_c = \frac{P}{A} = \frac{65{,}000}{2.04} = 31{,}860 \text{ psi}$$

Therefore,

$$\text{M.S.} = \frac{39{,}570}{31{,}860} - 1 = 0.241$$

Earlier in this article, the statement was made that form factors are often introduced into the calculations for odd-shaped sections used as columns. A form factor is considered unnecessary in the H section just calculated as it is a heavy forging symmetrical about both axes. If this H section had been a type shown in Fig. 10:7, where the relatively thin flanges might wave and twist under a compressive force, a form factor would have been necessary. It is impossible to set forth definite rules as to when a form

factor is required; as pointed out previously, always consult the stress department when in doubt.

If the designer could have used a round aluminum alloy tube of the same cross-sectional area as the forged H section, the strength would have been greater owing to a higher value for ρ. The area of this section shown in Fig. 10:6 is 2.04 sq. in. and the length 24.75 in. A $4\frac{1}{4}$—0.134 tube has an area of 1.733 sq. in. (considerably less than the forged H section), and $\rho = 1.456$. $D/t = 31.75$.

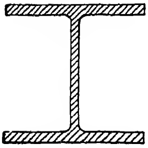


FIG. 10:7.

$$\frac{L'}{\rho} = \frac{24.75}{1.456} = 17.0$$

From Fig. 5-1, page 5-15 of ANC-5,

$$F_c = 42{,}900 \text{ psi,} \qquad \text{when} \qquad L'/\rho = 17$$
$$F_{cc} = 45{,}100 \text{ psi,} \qquad \text{when} \qquad D/t = 31.75$$

Hence the tube would be critical as a column at 42,900 psi.

$$f_c = 31{,}860 \text{ psi determined previously in this article}$$
$$\text{M.S.} = \frac{42{,}900}{31{,}860} - 1 = 0.347$$

Comparing this margin of safety with the margin for the forged H section, it is found to be 51 per cent greater. From a strength standpoint, the designer should use tubes for columns and struts wherever possible; however, from a practical viewpoint, this is not always desirable. Tubes used as columns usually require fittings in the ends; these fittings not only make extra parts, which are undesirable from a production standpoint; but at times the total weight of the tube and fittings is more than a single forged or extruded member. When in doubt whether to use a tube or some other section in compression, consult a member of the production design group.

Problems

10:1. *a*. Determine ρ for a $1\frac{3}{4}$-0.035 tube.
b. Determine ρ for a 2-0.095 tube.
c. Determine ρ for a 2-0.035 tube.
d. Determine ρ for a 1-0.095 tube.
e. Determine ρ for a $1\frac{1}{8}$-0.083 tube.

10:2. Determine L'/ρ and state whether the following columns are "long" or "short."

a. $1\frac{3}{4}$—0.035 C.M. steel tube 58 in. long. $c = 1$.
b. Same as *a*, except $c = 2$.
c. 2—0.095 C.M. steel tube heat-treated to $F_{tu} = 125{,}000$ psi. Length = 60. $c = 1$.
d. Same as *c*, except $c = 2$.
e. 1—0.095 24ST aluminum alloy tube 26 in. long. $c = 1$.

10:3. A 24ST aluminum alloy tube 38 in. long carries a compressive load of 2650 lb. For purposes of clearance it is necessary to limit the diameter of the tube to $1\frac{1}{4}$ in. What wall thickness is necessary to have a positive margin of safety when $c = 1$? Use the equations on page 5-2 of ANC-5 for the solution of this problem and select only tubes from the sizes listed in ANC-5.

10:4. The following problems on chrome-moly steel tubes used as columns should be solved using the nomogram in Niles and Newell, page 304.

a. What size of tube 40 in. long is required when the load is 1450 lb.? $c = 1$.
b. What is the column strength of a 1—0.049 tube 45 in. long? $c = 1$.
c. Same as above, except $c = 2$.
d. A $1\frac{1}{8}$—0.058 tube must support 6000 lb. What is the maximum permissible length? $c = 1$.
e. Same as above, except $c = 2$.

10:5. Determine F_c for the following C.M. steel tubes using curves in ANC-5:

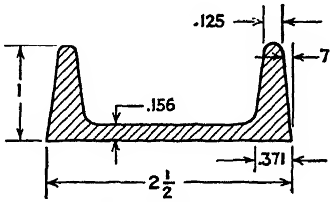
a. $1\frac{3}{4}$—0.035 tube; $L = 45$; $c = 1$.
b. $1\frac{3}{4}$—0.035 tube; $L = 45$; $c = 2$.
c. 2—0.095 tube; $L = 40$; $c = 1$.
d. 2—0.095 tube; $L = 40$; $c = 2$.
e. $1\frac{1}{8}$—0.083 tube; $L = 50$; $c = 1$.

10:6. Determine F_c for the tubes in Prob. 5 if they are heat-treated to $F_{tu} = 125{,}000$ psi.

10:7. A 2—0.058 24ST aluminum alloy tube 40 in. long supports a load of 8000 lb. $c = 1$. Find the M.S.

10:8. Find the margin of safety of a $1\frac{1}{2}$—0.049 24ST aluminum alloy tube 20 in. long subjected to a compressive load of 7500 lb. $c = 2$.

10:9. A $2\frac{1}{2}$—0.049 24ST aluminum alloy tube 19 in. long carries 15,500 lb. in compression. $c = 2$. Find the M.S.



PROB. FIG. 10:10.

10:10. Determine F_c for the section shown in the accompanying figure. $c = 2$ about the x-x axis but $c = 1$ about the y-y axis. The material is 14ST aluminum alloy forging and the length is 20 in. No form factor is required for this section.

CHAPTER 11

DESIGN OF THIN WEB BEAMS

11:1. Introduction. A "thin web beam" is the name applied to the conventional type of construction used in aircraft for beams. It consists essentially of suitable cap material (usually in the form of extruded angles or tees) separated by a thin sheet metal web reinforced by vertical stiffeners. It is customary to use extruded bulb angles or tees for them. Stiffeners are necessary for two reasons:

1. They break up the diagonal shear wrinkles.
2. They prevent the two caps from moving toward each other under load.

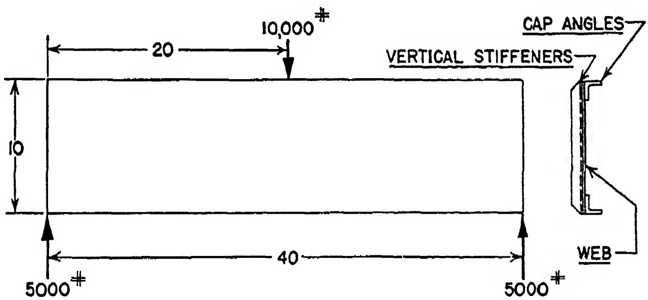


FIG. 11:1.

Without going into the beam theory, it is sufficient for this text simply to state that, when a beam of the type described above is loaded, diagonal shear wrinkles tend to form. The force that causes these shear wrinkles also attempts to draw the two caps together, that is, this force tries to *decrease* the depth of the beam. To prevent this tendency, suitable stiffening material must be added to the webs with a good connection between the ends of these vertical stiffeners and the cap members.

Frequently, the layout man is required to design a thin web beam as shown in Fig. 11:1. Here a concentrated load is applied at the center of the beam, which is supported at the ends by heavy

extruded aluminum alloy angles riveted to the Alclad web. In the figure these supports are indicated by arrows at the ends.

The best method of explaining the solution of this beam is actually to work it out. Figure 11:1 gives the necessary dimensions and loads for this example.

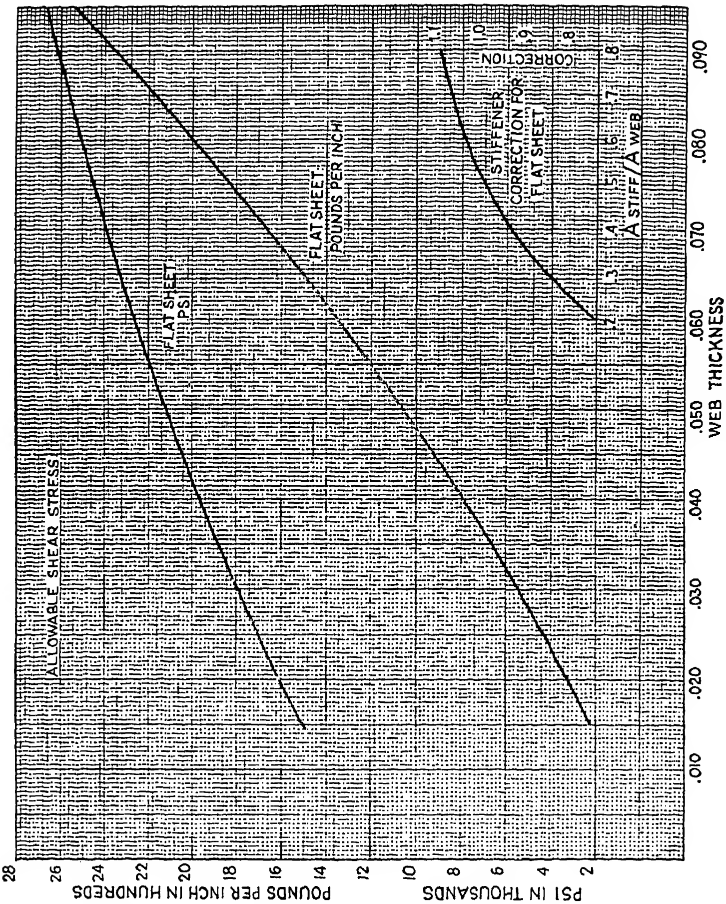


Fig. 11:2.

11:2. Thickness of Web. From Fig. 11:1 the reaction of 5000 lb. at each end and the depth of 10 in. are obtained. This means that the shear load in the web is $5000/10 = 500$ lb. per inch. The thickness of the web is determined from the graph in Fig. 11:2, where the curve marked "flat sheet pounds per inch"

is used. On the left-hand edge of the graph, find the line for 5, which in this case is 500 lb. per inch. Follow this line until it intersects the curve; then reading at the bottom of the sheet, it is found that the web thickness should be 0.0283. Since there is no standard aluminum alloy sheet gage of 0.0283, use the next heavier gage, which is 0.032. Checking back, it is found that an 0.032-in. sheet is good for 590 lb. per inch. This graph makes no mention of the material used. It applies equally well to any of the commonly used alloys of aluminum, as well as to Alclad sheet, but should not be used for soft stock, or for soft stock heat-treated. Of course, the various alloys have some

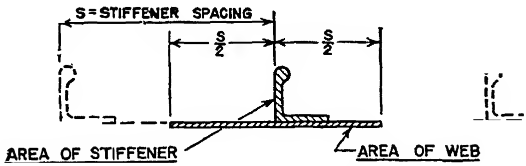


FIG. 11:3.

differences in strength; however this graph is not sufficiently accurate to consider such small differences. In this example a 24ST Alclad web is used, as that is the alloy most commonly used at present.

The curves for flat sheet in Fig. 11:2 are based on the assumption that

$$\frac{\text{Cross-sectional area of stiffener}}{\text{Cross-sectional area of web}} = 0.5$$

where the areas are as shown in Fig. 11:3.

If this ratio is anything other than 0.5, certain corrections should be made to the allowable shear values obtained from these curves. If the spacing is such that the ratio is greater than 0.5, it is reasonable to expect the web-stiffener combination to be stronger than the figure found from the graph. Conversely, if the ratio is less than 0.5, the values obtained from the curves should be decreased. This correction curve is found in the lower right-hand corner of the graph and is explained more fully in Art. 11:4.

11:3. Spacing of Vertical Stiffeners. In the range of sizes of beams used in aircraft, it is customary to place the vertical stiffeners from 6 to 10 in. apart. There are no actual calculations

for the designer to make; the spacing is largely a matter of "practiced eye." Once the spacing is decided upon, the layout man will use this spacing in subsequent calculations. Eight inches will be arbitrarily chosen as the stiffener spacing for this example; 9-in. spacing is considered by many to be a good average value; however 8-in. spacing will be used as the span of 40 in. is divisible by 8 and not by 9.

11:4. Area of Vertical Stiffeners. The areas referred to in the following empirical equation are cross-sectional areas as shown in Fig. 11:3. The area of each vertical stiffener is determined by an empirical formula based on actual test data.

$$A = Stk \tag{11:1}$$

where A = area of stiffener
S = spacing of stiffeners
t = thickness of web
$k = \dfrac{\text{area of stiffener}}{\text{area of web between stiffeners}}$

It has been found that, when $k = 0.40$, the highest weight-strength ratio in the stiffener-web combination is obtained. Hence, it is good practice to make an initial assumption that $k = 0.4$. The designer should always keep between $k = 0.4$ and $k = 0.5$, with $k = 0.4$ preferable. Having made this assumption, solve Eq. (11:1). Substituting the above values gives

$$A = 8 \times 0.032 \times 0.4 = 0.1024 \text{ sq. in.}$$

Now use the curve in the lower right-hand corner of Fig. 11:2, the one marked "stiffener correction for flat sheet." Knowing that the ratio of area of stiffener to area of web is 0.4, find along the lower edge of this curve the point marked 0.4. Following up until intersecting the curve and reading across to the right-hand border, a correction factor of 0.94 is determined. This means that, in using the proposed web and stiffener combination, it is good for

$$0.94 \times 590 = 555 \text{ lb. per inch shear}$$

where 590 is the value obtained in Art. 11:2 for the strength of a 0.032 web.

Now find an available extrusion having an area of 0.1024 sq. in., or very close to it. Before the layout man decides to design a new extrusion, not only the standards book but also the standards group should be consulted concerning all available sections (see Art. 6:4). It is preferable to use as stiffening angles extrusions with a bulb on one leg, the leg without the bulb being attached to the web, or tee sections.

Looking in the standards book, a $\frac{3}{4} \times 1$ bulb angle, Alcoa 15643 (see Appendix) is found, which has an area of 0.110 sq. in. As this area is very close to the desired one, it will probably be satisfactory. Now repeat some of the calculations in this article, using the actual angle.

$$k = \frac{\text{area of stiffener}}{\text{area of web}} \qquad \frac{0.110}{8 \times 0.032} = \frac{0.110}{0.256} = 0.43$$

From the curve in the lower right-hand corner of Fig. 11:2, when the ratio is 0.43, the correction factor is 0.96.

$$0.96 \times 590 = 567 \text{ lb. per inch}$$

Therefore, with the actual web and stiffener combination, the strength is 567 lb. per inch, which is satisfactory since the actual shear load is 500 lb. per inch (see Art. 11:2).

If the designer uses a stiffener spacing greater than 9 in., he is required to use an additional correction factor.

$$F = \frac{3}{\sqrt{S}} \tag{11:2}$$

where F = correction factor used when S is greater than 9 in.
S = spacing of stiffeners

To illustrate: Return to the example in Fig. 11:1. However, a stiffener spacing of 10 in. will be specified. Again assume that $k = 0.40$ in Eq. (11:1). Solving this equation by using a spacing of 10 in. gives

$$A = 10 \times 0.032 \times 0.4 = 0.128 \text{ sq. in.}$$

In the standards book, a $\frac{3}{4} \times 1$ extruded bulb angle, Alcoa 15644 (see Appendix), is found, having an area of 0.127 sq. in. Ordinarily, the designer should choose extruded sections having an area equal to or greater than the required area, but as this area is only 0.001 sq. in. less than the desired area, it will be

satisfactory. Having chosen the extruded section, find the ratio of stiffener area to web area.

$$k = \frac{0.127}{10 \times 0.032} = \frac{0.127}{0.32} = 0.397, \text{ say } 0.40$$

Referring to Fig. 11:2, the correction factor is 0.94 when the stiffener-web ratio is 0.40. Now calculate the additional correction factor mentioned in Eq. (11:2). Substituting values,

$$F = \frac{3}{\sqrt{10}} = \frac{3}{3.162} = 0.949$$

Since the correction factors have been obtained, the actual strength of the web-stiffener combination may be determined. It will be recalled that in Art. 11:2 it was found that the 0.032 web was good for 590 lb. per inch shear. Correcting it for the example with the 10-in. stiffener spacing, we write

$$590 \times 0.94 \times 0.949 = 526 \text{ lb.}$$

This means that the web-stiffener combination with the 10-in. spacing is good for 526 lb. shear per inch. Returning to the original example with the 8-in. stiffener spacing, proceed with its solution.

11:5. Area of Cap Material. To determine the area required, use the following equation:

$$A = \frac{M}{dF} \qquad (11:3)$$

where A = area of upper cap which is also the area of the lower cap
M = maximum bending moment
d = approximate distance between centroids of caps
F = allowable stress

In this example,

$$M = \frac{PL}{4} = \frac{10,000 \times 40}{4} = 100,000 \text{ (see Fig. 11:1)}$$

d = 9 approximately
F = 30,000 (see following paragraph for explanation)

Therefore,

$$A = \frac{100,000}{9 \times 30,000} = 0.37 \text{ sq. in.}$$

No definite figure can be given for the allowable stress F, as it varies with the shape and size of the extruded section used for the cap. The value is from 20,000 to 40,000 psi, with the majority of the sections using values between 25,000 and 35,000 psi. The heavier the section, the higher the allowable. In this example, the average value was used as it might be considered an average section. The designer should either use a conservative figure for this value or else check with the stress department, preferably the latter.

In the standards book, a $1\frac{1}{4} \times 1\frac{1}{2}$ bulb angle, Alcoa 8493, was found, having an area of 0.354 sq. in. Although this is a little less than the area required according to our calculations, it will tentatively be specified for this example.

In order to check the section, investigate the beam for maximum bending stresses, using these cap angles.

It will be remembered that the bending stress is given by Eq. (9:1),

$$f_b = \frac{My}{I}$$

The bending moment M has already been determined in this article, in order to obtain an approximate area. By inspection, the neutral axis of the entire section is at the center since this is a symmetrical section. The value for I is determined by Eq. (9:2),

$$I = \Sigma I_0 + \Sigma A d^2$$

It is common practice to neglect I_0 of the extruded caps in deep sections such as this since it is very small compared to the total moment of inertia. In thin web beams, it is also customary to neglect the web in the calculations for the moment of inertia. This simplifies Eq. (9:2) considerably, leaving only Ad^2 for the layout man to compute.

Figure 11:4 shows the section drawn to scale. As explained previously,

$$I = \Sigma A d^2$$

The area of the extruded cap angle (0.354 sq. in.) and the location of its neutral axis are obtained from the standards book (see

Appendix). Substituting the values in this equation,

$$I = 2 \times 0.354 \times 4.634^2 = 15.18$$

Now substituting this value in the bending stress equation,

$$f_b = \frac{My}{I} = \frac{100{,}000 \times 5}{15.18} = 32{,}950 \text{ psi}$$

An allowable stress of 30,000 psi was assumed; however, this is a little over that figure. Since the calculations for the value of I have been conservative, it is considered satisfactory for this example. In the discussion of cap material area, the assumption was made that the beam was to be symmetrical, that is, it was to have the same area for cap material on the top as on the bottom. This is necessary when there is a reversal of load; however, if the loads always act in one direction, it would be more efficient to have less area on the tension side than on the compression side as the allowable in tension is higher than in compression. However, it is often better from a production standpoint to use the same extruded section top and bottom; hence the designer should consult a member of the production design group before using an unsymmetrical section for the beam.

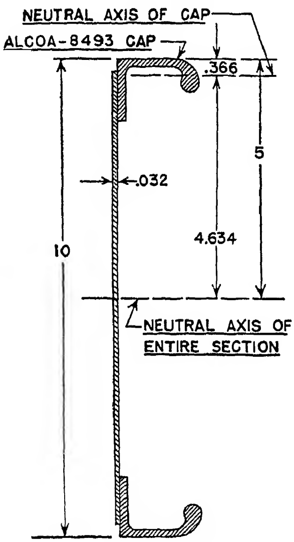


FIG. 11:4.

11:6. Rivets Attaching Webs to Caps. Without explaining the theory of thin web beams and the various internal forces within them, the student should understand one simple fact before continuing this study. By the beam theory, the vertical shear at any point is equal to the horizontal shear as shown in Fig. 11:5. In the example shown in Fig. 11:1, the vertical shearing force is 500 lb. per inch. Call this force S in Fig. 11:5. There is another shearing force acting at 90 deg. to the original force of the same magnitude. That is,

$$S = S' = 500 \text{ lb. per inch}$$

When this horizontal shearing force has been found, the number of rivets required to transmit this force into the flange material may be determined.

When the web material is 0.040 or less, a 1.2 factor should be applied to take care of the wrinkling of the web. In this example, $S' = 500$ lb. per inch. Since the web is 0.032, this correction factor must be applied.

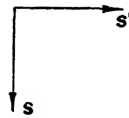


FIG. 11:5.

Design load $= 1.2 \times 500 = 600$ lb. per inch. Assume the AD rivets are in single shear. Choosing the rivets as outlined in Chap. 8, $\frac{5}{32}$-in.-diameter rivets critical in bearing on the 0.032 web at 409 lb. should be specified.

$$\text{Required spacing} = \frac{409}{600} = 0.682 \text{ in. O.C. [from Eq. (8:3)]}$$
$$\text{say } \frac{5}{8} \text{ in. O.C. (see Art. 8:8)}$$

This spacing is used across the top as well as across the bottom.

11:7. Rivets Attaching Webs to End Supports. The vertical shearing force is the force to use for this condition. In Art. 11:6, the design shear load was found to be 600 lb. per inch. Therefore, the same rivets and spacing will be used at the ends as were used across the top and bottom.

11:8. Rivets in Web Splice. Here again use the same vertical shearing force for the splice. The student should realize that there are many design problems where the vertical shear is *not* constant across the beam as it is in this case. But for this particular example, use the same vertical shearing force of 600 lb. per inch for the splice. Therefore, the same rivets and spacing will be specified here as were used around the four sides of this beam.

11:9. Rivets Attaching Vertical Stiffeners to Caps. This is determined by an empirical formula based on actual tests.

$$\text{Strength required} = 7500A \tag{11:4}$$

where A = area of stiffener

In Art. 11:4, the area of the stiffener was found to be 0.110 sq. in. Substituting this in Eq. (11:4) gives

$$\text{Strength required} = 7500 \times 0.110 = 825 \text{ lb.}$$

Checking the dimensions of the stiffener (Alcoa 15643, see Appendix) the thickness of the leg attached to the web and cap is

found to be 0.051. As this is less than the leg of the cap angle, the bearing of the rivet on 0.051 should be investigated, as well as the shear of the rivet itself. In this case, the bearing on the 0.032 web is not considered as the attachment of the stiffener to the cap angle is being investigated. Specify $\frac{5}{32}$-in.-diameter AD rivets, which are critical in single shear at 518 lb. Therefore, use two $\frac{5}{32}$-in. rivets in the end of each stiffener.

At times, it will be impossible to place two rivets through the end of the vertical stiffener into the cap. In such cases, it is customary to drive one rivet through the end of the stiffener into the cap and the second as close to the cap as possible, the web acting as a gusset at the ends. Of course, the designer will have many problems in which only one large rivet through the end is necessary. In such cases, it will be unnecessary to crowd another rivet up close to the cap as described above.

11:10. Rivets Attaching Vertical Stiffeners to Web. Again an empirical formula based on actual tests is used.

$$\text{Strength required} = \frac{25{,}000A}{S} \qquad (11:5)$$

where A = area of stiffener[1]
S = spacing of stiffener

[1] At times, the designer will encounter beams requiring double stiffeners because of high loads, these stiffeners being located on either side of the web, back to back. In cases where double stiffeners are necessary, use only the area of *one* stiffener in this empirical formula; for the remaining empirical equations in this chapter use the *total* area of stiffeners.

From a purely theoretical standpoint, there is no doubt that double stiffeners are superior to one large stiffener. On the compression side of the beam, there is a tendency for the cap to roll and twist, with double stiffeners offering more restraint to this twisting than a single stiffener. Since stiffeners are actually columns, the double stiffener combination has the neutral axis of the column on the center line of the web, which is highly desirable. In the case of single stiffeners, the neutral axis is not on the center line of the web and this eccentricity may become serious when the large angles are used. However, the layout man must not lose sight of the fact that from a production standpoint, one stiffener is preferable as it keeps the number of parts down to a minimum.

This question of one vs. two stiffeners is quite controversial, one which the layout man should not attempt to solve himself. He should consult the stress department or a member of the production design group.

Substituting values, we have

$$\text{Strength required} = \frac{25{,}000 \times 0.110}{8} = 344 \text{ lb. per inch}$$

AD rivets of $\frac{5}{32}$-in. diameter should be specified; these are critical in bearing on the 0.032 web at 409 lb.

$$\text{Spacing required} = {}^{409}\!/_{344} = 1.18, \text{ say } 1\tfrac{3}{16} \text{ in. O.C.}$$

11:11. Rivets Attaching Cover Plates to Cap Material. At times, it will be necessary to have cover plates on the cap material in order to obtain sufficient area. This condition should be avoided wherever possible and should be used only after consulting a member of the production design group. When it is deemed necessary to use this type of construction, the cover plates should be securely riveted to the cap angles. The determination of these rivets is quite complex, and it is recommended that the layout man refer this problem to the stress department for solution.

In the example in Fig. 11:1, it was unnecessary to have cover plates; hence this problem did not arise. It is mentioned here, however, as the layout man should know what to do in the event he is faced with this condition.

11:12. Summary. Now collect the information and data calculated for this example. The problem was to design a thin web beam having the general outline and loads as shown in Fig. 11:1.

Web = 0.032 24ST Alclad sheet

Spacing of vertical stiffeners	= 8 in. O.C.
Vertical stiffeners	= Alcoa 15643 extruded section
Cap members	= Alcoa 8493 extruded section
Attachment of web to cap Attachment of web to end support Splice in web (if any)	= $\frac{5}{32}$-in.-diameter AD rivets, $\frac{5}{8}$ in. O.C.
Attachment of vertical stiffeners to caps	= two $\frac{5}{32}$-in.-diameter rivets in each end
Attachment of vertical stiffeners to web	= $\frac{5}{32}$-in.-diameter AD rivets $1\frac{3}{16}$ in. O.C.

Another example will be solved to familiarize the student thoroughly with the method. Design a beam of the same general type and loading condition as was previously considered; this

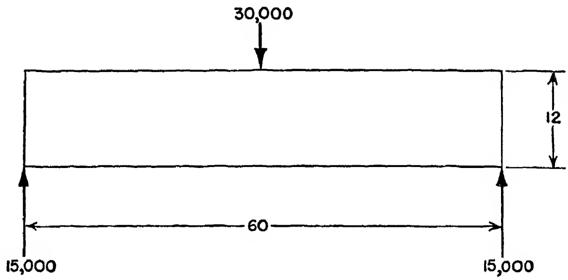


FIG. 11:6.

beam being shown in Fig. 11:6. The same procedure will be followed here as for the beam shown in Fig. 11:1.

Thickness of Web.

$$\text{Shear load} = \frac{15{,}000}{12} = 1250 \text{ lb. per inch}$$

From Fig. 11:2, we find

$$\text{Web required} = 0.064$$

which is good for 1470 lb. per inch. (0.057 is good for 1250 lb. per inch, but is not considered a standard gage.)

Spacing of Stiffeners. Since it is customary to have equal spacing for stiffeners and as 9 in. is a good average spacing, use seven equal spaces at 8.56 in. each. This would be referred to as $8\frac{9}{16}$ on centers (see Art. 11:3).

Area of Stiffeners. From Eq. (11:1), we find

$$A = 8.56 \times 0.064 \times 0.4 = 0.219 \text{ sq. in.}$$

This presents the problem that was discussed in the footnote to Art. 11:10, whether one or two stiffeners should be used. In this case, assume the airplane to be a high production model; hence to keep the number of parts down to a minimum, use a single stiffener. Realizing there will be a high end load to transfer into the caps, choose the tee section Alcoa 15272 having an area of 0.230 sq. in. The tee section has a decided advantage in its attachment to the cap, as rivets may be driven in each leg of the

tee; whereas, in an angle, it is seldom possible to place more than two rivets through the stiffener into the cap.

$$k = \frac{\text{actual area of stiffener}}{\text{actual area of web}} = \frac{0.230}{8.56 \times 0.064} = 0.420$$

From the correction chart in Fig. 11:2, the correction factor for this case is 0.95. Therefore, this web-stiffener combination is good for

$$0.95 \times 1470 = 1396 \text{ lb. per inch}$$

Area of Cap Material. From Eq. (11:3),

$$A = \frac{M}{dF}$$

where $M = \dfrac{30{,}000 \times 60}{4} = 450{,}000$ in.-lb.

$d = 11$ (approximately)

$F = 35{,}000$ psi (this figure used since the cap will be fairly large)

Therefore,

$$A = \frac{450{,}000}{11 \times 35{,}000} = 1.168 \text{ sq. in.}$$

Assume that the layout man is unable to find an existing extrusion that is satisfactory, and must design a new one. As this is a large area, two extrusions could be used back to back with a cover plate; however, that is not recommended because of the number of parts and rivets necessary for their attachment. A tee section would be considered good design in cases such as this.

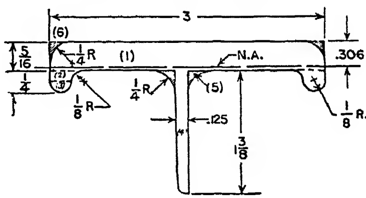


Fig. 11:7.

The first step will be to design an extruded tee having an area of 1.168 sq. in. or more. Such a tee is shown in Fig. 11:7. In determining the area, care must be used not to make as many approximations as are permissible in the calculations for the moment of inertia. The calculations for the area and the location of the neutral axis are shown on the following page. The upper surface of the cap is used as the base line.

			Area	× Arm	= Moment
(1)	3 × 0.3125	=	0.9375	× 0.156 =	0.1463
(2)	2 × 0.25 × 0.125	=	0.0625	× 0.375 =	0.0234
(3)	2 × ½ × 0.0491	=	0.0491	× 0.491 =	0.0241
(4)	0.125 × 1.375	=	0.1719	× 1.000 =	0.1719
(5)	2 × 0.0134	=	0.0268	× 0.39 =	0.0105
			1.2478	=	0.3762
(6)	−2 × 0.0134	=	−0.0268	× 0.08 =	−0.0021
			1.2210	× 0.306 =	0.3741

1.2210 = area of section
0.306 = neutral axis of section

The inertia of this section will not be determined as it will be neglected in the calculations for the moment of inertia of the beam. The cross section of the beam is shown in Fig. 11:8. As explained in Art. 11:5, consider only the Ad^2 in calculating the moment of inertia of the entire section.

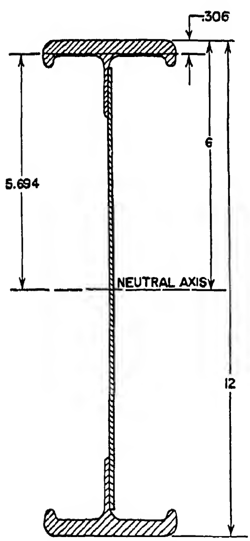


Fig. 11:8.

$$I = \Sigma Ad^2 = 2 \times 1.221 \times 5.694^2 = 79.3$$

$$f_b = \frac{My}{I} = \frac{450{,}000 \times 6}{79.3} = 34{,}050 \text{ psi}$$

This actual stress is well within the allowable for this section; hence it is considered satisfactory.

Rivets Attaching Web to Caps and to End Supports. In this example, the shearing force is 1250 lb. per inch with the rivets in single shear. Although ¼-in. rivets would be more efficient, for practical considerations, it will be better to specify $\frac{3}{16}$-in.-diameter AD rivets, which are critical in single shear at 745 lb.

$$\text{Spacing} = {}^{745}\!/_{1250} = 0.596, \text{ say } {}^{9}\!/_{16} \text{ in. O.C.}$$

See Fig. 11:9 for the rivet pattern.

Rivets Attaching Vertical Stiffeners to Caps. From Eq. (11:4),

$$\text{Strength required} = 7500 \times \text{area of stiffener}$$
$$= 7500 \times 0.230 = 1725 \text{ lb.}$$

The thickness of the stiffener leg is approximately 0.064; therefore, four $\frac{5}{32}$-in. AD rivets will be needed, and are critical in single shear at 518 lb. each = 2072 lb.

Rivets Attaching Stiffeners to Web. From Eq. (11:5),

$$\text{Strength required} = \frac{25{,}000A}{S} = \frac{25{,}000 \times 0.230}{8.563} = 671 \text{ lb.}$$

Use $\frac{3}{16}$-in. AD rivets critical in single shear at 745 lb.

$$\text{Spacing} = {}^{745}\!/_{671} = 1.11, \text{ say } 1\tfrac{1}{16} \text{ in. O.C.}$$

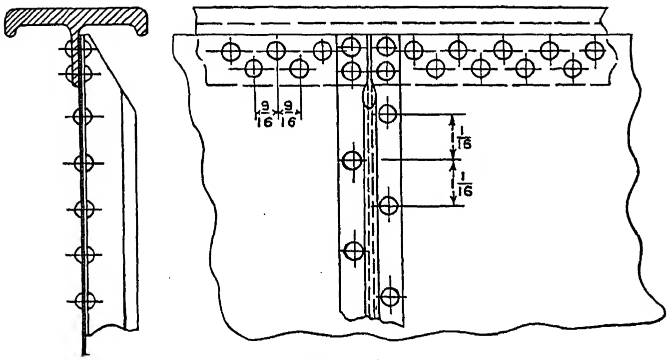


FIG. 11:9.

Now summarize the results of the calculations made for the beam shown in Fig. 11:6.

Web = 0.064 24ST Alclad sheet

Spacing of vertical stiffeners	= $8\frac{9}{16}$ in. O.C.
Vertical stiffeners	= S15272 extruded tee
Cap members, area	= 1.221 sq. in.
Attachment of web to caps Attachment of web to end support	= $\frac{3}{16}$-in.-diameter AD rivet $\frac{9}{16}$ in. O.C.
Attachment of vertical stiffeners to caps	= four $\frac{5}{32}$-in.-diameter AD rivets[1] in each end
Attachment of vertical stiffeners to web	= $\frac{3}{16}$-in.-diameter AD rivets $1\frac{1}{16}$ in. O.C.

[1] Although $\frac{5}{32}$-in.-diameter rivets may be used, specify $\frac{3}{16}$-in.-diameter rivets since the balance of rivets required are of that diameter. It is poor design to mix sizes of rivets unnecessarily.

Having summarized the results of the calculations, lay out a portion of the beam showing how the rivets would be placed in order to maintain the required spacing, as in Fig. 11:9.

Thus far in this chapter, only beams having a very simple loading condition have been considered. Obviously, when the loading condition changes, the design must change. The student probably noticed that the design of the web was based on the shearing load. In the following article more complex loading conditions and the method of determining the shear for them will be considered. After the shear has been determined for any problem, the actual design of the beam is relatively simple.

11:13. Shear Diagrams and Their Application to Thin Web Beams. It is necessary for the designer to understand shear

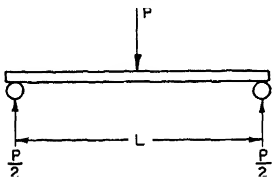


FIG. 11:10.

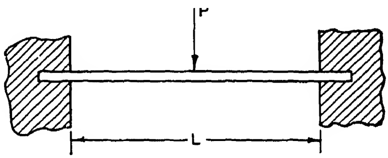


FIG. 11:11.

diagrams and be able to plot shear curves, in order to design intelligently a thin web beam of any but the simplest loading condition. In many instances, he will have a problem where there are two or more concentrated loads on a beam, such as a floor support; or he may have a uniformly distributed load as a support for a baggage compartment floor.

One of the simplest loading conditions is that of a single concentrated load acting in the center of a simple beam. A simple beam is one that has no end restraint, as a girder supported by rollers at the ends. Such a beam is shown in Fig. 11:10.

If the same beam were set in concrete, as shown in Fig. 11:11, the ends would be restrained by their method of attachment. This end restraint does not affect the shear curve; however, it does change the bending moment diagram considerably. Hence, the designer should be able to recognize the difference between a simple and a restrained beam. Fortunately, most of the beams encountered by the average layout man are considered to be simple beams. The end attachments offer some restraint; however, it is customary to calculate the stresses on the assumption that it is actually a simple beam with no end restraint. This

simplifies the calculations in many cases and is also conservative, a desirable feature in *any* calculations.

In the event the designer encounters a restrained, or "fixed," beam as some textbooks refer to it, or a beam having three or more supports, commonly referred to as a "continuous beam," he should check with the stress department for the determination of the bending moment.

Returning to the simple beam shown in Fig. 11:10 with a single concentrated load, the shear diagram will be drawn. To do this, it will be necessary to calculate the shear at any point, which is quite simple. The shear at any point is the algebraic sum of the vertical components of all the forces on either side of that point.

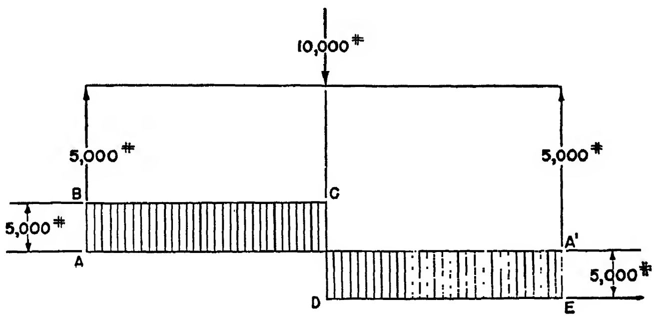


FIG. 11:12.

The shear is positive when the algebraic sum of all forces acting on the beam to the *left* of the section is upward. In other words, always think of an upward force from the left or a downward force from the right as positive shear.

Figure 11:12 is the same beam as shown in Fig. 11:1 and is repeated here for convenience. No depth is shown in Fig. 11:12 since only loads are considered. The shear diagram is plotted directly below the beam.

To plot this shear diagram, a horizontal line A-A' is drawn, which serves as the reference or base line. Start at the left support where there is a force of 5000 lb. acting upward, and plot the point B to some convenient scale. Locate B above the base line as the upward force to the left is positive. Moving to the right, no forces are encountered until the center is reached, where the downward force of 10,000 lb. is located. At the point of application of this 10,000-lb. load, locate point C, as the shear

remains constant at 5000 lb. until this downward force is reached. The instant this force is passed, the algebraic sum of the forces to the left becomes -5000 lb.; hence plot the point D to the same scale, 5000 lb. below the line, as a downward force to the left is negative. Continuing to the right, there are no forces until the right-hand support is reached. Therefore, the shear remains constant at -5000 lb. until that support is reached where the point E is plotted to scale. At the point of this reaction, the algebraic sum of the forces becomes 0; therefore the final point for the shear diagram is A'. Connecting A, B, C, etc. with straight lines gives the shear diagram as shown in Fig. 11:12. It is customary to crosshatch the area within the diagram to

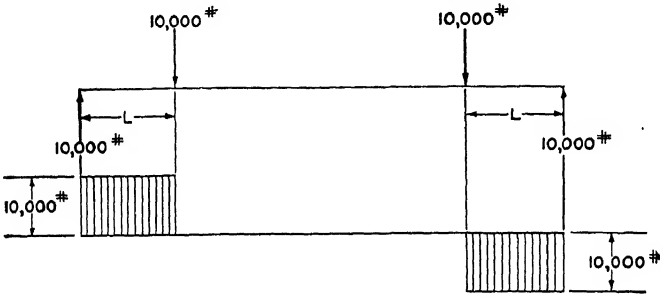


FIG. 11:13.

make it stand out more clearly. From this diagram, it may be seen that the shear value remains constant throughout the length of the beam: one side being $+5000$ lb., the other side -5000 lb.

In the application of these shear diagrams to the design of thin web beams, the sign of the force may be neglected; that is, the same web will have to be specified whether the load is $+5000$ or -5000 lb.

Now consider a simple beam having two equal symmetrically placed loads as shown in Fig. 11:13. To plot this shear diagram, follow the method used for Fig. 11:12. Starting at the left, there is a positive shear of 10,000 lb., which remains constant until the left-hand downward force of 10,000 lb. is reached, where the algebraic sum of the forces becomes zero. The shear remains zero until the right-hand downward force is reached, where it becomes $-10,000$ lb. Then it remains constant until the right-hand support is reached, where it returns to zero.

It was learned in Art. 11:2 that the gage of the web is dependent upon the shear. In Fig. 11:13, since the shear is 0 through the center of the beam, apparently no web is needed. However, a beam would never be designed without a web; a very thin sheet, say 0.020 or 0.025, would be used through this center part where the shear is zero. Then at the points where the load is applied, a web of suitable thickness would be spliced to carry the required shear. A good reason for having some web through the portion of the beam where the shear is zero is that under various attitudes of flight, the loads change. The loads given in the problem are the design loads which, in all cases, are the most severe of the various loads. With the actual loads changing with the attitude of flight, there may be conditions where there would be an unsymmetrical loading, in which case, the shear is not zero. It is not practical to attempt to calculate all these varying loads; hence use a web with stiffeners to take care of any such possibility.

Another purpose of web and stiffeners through this area is that the stiffeners tend to stabilize the caps. The caps under compression have a tendency to twist and deflect out of shape. Vertical stiffeners, being securely attached to the caps, restrain this tendency.

There are times when it would be advisable to carry the same web across the entire beam; although it may be unnecessary from a strength standpoint, it is done to keep the number of parts down to a minimum. There is no definite rule to follow; the designer should calculate the weight saved by splicing on a thin web, and then consult the supervisor or a member of the production design group.

In the event it is considered advisable to use a thin web for part of the beam, a vertical stiffener should be placed at the splice; that is, locate the splice so that it falls on a vertical stiffener. This saves driving unnecessary rivets, as well as stiffening the joint itself.

Another type of loading frequently encountered by the layout man is a simple beam with a uniform load, such as a baggage compartment floor support. A beam of this type is shown in Fig. 11:14. The uniform load w is 70 lb. per inch. The total load on the beam is $50 \times 70 = 3500$ lb.; hence, each reaction is $3500/2 = 1750$ lb.

To draw the shear diagram, proceed as in the previous examples by starting with the left-hand reaction of 1750 lb. This load is plotted to some convenient scale, which locates point *B*. Moving across to the right, this shear will constantly decrease as there is a continuous uniform downward load. Since the load is uniform, the shear curve will be a straight line. To obtain this line, plot the right-hand reaction of 1750 lb., and as an upward load on the right is considered negative, point *C* is located below the reference line. Connecting *B* and *C* gives the line *B-C*, which forms the shear diagram.

Since the web is determined by the shear load, the web may be decreased until the center is reached, where the shear is zero. In

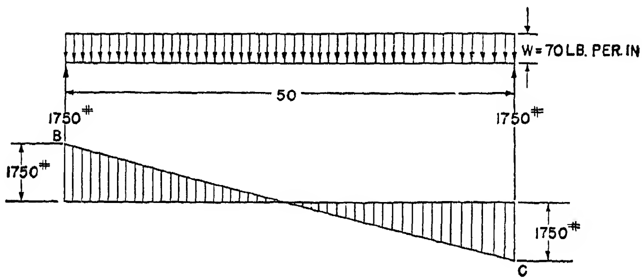


FIG. 11:14.

baggage compartment floors, this is never done as there is no certainty of having a uniform load at all times. With variations in cargo, at times there will be loads that are definitely not uniform; hence the shear diagram will change with each alteration in load.

Many of the loading conditions encountered by the designer are shown in Niles and Newell, Vol. I, pages 94 to 107. Information is given there that will enable the layout man to determine the shear and bending moment diagrams with a minimum of effort. He should consult the stress department for any loading conditions not covered by that text.

Return to the loading conditions shown in Fig. 11:13, and design such a beam. The dimensions and loads on this beam are shown in Fig. 11:15.

Without thoroughly going into this example, it will be decided to use a thin web between points of concentrated load in order to illustrate the method used in such cases; heavy webs will be used

between the points where the load is applied and the reaction. In the design of this beam, follow the method used in the first part of this chapter.

Design of Webs.

$$\text{Shear} = \frac{13{,}000}{8} = 1625 \text{ lb. per inch}$$

From the curves in Fig. 11:2, it is found that 0.072 will carry 1730 lb. per inch shear. Therefore, specify 0.072 Alclad sheet for the web between the points of application of the load and the reaction. For the web between loads where the shear is zero, arbitrarily specify 0.020 Alclad sheet with 4-in.-diameter lightening holes.

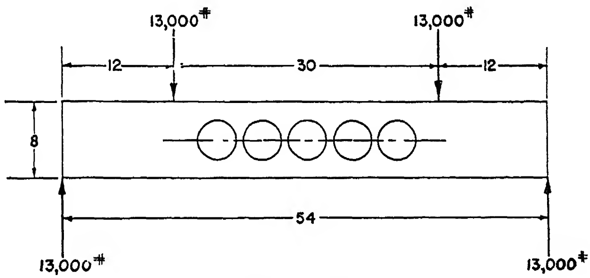


FIG. 11:15.

Spacing of Vertical Stiffeners. By inspection of Fig. 11:15, it may be seen that under the point of application of the load a heavy member or fitting should be provided to distribute this 13,000-lb. load into the web. This member will also serve as a stiffener.

As there are 12 in. between this load and the reaction, that panel should be broken up with one stiffener, giving two equal bays of 6 in. each.

Between the loads where the 0.020 web is specified, stiffeners are necessary for the reason explained previously in this article.

Area of Stiffeners. Applying Eq. (11:1),

$$A = 6 \times 0.072 \times 0.4 = 0.1728$$

This, of course, applies only to these two stiffeners on the heavy web. Looking in the standards book (see Appendix), a bulb

angle is found that will be satisfactory, Alcoa 8640, whose area is 0.177 sq. in. Therefore,

$$k = \frac{\text{actual area of stiffener}}{\text{actual area of web}} = \frac{0.177}{6 \times 0.072} = 0.41$$

From the correction curve in Fig. 11:2, the correction factor is 0.942 when the ratio is 0.41.

The web-stiffener combination is then good for

$$1730 \times 0.942 = 1630 \text{ lb. per inch.}$$

Since there is 1625 lb. per inch, the web-stiffener combination is satisfactory.

For the stiffeners on the portion of the beam with the thin web, use the $\frac{5}{8} \times \frac{7}{8}$ bulb angle Alcoa 16692 having an area of 0.097 sq. in. This section is arbitrarily chosen; no fixed rules govern the choice of this stiffener, as is true for the determination of the stiffeners on the stressed web.

Area of Caps. From Eq. (11:3),

$$A = \frac{M}{dF}$$

In the example,

$$M^* = 12 \times 13{,}000 = 156{,}000 \text{ in.-lb.}$$

Substituting these values gives

$$A = \frac{156{,}000}{7 \times 30{,}000} = 0.743 \text{ sq. in.}$$

In the selection of the cap for this beam, the designer should keep several points in mind. From the area required, he can see that one single angle will be too large to be practical and excessive eccentricity will be introduced; hence a tee section would be the logical choice as it reduces the number of parts required.

Another point to consider is the method of distributing the 13,000-lb. load into the web. This is a subject that has not been discussed as yet, so it would be well to do so at the present time. The load may be applied to this beam by another beam attaching

* From Niles and Newell, Vol. I, p. 100.

to the web and caps with a large angle. In cases such as this, the angle distributes the load into the web.

The 13,000-lb. load may, however, be applied by a fitting resting on top of the caps, or it may be suspended from the lower caps. In cases such as this, the designer must provide means of transferring this load concentrated on the caps into the web. This is usually done by means of a fitting securely fastened to the web that will nest up under the caps. In Fig. 11:15, the load is concentrated on the upper cap; hence, provision must be made to transfer this load through the caps into a fitting that will distribute the load into the web.

When choosing the cap material, it must be remembered that this fitting will be nesting under the cap; therefore, the cap should be wide enough to permit bolts to pass through the cap joining the fitting which receives the load on the upper side, and the fitting nesting under the cap. To keep eccentricities down to a minimum, provide fittings on each side of the web to effect this transfer of load.

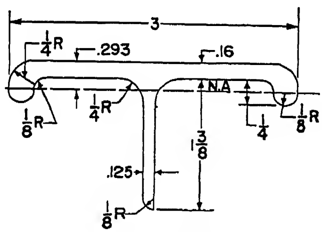


Fig. 11:16.

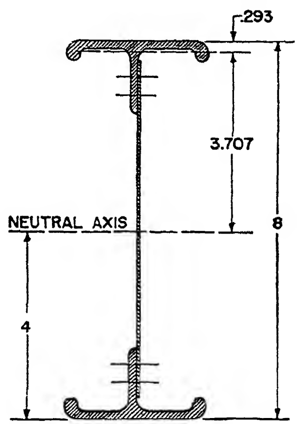


Fig. 11:17.

There may be similar problems of transferring loads at the end supports; however, there are usually heavy angles supporting the beam at the ends, which eliminate this particular problem.

With a mental picture of what is required, the cap material may be intelligently selected. Assume that there is no suitable existing extrusion; hence a new one is designed, as shown in Fig. 11:16. Without going into detail again as to the method of calculating the area and location of the neutral axis, the area has been found to be 0.767 sq. in. and the neutral axis is 0.293 in. below the upper surface of the cap. Figure 11:17 is a cross

section of the beam. As usual, neglect the inertia of the caps and web, using only the Ad^2 of the caps.

$$I = 2 \times 0.767 \times 3.707^2 = 21.10$$
$$f_b = \frac{My}{I} = \frac{156{,}000 \times 4}{21.10} = 29{,}600 \text{ psi}$$

Rivets Attaching Web to Caps and to End Supports. Since the horizontal shearing force is the same as the vertical shearing force and since the vertical shearing force was found to be 1625 lb., the load per inch is known which must be transferred from the web into the cap and into the end supports. The web is 0.072 24ST Alclad sheet; so for practical considerations, specify $\frac{3}{16}$-in.-diameter AD rivets which are critical in single shear at 745 lb. The spacing is

$$\text{Spacing} = \frac{745}{1625} = 0.458, \text{ say } \frac{7}{16} \text{ in. O.C.}$$

(See Fig. 11:19 for this pattern.)

This spacing applies only to the highly stressed portion of the web. For that portion of the beam with the 0.020 web, it is customary arbitrarily to make the connection between the web and the caps as strong as the web itself. From the graph in Fig. 11:2, it may be seen that an 0.020 web is good for 320 lb. per inch. Since there are lightening holes in the web, the actual strength is considerably less than this figure. The total depth is 8 in. and 4-in.-diameter lightening holes are specified; hence, approximately one-half of the sheet is removed. This means that the web is not good for 320 lb. but for one-half of that amount, 160 lb. per inch. In the selection of rivets, the designer should remember that it is not practical to try to drive heavy rivets through thin sheet. Since the web is 0.020, $\frac{3}{32}$-in.-diameter rivets should be used according to the rivet tables in ANC-5. This diameter is specified since, in 0.020 sheet, $\frac{3}{32}$-in. rivets are the largest that can be driven (see ANC-5, page 5-20). These $\frac{3}{32}$-in. AD rivets will be critical in bearing on the 0.020 sheet at 153 lb. The spacing should, therefore, be

$$\frac{153}{160} = 0.957, \text{ say, } \frac{15}{16} \text{ in. O.C.}$$

The 1.2 factor, mentioned in Art. 11:6 for use with webs 0.040 thick or less, may be neglected in cases such as this where the load is negligible.

Rivets Attaching Vertical Stiffeners to Caps. From Eq. (11:4),

$$\text{Strength required} = 7500 \times \text{area of stiffeners}$$

The area of the vertical stiffener on the heavy web has been found to be 0.177 sq. in. Substituting this in the above equation gives

$$\text{Strength required} = 7500 \times 0.177 = 1300 \text{ lb.}$$

Use two $\frac{3}{16}$-in.-diameter AD rivets critical in single shear at 745 lb. = 2 × 745 = 1490 lb.

As already mentioned this applies only to the stiffeners on the heavy web. For the stiffeners on the thin web, arbitrarily specify two $\frac{3}{32}$-in. AD rivets in the ends. This diameter is used to be consistent with the balance of the rivets through the 0.020 web into the cap material.

Rivets Attaching Vertical Stiffeners to Web. Using Eq. (11:5), we have

$$\text{Strength required} = \frac{25{,}000A}{S}$$

Substituting values,

$$\text{Strength required} = \frac{25{,}000 \times 0.177}{6} = 737 \text{ lb. per inch}$$

Using $\frac{3}{16}$-in.-diameter AD rivets critical in single shear at 745 lb.,

$$\text{Spacing required} = {}^{745}\!/_{737} = 1.011, \text{ say 1 in. O.C.}$$

For the stiffeners on the thin web, again specify $\frac{3}{32}$-in.-diameter AD rivets in order not to mix the sizes unnecessarily on this thin web. They will be spaced $1\frac{1}{4}$ in. O.C., which is considered the maximum spacing in a design such as this, as pointed out previously.

Summary. Now collect the design information as calculated on the previous pages for the beam shown in Fig. 11:15.

	Through highly loaded portion of beam	Through center of beam
Web	0.072 24ST Alclad	0.020 24ST Alclad
Spacing of vertical stiffeners	6 in. O.C.	Approx. 6 in. O.C.
Vertical stiffeners	Alcoa-8640	Alcoa-16692
Cap members, area	0.767 sq. in.	0.767 sq. in.
Attachment of web to caps	3/16 in. dia., 7/16 in. O.C.	3/32 in. dia., 15/16 in. O.C.
Attachment of web to end supports.	3/16 in. dia., 7/16 in. O.C.	
Attachment of vertical stiffeners to caps	two 3/16-in.-dia. rivets	two 3/32-in.-dia. rivets
Attachment of vertical stiffeners to webs.	3/16 in. dia., 1 in. O.C.	3/32-in.-dia., 1¼ in. O.C.

Transfer of Concentrated Load on Upper Cap into Web. As was pointed out previously, the concentrated load applied to the upper side of the top cap must be transferred into the web. This transfer may be effected by the use of channel fittings, as shown in Fig. 11:18. These fittings would probably be aluminum alloy forgings if the airplane were a high production model; or they might be machined from bar stock if it were an experimental model.

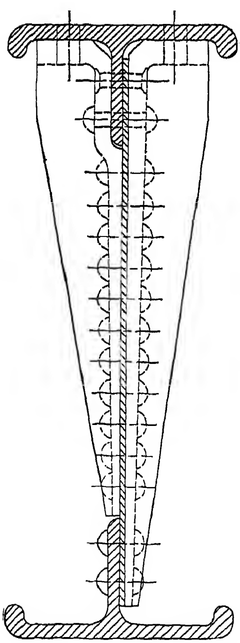


FIG. 11:18.—These fittings are designed for a compression load only.

By inspection it may be seen that most of the rivets attaching these fittings to the web are in double shear; hence, caution must be used in the calculations, some rivets being in single shear and others in double shear.

Throughout this text, the layout man has been cautioned to obtain his factors and required margins of safety from the stress department. For this problem, assume that the customer requires a 20 per cent fitting factor; hence, our design load will be the 13,000-lb. load given in Fig. 11:15, multiplied by this 1.20 fitting factor. This is usually written

$$\text{Design load} = 13{,}000 \times 1.2 = 15{,}600 \text{ lb.}$$

This is the load used to determine the number of attaching rivets and sizes of bolts, also the load used to check the cross-sectional area required in the fittings themselves.

To be consistent with other rivets through the 0.072 web, specify $\frac{3}{16}$-in.-diameter AD rivets, which will be critical in bearing on the 0.072 web at 1107 lb. for those rivets in double shear. The rivets in single shear will be critical in single shear at 745 lb. The total number of rivets must be good for a higher load than the design load; that is, a positive margin of safety must be maintained over the design load.

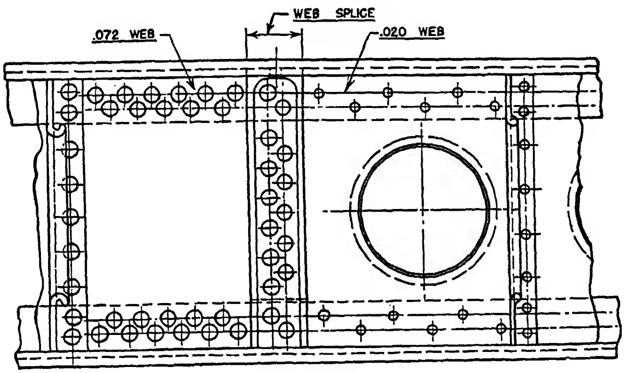


FIG. 11:19.

Our connection will require 13 rivets in double shear and two rivets in single shear:

$$\begin{aligned} 13 \times 1107 &= 14{,}391 \text{ lb.} \\ 2 \times 745 &= \underline{1{,}490} \text{ lb.} \\ \text{Total} &= 15{,}881 \text{ lb.} \end{aligned}$$

As the design load is 15,600 lb., this connection is satisfactory. Notice how the tee-section caps rest directly on the face of these fittings, and how provision is made for anchoring these channel fittings to the cap and to a fitting on the upper side receiving the concentrated loads.

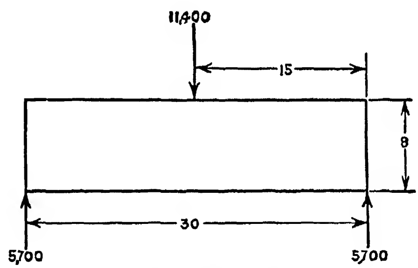
The designer should use care and be sure that the fitting is strong enough to carry the load imposed. He should check at least two sections to make sure that the P/A values do not exceed the allowable.

Fittings such as these should be placed under each of the concentrated loads. They would have to be used at the ends to take the load from the web into the lower cap, if it were not for the fact that the webs are supported at the ends by heavy attachment angles.

The splice between the 0.020 and the 0.072 webs is located directly under the fittings. As pointed out previously, this provides a good solid backing for the splice and also eliminates extra rivets. In Fig. 11:18, the 0.020 web is not shown. Figure 11:19 is a view of a portion of the beam showing the arrangement of the rivet patterns, etc.

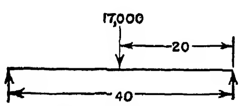
Problems

11:1. Design a thin web beam for the conditions shown in the accompanying figure. All the calculations are to be shown as well as a small sketch of a portion of the beam similar to Fig. 11:9. On this sketch show all the pertinent data, such as web

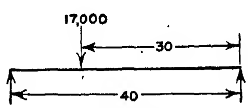


PROB. FIG. 11:1.

gage, spacing and size of stiffeners, cross section through cap members with the number of section used or the area if no standard section is specified, and the size and spacing of all attaching rivets.



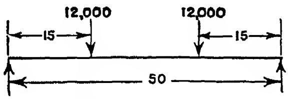
PROB. FIG. 11:2.



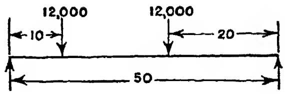
PROB. FIG. 11:3.

11:2–11:6. Calculate the reactions and plot the shear diagrams for the problems shown in the accompanying figures. The shear diagrams are to be drawn neatly to some convenient scale.

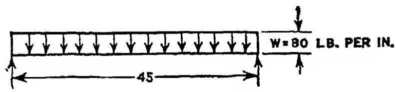
Any calculations necessary are to be on the same sheet as the shear diagram.



PROB. FIG. 11:4.



PROB. FIG. 11:5.



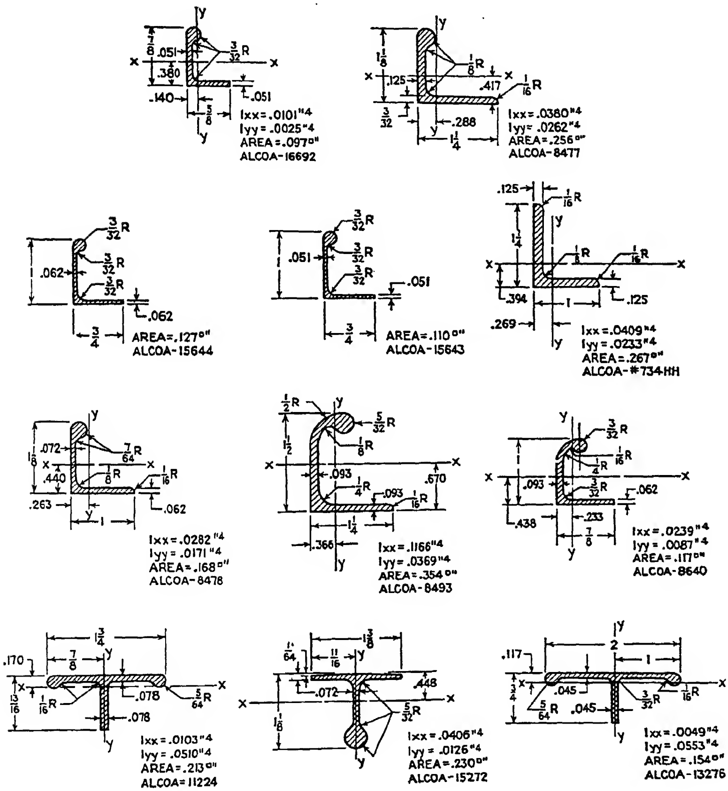
PROB. FIG. 11:6.

11:7. Design a thin web beam for the loading condition shown in Prob. 11:4; the depth of the beam is 10 in. The load is applied by fittings attached to the *underside* of the lower cap members, and suitable fittings are to be provided to distribute these concentrated loads into the web. The beam is supported at the ends by heavy angles attached to the web. The web gage is to be reduced through a portion of the beam.

All calculations are to be on the layout itself, including necessary calculations for the fitting that distributes the load from the lower cap into the web. A fitting factor of 1.2 is required for this fitting.

APPENDIX

TABLES USEFUL TO THE LAYOUT MAN



BEND ALLOWANCES FOR B. & S. GAGE ALUMINUM ALLOY SHEET

R \ T	.014	.016	.018	.020	.022	.025	.029	.032	.036	.040	.045	.051	.057	.064	.072	.081	.091	.114	.128	5/32	3/16	T / R
1/32	.00066	.00067	.00068	.00070	.00072	.00074	.00077	.00079	.00082													1/32
1/16	.00120	.00121	.00123	.00125	.00126	.00129	.00131	.00135	.00137	.00140	.00144	.00149	.00153	.00159	.00165							1/16
3/32	.00174	.00176	.00177	.00179	.00180	.00183	.00186	.00188	.00191	.00195	.00199	.00203	.00208	.00213	.00220	.00226	.00234					3/32
1/8	.00229	.00230	.00232	.00234	.00235	.00238	.00240	.00243	.00246	.00249	.00253	.00258	.00262	.00268	.00274	.00281	.00289	.00307	.00317			1/8
5/32	.00283	.00285	.00286	.00288	.00290	.00292	.00295	.00297	.00300	.00304	.00308	.00312	.00317	.00322	.00328	.00335	.00343	.00362	.00372	.00394		5/32
3/16	.00338	.00339	.00341	.00342	.00344	.00347	.00349	.00352	.00355	.00358	.00362	.00367	.00371	.00377	.00383	.00390	.00398	.00416	.00426	.00449	.00473	3/16
7/32	.00392	.00394	.00395	.00397	.00398	.00401	.00403	.00406	.00409	.00412	.00417	.00421	.00426	.00431	.00437	.00444	.00452	.00471	.00481	.00503	.00528	7/32
1/4	.00447	.00448	.00450	.00451	.00454	.00456	.00458	.00461	.00464	.00467	.00471	.00476	.00480	.00486	.00492	.00499	.00507	.00525	.00535	.00558	.00582	1/4
9/32	.00501	.00501	.00504	.00506	.00507	.00510	.00512	.00515	.00518	.00521	.00526	.00530	.00535	.00540	.00546	.00553	.00561	.00579	.00590	.00612	.00636	9/32
5/16	.00556	.00557	.00559	.00560	.00562	.00564	.00567	.00570	.00573	.00576	.00580	.00584	.00589	.00595	.00601	.00608	.00616	.00634	.00644	.00667	.00691	5/16
11/32	.00610	.00612	.00613	.00615	.00616	.00619	.00621	.00624	.00627	.00630	.00634	.00639	.00644	.00649	.00655	.00662	.00670	.00688	.00699	.00721	.00745	11/32
3/8	.00665	.00666	.00668	.00669	.00671	.00673	.00676	.00679	.00682	.00685	.00689	.00693	.00698	.00704	.00710	.00717	.00725	.00743	.00753	.00776	24SO .00800	3/8
13/32	.00719	.00721	.00722	.00724	.00725	.00728	.00730	.00733	.00736	.00739	.00743	.00748	.00753	.00758	.00764	.00771	.00779	.00797	.00808	.00830	.00854	13/32
7/16	.00774	.00775	.00777	.00778	.00780	.00782	.00785	.00787	.00791	.00794	.00798	.00802	.00807	.00812	.00819	.00826	.00834	.00852	.00862	.00884	.00909	7/16
15/32	.00828	.00829	.00831	.00833	.00834	.00837	.00839	.00842	.00845	.00848	.00852	.00857	.00862	.00867	.00873	.00880	.00888	.00906	.00917	.00939	.00963	15/32
1/2	.00883	.00884	.00885	.00887	.00889	.00891	.00894	.00896	.00899	.00903	.00907	.00911	.00916	.00921	.00928	.00935	.00943	.00961	.00971	.00993	.01018	1/2
17/32	.00937	.00938	.00940	.00942	.00943	.00946	.00948	.00951	.00954	.00957	.00961	.00966	.00970	.00976	.00982	.00989	.00997	.01015	.01025	.01048	.01072	17/32
9/16	.00992	.00993	.00994	.00996	.00998	.01000	.01002	.01005	.01008	.01012	.01016	.01020	.01025	.01030	.01037	.01043	.01051	.01070	.01080	.01102	.01127	9/16
19/32	.01046	.01047	.01048	.01051	.01052	.01055	.01057	.01060	.01063	.01065	.01070	.01073	.01079	.01083	.01091	.01098	.01105	.01124	.01133	.01157	.01181	19/32
5/8	.01100	.01102	.01103	.01105	.01107	.01109	.01112	.01114	.01117	.01121	.01125	.01129	.01134	.01139	.01146	.01152	.01160	.01179	.01189	.01211	17ST .01236	5/8
21/32	.01155	.01156	.01158	.01160	.01161	.01164	.01166	.01170	.01172	.01175	.01179	.01183	.01188	.01193	.01200	.01207	.01214	.01233	.01245	.01266	.01290	21/32
11/16	.01209	.01211	.01212	.01214	.01216	.01218	.01220	.01223	.01226	.01230	.01234	.01238	.01243	.01248	.01254	.01261	.01269	.01288	.01298	.01320	.01345	11/16
23/32	.01264	.01265	.01267	.01268	.01269	.01273	.01275	.01276	.01281	.01283	.01288	.01291	.01297	.01301	.01309	.01316	.01322	.01342	.01351	.01374	.01399	23/32
3/4	.01318	.01320	.01321	.01323	.01324	.01327	.01329	.01332	.01335	.01338	.01343	.01347	.01352	.01357	.01363	.01370	.01378	.01396	.01407	.01429	.01454	3/4
25/32	.01373	.01374	.01376	.01378	.01379	.01381	.01384	.01386	.01390	.01392	.01397	.01401	.01406	.01411	.01418	.01425	.01432	.01451	.01461	.01484	.01508	25/32
13/16	.01427	.01429	.01430	.01432	.01433	.01436	.01438	.01441	.01444	.01447	.01451	.01456	.01461	.01466	.01472	.01479	.01487	.01505	.01516	.01538	.01562	13/16
27/32	.01482	.01483	.01485	.01486	.01487	.01490	.01493	.01494	.01499	.01501	.01506	.01509	.01515	.01519	.01527	.01534	.01540	.01560	.01569	.01593	24ST .01617	27/32
7/8	.01536	.01538	.01539	.01541	.01542	.01545	.01547	.01550	.01553	.01556	.01560	.01565	.01570	.01575	.01581	.01588	.01596	.01614	.01625	.01647	.01671	7/8
29/32	.01591	.01592	.01594	.01595	.01596	.01599	.01602	.01604	.01608	.01611	.01615	.01619	.01624	.01629	.01636	.01643	.01650	.01669	.01679	.01701	24SRT .01726	29/32
15/16	.01645	.01646	.01648	.01650	.01651	.01654	.01656	.01659	.01662	.01665	.01669	.01674	.01679	.01684	.01690	.01697	.01705	.01723	.01734	.01756	.01780	15/16
31/32	.01700	.01701	.01702	.01704	.01705	.01708	.01711	.01712	.01717	.01718	.01724	.01727	.01733	.01737	.01745	.01752	.01758	.01778	.01787	.01810	.01835	31/32
1	.01754	.01755	.01757	.01759	.01760	.01763	.01765	.01768	.01771	.01774	.01778	.01783	.01788	.01793	.01799	.01806	.01814	.01832	.01843	.01865	.01889	1

Values given are based on the empirical formula $(0.01743R + 0.0078T) \times$ number of degrees; R = radius of bend; T = thickness of sheet in inches. Values given are for 1 deg. of the given radius $\times$ the thickness. Values omitted from table are not to be used as the bends are too sharp for satisfactory production.

Total developed length $= X + Y + Z$

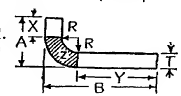
T = thickness of metal.

R = radius of bend.

Z = $B.A.$ from the chart.

$X = A - (R + T)$

$Y = B - (R + T)$



BEND ALLOWANCES FOR U.S.S. GAGE FERROUS SHEET

R \ T	.022	.028	.031*	.034	.038*	.050	.056*	.063	.070*	.078	.094	.109	.125	5/32	3/16	1/4	5/16	T / R
1/32	.00072	.00076	.00079	.00081	.00084	.00093	.00098											1/32
1/16	.00126	.00131	.00133	.00136	.00138	.00148	.00153	.00158	.00164									1/16
3/32	.00180	.00185	.00188	.00190	.00193	.00202	.00207	.00212	.00218	.00224								3/32
1/8	.00235	.00240	.00242	.00245	.00247	.00257	.00262	.00267	.00273	.00279	.00291							1/8
5/32	.00289	.00294	.00297	.00299	.00302	.00311	.00316	.00321	.00327	.00333	.00346	.00358						5/32
3/16	.00344	.00349	.00351	.00354	.00356	.00366	.00371	.00376	.00382	.00388	.00400	.00412	.00424					3/16
7/32	.00398	.00403	.00405	.00408	.00411	.00420	.00425	.00430	.00436	.00442	.00454	.00467	.00479					7/32
1/4	.00453	.00458	.00460	.00463	.00465	.00475	.00480	.00485	.00491	.00497	.00509	.00521	.00533	.00558				1/4
9/32	.00507	.00512	.00514	.00517	.00519	.00529	.00534	.00539	.00545	.00551	.00563	.00576	.00588	.00612				9/32
5/16	.00562	.00567	.00569	.00572	.00574	.00584	.00589	.00593	.00600	.00606	.00618	.00630	.00642	.00667	.00691			5/16
11/32	.00616	.00621	.00624	.00626	.00628	.00638	.00643	.00648	.00654	.00660	.00672	.00684	.00697	.00721	.00745			11/32
3/8	.00671	.00676	.00678	.00680	.00683	.00693	.00698	.00702	.00708	.00715	.00727	.00739	.00751	.00776	.00800	.00849		3/8
13/32	.00725	.00730	.00733	.00735	.00737	.00747	.00752	.00757	.00763	.00769	.00781	.00793	.00806	.00830	.00854	.00903		13/32
7/16	.00780	.00784	.00787	.00789	.00792	.00802	.00806	.00811	.00817	.00823	.00836	.00848	.00860	.00884	.00909	.00958	.01006	7/16
15/32	.00834	.00839	.00841	.00844	.00846	.00856	.00861	.00866	.00872	.00878	.00890	.00902	.00915	.00939	.00963	.01012	.01061	15/32
1/2	.00889	.00893	.00896	.00898	.00901	.00910	.00915	.00920	.00926	.00932	.00945	.00957	.00969	.00993	.01018	.01067	.01115	1/2
17/32	.00943	.00948	.00950	.00953	.00955	.00965	.00970	.00975	.00981	.00987	.00999	.01011	.01023	.01048	.01072	.01121	.01170	17/32
9/16	.00998	.01002	.01005	.01007	.01010	.01019	.01024	.01029	.01035	.01041	.01054	.01066	.01078	.01102	.01127	.01175	.01224	9/16
19/32	.01052	.01057	.01059	.01062	.01064	.01074	.01079	.01084	.01090	.01096	.01108	.01120	.01132	.01157	.01181	.01230	.01279	19/32
5/8	.01106	.01111	.01114	.01116	.01119	.01128	.01133	.01138	.01144	.01150	.01163	.01175	.01187	.01211	.01236	.01284	.01333	5/8
21/32	.01161	.01166	.01168	.01171	.01173	.01183	.01188	.01193	.01199	.01205	.01217	.01229	.01241	.01266	.01290	.01339	.01388	21/32
11/16	.01215	.01220	.01223	.01225	.01228	.01237	.01242	.01247	.01253	.01259	.01271	.01284	.01296	.01320	.01345	.01393	.01442	11/16
23/32	.01270	.01274	.01277	.01280	.01282	.01292	.01297	.01302	.01308	.01314	.01326	.01338	.01350	.01374	.01399	.01448	.01497	23/32
3/4	.01324	.01329	.01332	.01335	.01337	.01346	.01351	.01356	.01362	.01368	.01380	.01393	.01405	.01429	.01454	.01502	.01551	3/4
25/32	.01379	.01384	.01386	.01389	.01391	.01401	.01406	.01410	.01417	.01423	.01435	.01448	.01459	.01484	.01508	.01557	.01605	25/32
13/16	.01433	.01438	.01441	.01443	.01445	.01455	.01460	.01465	.01471	.01477	.01489	.01502	.01514	.01538	.01562	.01611	.01660	13/16
27/32	.01488	.01493	.01495	.01497	.01500	.01510	.01515	.01519	.01525	.01532	.01544	.01556	.01568	.01593	.01617	.01666	.01714	27/32
7/8	.01542	.01547	.01550	.01552	.01554	.01564	.01569	.01574	.01580	.01586	.01598	.01610	.01623	.01647	.01671	.01720	.01769	7/8
29/32	.01597	.01602	.01604	.01606	.01609	.01619	.01624	.01628	.01634	.01641	.01653	.01665	.01677	.01701	.01726	.01775	.01823	29/32
15/16	.01651	.01656	.01658	.01661	.01663	.01673	.01678	.01683	.01689	.01695	.01707	.01719	.01732	.01756	.01780	.01829	.01878	15/16
31/32	.01706	.01710	.01713	.01715	.01718	.01728	.01732	.01737	.01743	.01749	.01762	.01774	.01786	.01810	.01835	.01884	.01932	31/32
1	.01760	.01765	.01767	.01770	.01772	.01782	.01787	.01792	.01798	.01804	.01816	.01828	.01841	.01865	.01889	.01938	.01987	1

* Not standard for production.

Values given are based on the empirical formula $(0.01743R + 0.0078T) \times$ number of degrees; R = radius of bend; T = thickness of sheet in inches. Values given are for 1 deg. of the given radius $\times$ the thickness. Values omitted from table are not to be used as the bends are too sharp for satisfactory production.

Total developed length = $X + Y + Z$

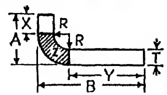
T = thickness of metal.

R = radius of bend.

Z = $B.A.$ from the chart.

$X = A - (R + T)$

$Y = B - (R + T)$



Bend Allowances for B. & S. Gage Magnesium Alloy Sheet

R \ T	.014	.016	.018	.020	.022	.025	.029	.032	.036	.040	.045	.051	.057	.064	.072	.081	.091	.114	.128	5/32	3/16	T / R
1/32	.00066	.00067	.00068	.00070	.00072	.00074	.00077	.00079	.00082													1/32
1/16	.00120	.00121	.00123	.00125	.00126	.00129	.00131	.00135	.00137	.00140	.00144	.00149	.00153	.00159	.00165							1/16
3/32	.00174	.00176	.00177	.00179	.00180	.00183	.00186	.00188	.00191	.00195	.00199	.00203	.00208	.00213	.00220	.00226	.00234					3/32
1/8	.00229	.00230	.00232	.00234	.00235	.00238	.00240	.00243	.00246	.00249	.00253	.00258	.00262	.00268	.00274	.00281	.00289	.00307	.00317			1/8
5/32	.00283	.00285	.00286	.00288	.00290	.00292	.00295	.00297	.00300	.00304	.00308	.00312	.00317	.00322	.00328	.00335	.00343	.00362	.00372	.00394		5/32
3/16	.00338	.00339	.00341	.00342	.00344	.00347	.00349	.00352	.00355	.00358	.00362	.00367	.00371	.00377	.00383	.00390	.00398	.00416	.00426	.00449	.00473	3/16
7/32	.00392	.00394	.00395	.00397	.00398	.00401	.00403	.00406	.00409	.00412	.00417	.00421	.00426	.00431	.00437	.00444	.00452	.00471	.00481	.00503	.00528	7/32
1/4	.00447	.00448	.00450	.00451	.00454	.00456	.00458	.00461	.00464	.00467	.00471	.00476	.00480	.00486	.00492	.00499	.00507	.00525	.00535	.00558	.00582	1/4
9/32	.00501	.00501	.00504	.00506	.00507	.00510	.00512	.00515	.00518	.00521	.00526	.00530	.00535	.00540	.00546	.00553	.00561	.00579	.00590	.00612	.00636	9/32
5/16	.00556	.00557	.00559	.00560	.00562	.00564	.00567	.00570	.00573	.00576	.00580	.00584	.00589	.00595	.00601	.00608	.00616	.00634	.00644	.00667	.00691	5/16
11/32	.00610	.00612	.00613	.00615	.00616	.00619	.00621	.00624	.00627	.00630	.00634	.00639	.00644	.00649	.00655	.00662	.00670	.00688	.00699	.00721	.00745	11/32
3/8	.00665	.00666	.00668	.00669	.00671	.00673	.00676	.00679	.00682	.00685	.00689	.00693	.00698	.00704	.00710	.00717	.00725	.00743	.00753	.00776	.00800	3/8
13/32	.00719	.00721	.00722	.00724	.00725	.00728	.00730	.00733	.00736	.0073	.00743	.00748	.00753	.00758	.00764	.00771	.00779	.00797	.00808	.00830	.00854	13/32
7/16	.00774	.00775	.00777	.00778	.00780	.00782	.00785	.00787	.00791	.00794	.00798	.00802	.00807	.00812	.00819	.00826	.00834	.00852	.00862	.00884	.00909	7/16
15/32	.00828	.00829	.00831	.00833	.00834	.00837	.00839	.00842	.00845	.00848	.00852	.00857	.00862	.00867	.00873	.00880	.00888	.00906	.00917	.00939	.00963	15/32
1/2	.00883	.00884	.00885	.00887	.00889	.00891	.00894	.00896	.00899	.00903	.00907	.00911	.00916	.00921	.00928	.00935	.00943	.00961	.00971	.00993	.01018	1/2
17/32	.00937	.00938	.00940	.00942	.00943	.00946	.00948	.00951	.00954	.00957	.00961	.00966	.00970	.00976	.00982	.00989	.00997	.01015	.01025	.01048	.01072	17/32
9/16	.00992	.00993	.00994	.00996	.00998	.01000	.01002	.01005	.01008	.01012	.01016	.01020	.01025	.01030	.01037	.01043	.01051	.01070	.01080	.01102	.01127	9/16
19/32	.01046	.01047	.01049	.01051	.01052	.01055	.01057	.01060	.01063	.01065	.01070	.01073	.01079	.01083	.01091	.01098	.01105	.01124	.01133	.01157	.01181	19/32
5/8	.01100	.01102	.01103	.01105	.01107	.01109	.01112	.01114	.01117	.01121	.01125	.01129	.01134	.01139	.01146	.01152	.01160	.01179	.01189	.01211	.01236	5/8
21/32	.01155	.01156	.01158	.01160	.01161	.01164	.01166	.01170	.01172	.01175	.01179	.01183	.01188	.01193	.01200	.01207	.01214	.01233	.01245	.01266	.01290	21/32
11/16	.01209	.01211	.01212	.01214	.01216	.01218	.01220	.01223	.01226	.01230	.01234	.01238	.01243	.01248	.01254	.01261	.01269	.01288	.01298	.01320	.01345	11/16
23/32	.01264	.01265	.01267	.01268	.01269	.01273	.01275	.01276	.01281	.01283	.01288	.01291	.01297	.01301	.01309	.01316	.01322	.01342	.01351	.01374	.01399	23/32
3/4	.01318	.01320	.01321	.01323	.01324	.01327	.01329	.01332	.01335	.01338	.01343	.01347	.01352	.01357	.01363	.01370	.01378	.01396	.01407	.01429	.01454	3/4
25/32	.01373	.01374	.01376	.01378	.01379	.01381	.01384	.01386	.01390	.01392	.01397	.01401	.01406	.01411	.01418	.01425	.01432	.01451	.01461	.01484	.01508	25/32
13/16	.01427	.01429	.01430	.01432	.01433	.01436	.01438	.01441	.01444	.01447	.01451	.01456	.01461	.01466	.01472	.01479	.01487	.01505	.01516	.01538	.01562	13/16
27/32	.01482	.01483	.01485	.01486	.01487	.01490	.01493	.01494	.01499	.01501	.01506	.01509	.01515	.01519	.01527	.01534	.01540	.01560	.01569	.01593	.01617	27/32
7/8	.01536	.01538	.01539	.01541	.01542	.01545	.01547	.01550	.01553	.01556	.01560	.01565	.01570	.01575	.01581	.01588	.01596	.01614	.01625	.01647	.01671	7/8
29/32	.01591	.01592	.01594	.01595	.01596	.01599	.01602	.01604	.01608	.01611	.01615	.01619	.01624	.01629	.01636	.01643	.01650	.01669	.01679	.01701	.01726	29/32
15/16	.01645	.01646	.01648	.01650	.01651	.01654	.01656	.01659	.01662	.01665	.01669	.01674	.01679	.01684	.01690	.01697	.01705	.01723	.01734	.01756	.01780	15/16
31/32	.01700	.01701	.01702	.01704	.01705	.01708	.01711	.01712	.01717	.01718	.01724	.01727	.01733	.01737	.01745	.01752	.01758	.01778	.01787	.01810	.01835	31/32
1	.01754	.01755	.01757	.01759	.01760	.01763	.01765	.01768	.01771	.01774	.01778	.01783	.01788	.01793	.01799	.01806	.01814	.01832	.01843	.01865	.01889	1

Values given are based on the empirical formula $(0.01743R + .0078T) \times$ number of degrees; R = radius of bend; T = thickness of sheet in inches. Values given are for 1 deg. of the given radius $\times$ the thickness. Values omitted from table are not to be used as the bends are too sharp for satisfactory production.

Upper heavy staggered line indicates Dowmetal MA, Alcoa AM3S-O. Lower heavy staggered line indicates Dowmetal MH, Alcoa AM3S-H.

Total developed length = $X + Y + Z$

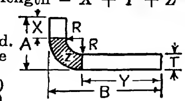
T = thickness of metal.

R = radius of bend.

Z = $B.A.$ from the chart.

$X = A - (R + T)$

$Y = B - (R + T)$



DECIMALS, LOGARITHMS, CIRCUMFERENCE, AREA, SQUARE, SQUARE ROOT, CUBE, CUBE ROOT, FOURTH POWER, FOURTH ROOT, FIFTH POWER, AND FIFTH ROOT OF NUMBERS FROM 1/32 TO 12 INCHES BY THIRTY-SECONDS

N	Decimal	Logarithm	Circumference	Area	N^2	$\sqrt{N}$
1/32	.03125	8.4948500 − 10	.09817	.00077	.000976	.176777
1/16	.06250	8.7958800 − 10	.19635	.00307	.003906	.250000
3/32	.09375	8.9719713 − 10	.29453	.00690	.008789	.306186
1/8	.12500	9.0969100 − 10	.39270	.01227	.015625	.353553
5/32	.15625	9.1938200 − 10	.49088	.01917	.024414	.395285
3/16	.18750	9.2730013 − 10	.58905	.02761	.035156	.433013
7/32	.21875	9.3399481 − 10	.68723	.03758	.047852	.467707
1/4	.25000	9.3979400 − 10	.78540	.04909	.062500	.500000
9/32	.28125	9.4490925 − 10	.88358	.06213	.079102	.530330
5/16	.31250	9.4948500 − 10	.98175	.07670	.097656	.559017
11/32	.34375	9.5362427 − 10	1.079925	.09281	.118164	.586302
3/8	.37500	9.5740313 − 10	1.17810	.11045	.140625	.612372
13/32	.40625	9.6087934 − 10	1.27628	.12962	.165039	.637377
7/16	.43750	9.6409781 − 10	1.37445	.15033	.191406	.661438
15/32	.46875	9.6709413 − 10	1.47263	.17257	.219727	.684653
1/2	.50000	9.6989700 − 10	1.57080	.19635	.250000	.707107
17/32	.53125	9.7252989 − 10	1.66898	.22166	.282227	.728869
9/16	.56250	9.7501225 − 10	1.76715	.24851	.316406	.750000
19/32	.59375	9.7736036 − 10	1.86733	.27828	.354320	.770552
5/8	.62500	9.7958800 − 10	1.96350	.30680	.390625	.790570
21/32	.65625	9.8170693 − 10	2.06168	.33824	.430664	.810093
11/16	.68750	9.8372727 − 10	2.15985	.37122	.472656	.829156
23/32	.71875	9.8565779 − 10	2.25803	.40574	.516602	.847791
3/4	.75000	9.8750613 − 10	2.35620	.44179	.562500	.866026
25/32	.78125	9.8927900 − 10	2.45438	.47937	.610352	.883884
13/16	.81250	9.9098234 − 10	2.55255	.51849	.660156	.901388
27/32	.84375	9.9262138 − 10	2.65073	.55914	.711914	.918659
7/8	.87500	9.9420081 − 10	2.74890	.60132	.765625	.935415
29/32	.90625	9.9572480 − 10	2.84708	.64504	.821289	.951972
15/16	.93750	9.9719713 − 10	2.94525	.69029	.878906	.968246
31/32	.96875	9.9862117 − 10	3.04343	.73708	.938477	.984251
1	1.00000	.0000000	3.14160	.78540	1.000000	1.000000
1 1/32	1.03125	.0133640	3.23978	.83526	1.06348	1.01550
1 1/16	1.06250	.0263289	3.33795	.88665	1.12891	1.03078
1 3/32	1.09375	.0389181	3.43613	.93957	1.19629	1.04583
1 1/8	1.12500	.0511525	3.53430	.99402	1.26562	1.06064
1 5/32	1.15625	.0630518	3.63248	1.05000	1.33691	1.07529
1 3/16	1.18750	.0746336	3.73065	1.10754	1.41016	1.08973
1 7/32	1.21875	.0859146	3.82883	1.16659	1.48535	1.10397
1 1/4	1.25000	.0969100	3.92700	1.22719	1.56250	1.11804
1 9/32	1.28125	.1076339	4.02518	1.28931	1.64160	1.13192
1 5/16	1.31250	.1180993	4.12335	1.35298	1.72266	1.14564
1 11/32	1.34375	.1283186	4.22153	1.41817	1.80566	1.15920
1 3/8	1.37500	.1383027	4.31970	1.48489	1.89062	1.17260
1 13/32	1.40625	.1480626	4.41788	1.55316	1.97754	1.18449
1 7/16	1.43750	.1576079	4.51605	1.62296	2.06641	1.19896
1 15/32	1.46875	.1669479	4.61423	1.69429	2.15723	1.21192
1 1/2	1.50000	.1760913	4.71240	1.76715	2.25000	1.22474
1 17/32	1.53125	.1850461	4.81058	1.84155	2.34473	1.23744
1 9/16	1.56250	.1938200	4.90875	1.91748	2.44141	1.25000
1 19/32	1.59375	.2024202	5.00693	1.99495	2.54004	1.26244
1 5/8	1.62500	.2108534	5.10510	2.07395	2.64063	1.27475
1 21/32	1.65625	.2191259	5.20328	2.15448	2.74316	1.28696
1 11/16	1.68750	.2272438	5.30145	2.23655	2.84766	1.29904
1 23/32	1.71875	.2352127	5.39963	2.32015	2.95410	1.31101
1 3/4	1.75000	.2430380	5.49780	2.40529	3.06250	1.32288

N^3	$\sqrt[3]{N}$	N^4	$\sqrt[4]{N}$	N^5	$\sqrt[5]{N}$	N
.000031	.314880	.000001	.420448	.000000	.500000	1/32
.000244	.396850	.000015	.500000	.000000	.574349	1/16
.000824	.454280	.000077	.553341	.000007	.622866	3/32
.001953	.500000	.000244	.594604	.000031	.659764	1/8
.003815	.538611	.000596	.628717	.000093	.689865	5/32
.006592	.572357	.001236	.658037	.000232	.715485	3/16
.010468	.602536	.002290	.683891	.000501	.737887	7/32
.015625	.629960	.003906	.707107	.000977	.757858	1/4
.022247	.655185	.006257	.728238	.001760	.775923	9/32
.030518	.678604	.009537	.747674	.002980	.792447	5/16
.040619	.700510	.013963	.765704	.004800	.807697	11/32
.052734	.721125	.019776	.782542	.007416	.821876	3/8
.067047	.740624	.027238	.798359	.011065	.835139	13/32
.083740	.750147	.036636	.813288	.016028	.847609	7/16
.102997	.776808	.048280	.827438	.022631	.859386	15/32
.125000	.793701	.062500	.840896	.031250	.870551	1/2
.149933	.809903	.079652	.853738	.042315	.881170	17/32
.177978	.825482	.100113	.866025	.056314	.891301	9/16
.210378	.840494	.125543	.877811	.074541	.900992	19/32
.244141	.854988	.152588	.889140	.095368	.910282	5/8
.282623	.869007	.185472	.900052	.121716	.919208	21/32
.324951	.882587	.223404	.910580	.153590	.927800	11/16
.371308	.895762	.266878	.920756	.191819	.936086	23/32
.421875	.908560	.316406	.930605	.237305	.944088	3/4
.476838	.921008	.372530	.940151	.291039	.951827	25/32
.536377	.933128	.435806	.949415	.354092	.959323	13/16
.600677	.944941	.506822	.958415	.427631	.966591	27/32
.669922	.956466	.586182	.967168	.512909	.973647	7/8
.744293	.967719	.674516	.975690	.611280	.980505	29/32
.823974	.978717	.772476	.983995	.724196	.987175	15/16
.909150	.989473	.880739	.992094	.853216	.993670	31/32
1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1
1.09680	1.01031	1.13099	1.00772	1.16633	1.00617	1 1/32
1.19946	1.02041	1.27444	1.01527	1.35409	1.01220	1 1/16
1.30844	1.03030	1.43111	1.02266	1.56528	1.01808	1 3/32
1.42383	1.04004	1.60179	1.02987	1.80201	1.02384	1 1/8
1.54581	1.04958	1.78733	1.03696	2.06660	1.02946	1 5/32
1.67456	1.05896	1.98855	1.04390	2.36140	1.03497	1 3/16
1.81027	1.06817	2.20627	1.05070	2.68889	1.04036	1 7/32
1.95313	1.07722	2.44141	1.05737	3.05176	1.04564	1 1/4
2.10330	1.08612	2.69485	1.06392	3.45278	1.05082	1 9/32
2.26099	1.09490	2.96756	1.07035	3.89492	1.05589	1 5/16
2.42636	1.10350	3.26041	1.07666	4.38118	1.06088	1 11/32
2.59961	1.10809	3.57444	1.08287	4.91486	1.06576	1 3/8
2.78091	1.11949	3.91066	1.08834	5.49937	1.07056	1 13/32
2.97046	1.12859	4.27005	1.09497	6.13820	1.07526	1 7/16
3.16893	1.13671	4.65364	1.10087	6.83503	1.07992	1 15/32
3.37500	1.14472	5.06250	1.10668	7.59373	1.08447	1 1/2
3.59036	1.15261	5.49774	1.11240	8.41842	1.08895	1 17/32
3.81470	1.16040	5.96046	1.11804	9.31322	1.09336	1 9/16
4.04819	1.16808	6.45160	1.12358	10.2816	1.09770	1 19/32
4.29102	1.17567	6.97290	1.12905	11.3310	1.10197	1 5/8
4.54337	1.18316	7.52495	1.13444	12.4632	1.10618	1 21/32
4.80542	1.19055	8.10915	1.13975	13.6842	1.11032	1 11/16
5.07736	1.19786	8.72672	1.14499	14.9990	1.11440	1 23/32
5.35938	1.20507	9.37890	1.15016	16.4131	1.11843	1 3/4

N	Decimal	Logarithm	Circumference	Area	N^2	$\sqrt{N}$
1 25/32	1.78125	.2507249	5.59598	2.49196	3.17285	1.33464
1 13/16	1.81250	.2582780	5.69415	2.58015	3.28516	1.34629
1 27/32	1.84375	.2657021	5.79233	2.66990	3.39941	1.35785
1 7/8	1.87500	.2730013	5.89050	2.76118	3.51563	1.36931
1 29/32	1.90625	.2801799	5.98868	2.85398	3.63379	1.38067
1 15/16	1.93750	.2872417	6.08685	2.94832	3.75391	1.39194
1 31/32	1.96875	.2941906	6.18503	3.04419	3.87597	1.40312
2	2.00000	.3010300	6.28320	3.14160	4.00000	1.41421
2 1/32	2.03125	.3077634	6.38138	3.24055	4.12598	1.42522
2 1/16	2.06250	.3143940	6.47955	3.34102	4.25391	1.43614
2 3/32	2.09375	.3209249	6.57773	3.44303	4.38379	1.44698
2 1/8	2.12500	.3273589	6.67590	3.54657	4.51562	1.45774
2 5/32	2.15625	.3336991	6.77408	3.65165	4.64941	1.46842
2 3/16	2.18750	.3399481	6.87225	3.75827	4.78516	1.47902
2 7/32	2.21875	.3461084	6.97043	3.86641	4.92285	1.48955
2 1/4	2.25000	.3521825	7.06860	3.97609	5.06250	1.50000
2 9/32	2.28125	.3581729	7.16678	4.08730	5.20410	1.51038
2 5/16	2.31250	.3640817	7.26495	4.20005	5.34766	1.52069
2 11/32	2.34375	.3699113	7.36313	4.31433	5.49316	1.53093
2 3/8	2.37500	.3756636	7.46130	4.43014	5.64062	1.54110
2 13/32	2.40625	.3813408	7.55948	4.54778	5.79004	1.55121
2 7/16	2.43750	.3869446	7.65765	4.66638	5.94141	1.56125
2 15/32	2.46875	.3924771	7.75583	4.78680	6.09473	1.57123
2 1/2	2.50000	.3979400	7.85400	4.90875	6.25000	1.58114
2 17/32	2.53125	.4033351	7.95218	5.03224	6.40723	1.59099
2 9/16	2.56250	.4086639	8.05035	5.15725	6.56640	1.60074
2 19/32	2.59375	.4139281	8.14853	5.28381	6.72754	1.61051
2 5/8	2.62500	.4191293	8.24670	5.41190	6.89063	1.62018
2 21/32	2.65625	.4242690	8.34488	5.54152	7.05566	1.62980
2 11/16	2.68750	.4293485	8.44305	5.65368	7.19847	1.63936
2 23/32	2.71875	.4343693	8.54125	5.80536	7.39160	1.64886
2 3/4	2.75000	.4393327	8.63940	5.93959	7.56250	1.65931
2 25/32	2.78125	.4442400	8.73758	6.07534	7.73535	1.66005
2 13/16	2.81250	.4490925	8.83575	6.21264	7.91016	1.67705
2 27/32	2.84375	.4538914	8.93393	6.35146	8.08691	1.68634
2 7/8	2.87500	.4586378	9.03210	6.49183	8.26563	1.69558
2 29/32	2.90625	.4633330	9.13028	6.63372	8.44629	1.70477
2 15/16	2.93750	.4679779	9.22845	6.77715	8.62891	1.71392
2 31/32	2.96875	.4725736	9.32663	6.92211	8.81348	1.72301
3	3.00000	.4771213	9.42480	7.06860	9.00000	1.73205
3 1/32	3.03125	.4816218	9.52298	7.21663	9.18848	1.74105
3 1/16	3.06250	.4860761	9.62115	7.36620	9.37891	1.75000
3 3/32	3.09375	.4904852	9.71933	7.51729	9.57129	1.75891
3 1/8	3.12500	.4948500	9.81750	7.66993	9.76563	1.76777
3 5/32	3.15625	.4991714	9.91568	7.82409	9.96191	1.77658
3 3/16	3.18750	.5034502	10.01385	7.97979	10.16016	1.78536
3 7/32	3.21875	.5076873	10.11203	8.13702	10.36035	1.79409
3 1/4	3.25000	.5118834	10.21020	8.29579	10.56250	1.80278
3 9/32	3.28125	.5160393	10.30838	8.45609	10.76660	1.81142
3 5/16	3.31250	.5201559	10.40655	8.61793	10.97266	1.82003
3 11/32	3.34375	.5242338	10.50473	8.78129	11.18066	1.82859
3 3/8	3.37500	.5282738	10.60290	8.94620	11.39063	1.83712
3 13/32	3.40625	.5322765	10.70108	9.11264	11.60254	1.84560
3 7/16	3.43750	.5362427	10.79925	9.28061	11.81641	1.85405
3 15/32	3.46875	.5401730	10.89743	9.45011	12.03223	1.86246
3 1/2	3.50000	.5440680	10.99560	9.62115	12.25000	1.87083

N^3	$\sqrt[3]{N}$	N^4	$\sqrt[4]{N}$	N^5	$\sqrt[5]{N}$	N
5.65164	1.21220	10.0693	1.15526	17.9318	1.12240	1 25/32
5.95435	1.21925	10.7923	1.16030	19.5609	1.12630	1 13/16
6.26766	1.22622	11.5560	1.16527	21.3064	1.13016	1 27/32
6.59180	1.23310	12.3596	1.17017	23.1741	1.13397	1 7/8
6.92691	1.23992	13.2044	1.17502	25.1710	1.13772	1 29/32
7.27319	1.24666	14.0918	1.17981	27.3029	1.14133	1 15/16
7.63083	1.25333	15.0232	1.18453	29.5769	1.14509	1 31/32
8.00000	1.25983	16.0000	1.18911	32.0000	1.14870	2
8.38090	1.26645	17.0237	1.18834	34.5794	1.15227	2 1/32
8.77369	1.27291	18.0958	1.19839	37.3226	1.15579	2 1/16
9.17856	1.27931	19.2176	1.20390	40.2369	1.15927	2 3/32
9.59569	1.28564	20.3908	1.20737	43.3305	1.16271	2 1/8
10.0253	1.29191	21.6170	1.21178	46.6117	1.16611	2 5/32
10.4675	1.29812	22.8978	1.21615	50.0889	1.16946	2 3/16
10.9226	1.30428	24.2345	1.22047	53.7703	1.17279	2 7/32
11.3906	1.31037	25.6289	1.22475	57.6650	1.17608	2 1/4
11.8719	1.31641	27.0827	1.22898	61.7824	1.17933	2 9/32
12.3665	1.32239	28.5975	1.22316	66.1317	1.18254	2 5/16
12.8746	1.32832	30.1748	1.23731	70.7222	1.18572	2 11/32
13.3965	1.33420	31.8166	1.24141	75.5644	1.18887	2 3/8
13.9323	1.34003	33.5246	1.24545	80.6686	1.19198	2 13/32
14.4822	1.34580	35.3004	1.24949	86.0447	1.19503	2 7/16
15.0464	1.35153	37.1457	1.25349	91.7034	1.19811	2 15/32
15.6250	1.35721	39.0625	1.25743	97.6563	1.20112	2 1/2
16.2183	1.36284	41.0526	1.26134	103.9144	1.20411	2 17/32
16.8264	1.36843	43.1176	1.26522	110.4889	1.20707	2 9/16
17.4236	1.37397	45.1925	1.26906	117.2180	1.21000	2 19/32
18.0879	1.37946	47.4808	1.27286	124.6371	1.21290	2 5/8
18.7416	1.38492	49.7823	1.27664	132.2342	1.21578	2 21/32
19.3459	1.39032	51.8180	1.28037	139.2609	1.21862	2 11/16
20.0959	1.39569	54.6358	1.28408	148.5411	1.22145	2 23/32
20.7969	1.40102	57.1914	1.28775	157.2764	1.22424	2 3/4
21.5139	1.40631	59.8356	1.29140	166.4178	1.22701	2 25/32
22.2473	1.41155	62.5706	1.29501	175.9798	1.22976	2 13/16
22.9972	1.41676	65.3981	1.29859	185.9759	1.23245	2 27/32
23.7637	1.42193	68.3206	1.30215	196.4217	1.23517	2 7/8
24.5470	1.42707	71.3398	1.30567	207.3313	1.23785	2 29/32
25.3474	1.43216	74.4581	1.30917	218.7207	1.24050	2 15/16
26.1650	1.43722	77.6774	1.31263	230.6048	1.24312	2 31/32
27.0000	1.44125	81.0000	1.31607	243.0000	1.24573	3
27.8526	1.44724	84.4282	1.31949	255.9230	1.24832	3 1/32
28.7229	1.45220	87.9640	1.32288	269.3898	1.25088	3 1/16
29.6112	1.45712	91.6096	1.32624	283.4172	1.25349	3 3/32
30.5176	1.46201	95.3675	1.32957	298.0234	1.25594	3 1/8
31.4423	1.46687	99.2397	1.33289	313.2253	1.25845	3 5/32
32.3855	1.47169	103.2289	1.33617	329.0421	1.26093	3 3/16
33.3474	1.47649	107.3369	1.33944	345.4906	1.26339	3 7/32
34.3281	1.48125	111.5664	1.34268	362.5908	1.26583	3 1/4
35.3279	1.48595	115.9197	1.34589	380.3320	1.26826	3 9/32
36.3469	1.49068	120.3993	1.34908	398.8227	1.27067	3 5/16
37.3853	1.49536	125.0072	1.35223	417.9928	1.27305	3 11/32
38.4434	1.50000	129.7465	1.35540	437.8944	1.27542	3 3/8
39.5212	1.50462	134.6189	1.35853	458.5456	1.27778	3 13/32
40.6189	1.50920	139.6276	1.36164	479.9699	1.28011	3 7/16
41.7368	1.51376	144.7746	1.36472	502.1869	1.28243	3 15/32
42.8750	1.51830	150.0625	1.36778	525.2188	1.28474	3 1/2

N	Decimal	Logarithm	Circumference	Area	N^2	$\sqrt{N}$
$3\frac{17}{32}$	3.53125	.5479285	11.09378	9.62115	12.25000	1.87083
$3\frac{9}{16}$	3.56250	.5517549	11.19195	9.96783	12.69141	1.88746
$3\frac{19}{32}$	3.59375	.5555478	11.29013	10.14347	12.91504	1.89572
$3\frac{5}{8}$	3.62500	.5593080	11.38830	10.32065	13.14063	1.90394
$3\frac{21}{32}$	3.65625	.5630359	11.48648	10.49935	13.36816	1.91213
$3\frac{11}{16}$	3.68750	.5667320	11.58465	10.67960	13.59766	1.92029
$3\frac{23}{32}$	3.71875	.5703970	11.68283	10.86138	13.82910	1.92841
$3\frac{3}{4}$	3.75000	.5740313	11.78100	11.04469	14.06250	1.93649
$3\frac{25}{32}$	3.78125	.5776354	11.87918	11.22953	14.29785	1.94454
$3\frac{13}{16}$	3.81250	.5812099	11.97735	11.41591	14.53516	1.95256
$3\frac{27}{32}$	3.84375	.5847552	12.07553	11.60382	14.77441	1.96055
$3\frac{7}{8}$	3.87500	.5882717	12.17370	11.79328	15.01563	1.96850
$3\frac{29}{32}$	3.90625	.5917601	12.27188	11.98425	15.25879	1.97642
$3\frac{15}{16}$	3.93750	.5952206	12.37005	12.17677	15.50391	1.98431
$3\frac{31}{32}$	3.96875	.5986538	12.46823	12.37082	15.75098	1.99217
4	4.00000	.6020600	12.56640	12.56640	16.00000	2.00000
$4\frac{1}{32}$	4.03125	.6054397	12.66458	12.76352	16.25098	2.00779
$4\frac{1}{16}$	4.06250	.6087934	12.76275	12.96216	16.50390	2.01556
$4\frac{3}{32}$	4.09375	.6121213	12.86093	13.16235	16.75879	2.02330
$4\frac{1}{8}$	4.12500	.6154240	12.95910	13.36408	17.01563	2.03101
$4\frac{5}{32}$	4.15625	.6187017	13.05728	13.56732	17.27441	2.03869
$4\frac{3}{16}$	4.18750	.6219548	13.15545	13.77212	17.53516	2.04634
$4\frac{7}{32}$	4.21875	.6251838	13.25363	13.97843	17.79785	2.05396
$4\frac{1}{4}$	4.25000	.6283889	13.35180	14.18629	18.06250	2.06155
$4\frac{9}{32}$	4.28125	.6315706	13.44998	14.39568	18.32910	2.06912
$4\frac{5}{16}$	4.31250	.6347291	13.54815	14.60660	18.59766	2.07666
$4\frac{11}{32}$	4.34375	.6378648	13.64633	14.81905	18.86816	2.08417
$4\frac{3}{8}$	4.37500	.6409781	13.74450	15.03305	19.14063	2.09165
$4\frac{13}{32}$	4.40625	.6440692	13.84268	15.24857	19.41504	2.09911
$4\frac{7}{16}$	4.43750	.6471384	13.94085	15.46563	19.69141	2.10654
$4\frac{15}{32}$	4.46875	.6501861	14.03903	15.68423	19.96973	2.11394
$4\frac{1}{2}$	4.50000	.6532125	14.13720	15.90435	20.25000	2.12133
$4\frac{17}{32}$	4.53125	.6562180	14.23538	16.12601	20.53223	2.12867
$4\frac{9}{16}$	4.56250	.6592029	14.33355	16.34921	20.81641	2.13600
$4\frac{19}{32}$	4.59375	.6621674	14.43173	16.57394	21.10254	2.14330
$4\frac{5}{8}$	4.62500	.6651117	14.52990	16.80020	21.39063	2.15058
$4\frac{21}{32}$	4.65625	.6680364	14.62808	17.02799	21.68066	2.15784
$4\frac{11}{16}$	4.68750	.6709413	14.72625	17.25733	21.97266	2.16506
$4\frac{23}{32}$	4.71875	.6738270	14.82443	17.48819	22.26660	2.17227
$4\frac{3}{4}$	4.75000	.6766936	14.92260	17.72059	22.56250	2.17945
$4\frac{25}{32}$	4.78125	.6795415	15.02078	17.95452	22.86035	2.18661
$4\frac{13}{16}$	4.81250	.6823707	15.11895	18.18999	23.16016	2.19374
$4\frac{27}{32}$	4.84375	.6851817	15.21713	18.42698	23.46191	2.20085
$4\frac{7}{8}$	4.87500	.6879746	15.31530	18.66553	23.76563	2.20794
$4\frac{29}{32}$	4.90625	.6907497	15.41348	18.90559	24.07129	2.21501
$4\frac{15}{16}$	4.93750	.6935071	15.51165	19.14719	24.37890	2.22205
$4\frac{31}{32}$	4.96875	.6962472	15.60983	19.39033	24.68848	2.22907
5	5.00000	.6989700	15.70800	19.63500	25.00000	2.23607
$5\frac{1}{32}$	5.03125	.7016762	15.80618	19.88121	25.31348	2.24305
$5\frac{1}{16}$	5.06250	.7043650	15.90435	20.12895	25.62891	2.25000
$5\frac{3}{32}$	5.09375	.7070377	16.00253	20.37822	25.94629	2.25693
$5\frac{1}{8}$	5.12500	.7096939	16.10070	20.62903	26.26563	2.26385
$5\frac{5}{32}$	5.15625	.7123340	16.19888	20.88136	26.58691	2.27074
$5\frac{3}{16}$	5.18750	.7149581	16.29705	21.13524	26.91016	2.27761
$5\frac{7}{32}$	5.21875	.7175665	16.39523	21.39064	27.23535	2.28446
$5\frac{1}{4}$	5.25000	.7201593	16.49340	21.65759	27.56250	2.29129

N^3	$\sqrt[3]{N}$	N^4	$\sqrt[4]{N}$	N^5	$\sqrt[5]{N}$	N
44.0337	1.52280	155.4942	1.37083	549.0889	1.28702	$3\frac{17}{32}$
45.2132	1.52727	161.0719	1.37385	573.8186	1.28929	$3\frac{9}{16}$
46.4134	1.53173	166.7983	1.37685	599.4314	1.29155	$3\frac{19}{32}$
47.6348	1.53616	172.6762	1.37983	625.9512	1.29378	$3\frac{5}{8}$
48.8773	1.54056	178.7077	1.38280	653.4000	1.29601	$3\frac{21}{32}$
50.1414	1.54494	184.8964	1.38574	681.8055	1.29822	$3\frac{11}{16}$
51.4270	1.54929	191.2442	1.38854	711.2094	1.30041	$3\frac{23}{32}$
52.7344	1.55362	197.7539	1.39158	741.5771	1.30259	$3\frac{3}{4}$
54.0638	1.55792	204.4285	1.39447	772.9953	1.30475	$3\frac{25}{32}$
55.4153	1.56220	211.2709	1.39734	805.4703	1.30690	$3\frac{13}{16}$
56.7891	1.56646	218.2832	1.40020	839.0261	1.30904	$3\frac{27}{32}$
58.1856	1.57069	225.4691	1.40303	873.6928	1.31116	$3\frac{7}{8}$
59.6047	1.57490	232.8307	1.40585	909.4949	1.31353	$3\frac{29}{32}$
61.0467	1.57909	240.3712	1.40866	946.4616	1.31536	$3\frac{15}{16}$
62.5117	1.58326	248.0934	1.41144	984.6207	1.31744	$3\frac{31}{32}$
64.0000	1.58740	256.0000	1.41421	1024.0000	1.31951	4
65.5118	1.59152	264.0944	1.41697	1064.631	1.32156	$4\frac{1}{32}$
67.0471	1.59563	272.3787	1.41971	1106.539	1.32361	$4\frac{1}{16}$
68.6063	1.59971	280.8570	1.42243	1149.759	1.32564	$4\frac{3}{32}$
70.1895	1.60008	289.5317	1.42514	1194.318	1.32765	$4\frac{1}{8}$
71.7968	1.60781	298.4052	1.42783	1240.247	1.32966	$4\frac{5}{32}$
73.4285	1.61183	307.4818	1.43050	1287.580	1.33165	$4\frac{3}{16}$
75.0847	1.61583	316.7635	1.43316	1336.346	1.33363	$4\frac{7}{32}$
76.7656	1.61981	326.2539	1.43581	1386.579	1.33560	$4\frac{1}{4}$
78.4715	1.62378	335.9559	1.43844	1438.311	1.33756	$4\frac{9}{32}$
80.2024	1.62771	345.8730	1.44106	1491.580	1.33951	$4\frac{5}{16}$
81.9586	1.63163	356.0075	1.44366	1546.408	1.34145	$4\frac{11}{32}$
83.7403	1.63553	366.3637	1.44625	1602.841	1.34337	$4\frac{3}{8}$
85.5475	1.63942	376.9438	1.44883	1660.909	1.34528	$4\frac{13}{32}$
87.3806	1.64329	387.7516	1.45139	1720.648	1.34719	$4\frac{7}{16}$
89.2397	1.64713	398.7901	1.45394	1782.093	1.34908	$4\frac{15}{32}$
91.1250	1.65093	410.0625	1.45648	1845.281	1.35096	$4\frac{1}{2}$
93.0367	1.65478	421.5725	1.45900	1910.250	1.35283	$4\frac{17}{32}$
94.9749	1.65857	433.3229	1.46151	1977.036	1.35469	$4\frac{9}{16}$
96.9398	1.66235	445.3172	1.46400	2045.676	1.35654	$4\frac{19}{32}$
98.9317	1.66611	457.5591	1.46649	2116.211	1.35838	$4\frac{5}{8}$
100.9506	1.66986	470.0510	1.46896	2188.675	1.36021	$4\frac{21}{32}$
102.9968	1.67358	482.7978	1.47142	2263.115	1.36204	$4\frac{11}{16}$
105.0706	1.67729	495.8017	1.47386	2339.612	1.36385	$4\frac{23}{32}$
107.1719	1.68099	509.0664	1.47630	2418.065	1.36565	$4\frac{3}{4}$
109.3011	1.68467	522.5956	1.47872	2498.660	1.36744	$4\frac{25}{32}$
111.4583	1.68833	536.3930	1.48113	2581.391	1.36922	$4\frac{13}{16}$
113.6436	1.69198	550.4612	1.48353	2666.296	1.37100	$4\frac{27}{32}$
115.8575	1.69561	564.8052	1.48591	2753.425	1.37276	$4\frac{7}{8}$
118.0998	1.69922	579.4270	1.48829	2842.814	1.37452	$4\frac{29}{32}$
120.3708	1.70282	594.3308	1.49065	2934.508	1.37626	$4\frac{15}{16}$
122.6709	1.70641	609.5211	1.49301	3028.558	1.37800	$4\frac{31}{32}$
125.0000	1.70998	625.0000	1.49535	3125.000	1.37973	5
127.3584	1.71353	640.7723	1.49768	3223.886	1.38145	$5\frac{1}{32}$
129.7464	1.71707	656.8410	1.50000	3325.258	1.38316	$5\frac{1}{16}$
132.1639	1.72060	673.2100	1.50231	3429.163	1.38487	$5\frac{3}{32}$
134.6114	1.72411	689.8833	1.50461	3535.652	1.38656	$5\frac{1}{8}$
137.0888	1.72761	706.8638	1.50690	3644.767	1.38825	$5\frac{5}{32}$
139.9646	1.73109	724.1567	1.50917	3756.563	1.38993	$5\frac{3}{16}$
142.1345	1.73456	741.7643	1.51144	3871.082	1.39160	$5\frac{7}{32}$
144.7031	1.73801	759.6914	1.51370	3988.380	1.39326	$5\frac{1}{4}$

N	Decimal	Logarithm	Circumference	Area	N^2	$\sqrt{N}$
5 9/32	5.28125	.7227367	16.59158	21.90606	27.89160	2.29809
5 5/16	5.31250	.7252989	16.68975	22.16765	28.22466	2.30489
5 11/32	5.34375	.7278462	16.78793	22.42762	28.55566	2.31166
5 3/8	5.37500	.7303785	16.88610	22.69070	28.89063	2.31840
5 13/32	5.40625	.7328961	16.98428	22.95531	29.22754	2.32513
5 7/16	5.43750	.7353993	17.08245	23.22146	29.56641	2.33185
5 15/32	5.46875	.7378881	17.18063	23.48914	29.90723	2.33854
5 1/2	5.50000	.7403627	17.27880	23.75835	30.25000	2.34521
5 17/32	5.53125	.7428233	17.37798	24.02910	30.59473	2.35186
5 9/16	5.56250	.7452700	17.47515	25.30138	30.94141	2.35849
5 19/32	5.59375	.7477031	17.57333	24.57520	31.29004	2.36511
5 5/8	5.62500	.7501225	17.67150	24.85055	31.64063	2.37171
5 21/32	5.65625	.7525286	17.76968	25.12743	31.99316	2.37829
5 11/16	5.68750	.7549214	17.86785	25.40585	32.34766	2.38485
5 23/32	5.71875	.7573011	17.96603	25.68580	32.70410	2.39139
5 3/4	5.75000	.7596678	18.06420	25.06720	33.06250	2.39792
5 25/32	5.78125	.7620218	18.16238	26.25031	33.42285	2.40442
5 13/16	5.81250	.7643630	18.26055	26.53486	33.78516	2.41091
5 27/32	5.84375	.7666916	18.35873	26.82095	34.14941	2.41738
5 7/8	5.87500	.7690079	18.45690	27.10858	34.51563	2.42384
5 29/32	5.90625	.7713118	18.55508	27.39773	34.88379	2.43028
5 15/16	5.93750	.7736036	18.65325	27.68842	35.25391	2.43670
5 31/32	5.96875	.7758834	18.75143	27.98065	35.62598	2.44310
6	6.00000	.7781513	18.84960	28.27440	36.00000	2.44949
6 1/32	6.03125	.7804073	18.94778	28.56970	36.37598	2.45586
6 1/16	6.06250	.7826518	19.04595	28.86652	36.75391	2.46221
6 3/32	6.09375	.7848847	19.14413	29.16488	37.13379	2.46855
6 1/8	6.12500	.7871061	19.24230	29.46478	37.51563	2.47487
6 5/32	6.15625	.7893163	19.34048	29.76620	37.89941	2.48117
6 3/16	6.18750	.7915152	19.43865	30.06917	38.28516	2.48747
6 7/32	6.21875	.7937031	19.53683	30.37523	38.67485	2.49374
6 1/4	6.25000	.7958800	19.63500	30.67969	39.06250	2.50000
6 9/32	6.28125	.7980461	19.73318	30.98725	39.45410	2.50624
6 5/16	6.31250	.8002014	19.83135	31.29635	39.84766	2.51247
6 11/32	6.34375	.8023461	19.92953	31.60698	40.24316	2.51867
6 3/8	6.37500	.8044802	20.02770	31.91915	40.64063	2.52488
6 13/32	6.40625	.8066039	20.12588	32.23285	41.04004	2.53106
6 7/16	6.43750	.8087172	20.22405	32.54808	41.44141	2.53722
6 15/32	6.46875	.8108204	20.32223	32.86485	41.84473	2.54337
6 1/2	6.50000	.8129134	20.42040	33.18315	42.25000	2.54951
6 17/32	6.53125	.8149963	20.51858	33.50299	42.65723	2.55563
6 9/16	6.56250	.8170693	20.61675	33.82436	43.06641	2.56174
6 19/32	6.59375	.8191325	20.71493	34.14726	43.47754	2.56783
6 5/8	6.62500	.8211859	20.81310	34.47170	43.89063	2.57391
6 21/32	6.65625	.8232297	20.91128	34.79767	44.30566	2.57997
6 11/16	6.68750	.8252638	21.00945	35.12518	44.72266	2.58602
6 23/32	6.71875	.8272885	21.10763	35.45421	45.14160	2.59206
6 3/4	6.75000	.8293038	21.20580	35.78479	45.56250	2.59808
6 25/32	6.78125	.8313098	21.30398	36.11689	45.98535	2.60408
6 13/16	6.81250	.8333065	21.40215	36.45054	46.41016	2.61008
6 27/32	6.84375	.8352942	21.50033	36.78571	46.83691	2.61606
6 7/8	6.87500	.8372727	21.59850	37.12243	47.26563	2.62202
6 29/32	6.90625	.8392423	21.69668	37.46067	47.69629	2.62797
6 15/16	6.93750	.8412030	21.79485	37.80045	48.12891	2.63391
6 31/32	6.96875	.8431549	21.89303	38.14176	48.56348	2.63984
7	7.00000	.8450980	21.99120	38.48460	49.00000	2.64575

N^3	$\sqrt[3]{N}$	N^4	$\sqrt[4]{N}$	N^5	$\sqrt[5]{N}$	N
147.3025	1.74146	777.9414	1.51595	4108.503	1.39491	5 9/32
149.9435	1.74488	796.6314	1.51819	4232.104	1.39656	5 5/16
152.5943	1.74830	815.4257	1.52041	4357.431	1.39820	5 11/32
155.2871	1.75170	834.6685	1.52263	4486.343	1.39983	5 3/8
158.0114	1.75509	854.2491	1.52484	4618.284	1.40146	5 13/32
160.7674	1.75846	874.1726	1.52704	4753.315	1.40307	5 7/16
163.5552	1.76182	894.4424	1.52923	4891.492	1.40468	5 15/32
166.3750	1.76558	915.0625	1.53141	5032.844	1.40628	5 1/2
169.2271	1.76851	936.0375	1.53358	5177.457	1.40788	5 17/32
172.1116	1.77184	957.3709	1.53574	5325.376	1.40946	5 9/16
175.0287	1.77515	979.0666	1.53789	5476.654	1.41104	5 19/32
177.9785	1.77845	1001.130	1.54004	5631.356	1.41262	5 5/8
180.9613	1.78173	1023.562	1.54217	5789.523	1.41418	5 21/32
183.9773	1.78501	1046.371	1.54438	5951.235	1.41574	5 11/16
187.0266	1.78827	1069.558	1.54641	6116.535	1.41730	5 23/32
190.1094	1.79152	1093.129	1.54852	6285.492	1.41884	5 3/4
193.2259	1.79476	1117.087	1.55062	6458.159	1.42038	5 25/32
196.3762	1.79799	1141.437	1.55271	6634.603	1.42191	5 13/16
199.5606	1.80121	1166.182	1.55479	6814.876	1.42344	5 27/32
202.7793	1.80441	1191.329	1.55678	6999.058	1.42496	5 7/8
206.0324	1.80761	1216.879	1.55893	7187.192	1.42647	5 29/32
209.4201	1.81079	1242.838	1.56099	7379.351	1.42798	5 15/16
212.6426	1.81396	1269.211	1.56304	7575.603	1.42948	5 31/32
216.0000	1.81712	1296.000	1.56508	7776.000	1.43097	6
219.3926	1.82027	1323.212	1.56712	7980.622	1.43246	6 1/32
222.8206	1.82341	1350.850	1.56914	8189.528	1.43394	6 1/16
226.2840	1.82554	1378.918	1.57116	8402.782	1.43541	6 3/32
229.7832	1.82965	1407.423	1.57317	8620.466	1.43688	6 1/8
233.3182	1.83276	1436.365	1.57518	8842.622	1.43839	6 5/32
236.8894	1.83586	1465.754	1.57718	9069.353	1.43980	6 3/16
240.5092	1.83894	1495.744	1.57916	9301.658	1.44125	6 7/32
244.1406	1.84202	1525.879	1.58114	9536.744	1.44270	6 1/4
247.8211	1.84508	1556.626	1.58311	9777.557	1.44414	6 9/32
251.5384	1.84814	1587.836	1.58508	10023.22	1.44557	6 5/16
255.2926	1.85118	1619.512	1.58704	10273.78	1.44700	6 11/32
259.2840	1.85422	1651.681	1.58899	10529.47	1.44843	6 3/8
262.9128	1.85724	1684.285	1.59093	10789.95	1.44984	6 13/32
266.7791	1.86025	1717.401	1.50287	11055.77	1.45125	6 7/16
270.6831	1.86326	1750.981	1.59480	11326.66	1.45266	6 15/32
274.6250	1.86626	1785.063	1.59672	11602.91	1.45406	6 1/2
278.6050	1.86924	1819.639	1.59863	11884.52	1.45546	6 17/32
282.6233	1.87222	1854.716	1.60054	12171.57	1.45685	6 9/16
286.6820	1.87519	1890.297	1.60245	12464.15	1.45823	6 19/32
290.7754	1.87814	1926.387	1.60434	12762.31	1.45961	6 5/8
294.9096	1.88109	1963.012	1.60623	13066.50	1.46099	6 21/32
299.0828	1.88403	2000.116	1.60811	13375.78	1.46236	6 11/16
303.2951	1.88696	2037.764	1.60999	13691.23	1.46372	6 23/32
307.5471	1.88988	2075.943	1.61186	14012.62	1.46508	6 3/4
311.8382	1.89280	2114.652	1.61372	14339.98	1.46643	6 25/32
316.1692	1.89570	2153.945	1.61557	14673.75	1.46778	6 13/16
320.5401	1.89859	2193.696	1.61742	15013.12	1.46913	6 27/32
324.9512	1.90148	2234.040	1.61927	19359.03	1.47046	6 7/8
329.4025	1.90431	2274.936	1.62110	15711.28	1.47180	6 29/32
333.8943	1.90722	2316.392	1.62293	16069.97	1.47313	6 15/16
338.4268	1.91008	2358.512	1.62476	16435.88	1.47445	6 31/32
343.0000	1.91297	2401.000	1.62658	16807.00	1.47577	7

N	Decimal	Logarithm	Circum-ference	Area	N^2	$\sqrt{N}$
7 1/32	7.03125	.8470326	22.08938	38.82898	49.43848	2.65165
7 1/16	7.06250	.8489585	22.18755	39.17490	49.87891	2.65754
7 3/32	7.09375	.8508759	22.28573	39.52234	50.32129	2.66341
7 1/8	7.12500	.8527849	22.38390	39.87133	50.76563	2.66927
7 5/32	7.15625	.8546855	22.48208	40.22183	51.21191	2.67511
7 3/16	7.18750	.8565779	22.58025	40.57389	51.66016	2.68095
7 7/32	7.21875	.8584620	22.68043	40.92747	52.11035	2.68677
7 1/4	7.25000	.8603380	22.77660	41.28259	52.56250	2.69258
7 9/32	7.28125	.8622060	22.87478	41.63924	53.01660	2.69838
7 5/16	7.31250	.8640659	22.97295	41.99743	53.47266	2.70416
7 11/32	7.34375	.8659179	23.07113	42.35714	53.93066	2.70994
7 3/8	7.37500	.8677620	23.16930	42.71840	54.39063	2.71570
7 13/32	7.40625	.8695984	23.26748	43.08119	54.85254	2.72144
7 7/16	7.43750	.8714270	23.36565	43.44551	55.31641	2.72718
7 15/32	7.46875	.8732479	23.46383	43.81136	55.78223	2.73290
7 1/2	7.50000	.8750613	23.56200	44.17875	56.25000	2.73861
7 17/32	7.53125	.8768671	23.66018	44.54768	56.71973	2.74431
7 9/16	7.56250	.8786654	23.75835	44.91813	57.19141	2.75000
7 19/32	7.59375	.8804563	23.85653	45.29012	57.66504	2.75568
7 5/8	7.62500	.8822398	23.95470	45.66444	58.14063	2.76134
7 21/32	7.65625	.8840161	24.05288	46.03870	58.61816	2.76699
7 11/16	7.68750	.8857851	24.15105	46.41530	59.09766	2.77263
7 23/32	7.71875	.8875460	24.24923	46.79343	59.57910	2.77826
7 3/4	7.75000	.8893017	24.34740	47.17309	60.06250	2.78388
7 25/32	7.78125	.8910494	24.44558	47.55428	60.54785	2.78949
7 13/16	7.81250	.8927900	24.54375	47.93702	61.03516	2.79508
7 27/32	7.84375	.8945238	24.64193	48.32189	61.52520	2.80067
7 7/8	7.87500	.8962506	24.74010	48.70708	62.01563	2.80624
7 29/32	7.90625	.8979706	24.83828	49.09440	62.50879	2.81181
7 15/16	7.93750	.8996837	24.93645	49.48327	63.00391	2.81736
7 31/32	7.96875	.9013902	25.03463	49.87367	63.50098	2.82290
8	8.00000	.9030900	25.13280	50.26560	64.00000	2.82843
8 1/32	8.03125	.9047831	25.23098	50.65907	64.50098	2.83395
8 1/16	8.06250	.9064697	25.32915	51.05407	65.00391	2.83945
8 3/32	8.09375	.9081508	25.42733	51.45060	65.50879	2.84496
8 1/8	8.12500	.9098234	25.52550	51.84868	66.01563	2.85044
8 5/32	8.15625	.9114906	25.62368	52.24827	66.52441	2.85592
8 3/16	8.18750	.9131513	25.72185	52.64941	67.03516	2.86138
8 7/32	8.21875	.9148058	25.82003	53.05208	67.54785	2.86684
8 1/4	8.25000	.9164539	25.91820	53.45629	68.06250	2.87228
8 9/32	8.28125	.9180959	26.01638	53.86203	68.57910	2.87771
8 5/16	8.31250	.9197317	26.11455	54.26930	69.09766	2.88314
8 11/32	8.34375	.9213613	26.21273	54.67810	69.61816	2.88856
8 3/8	8.37500	.9229848	26.31090	55.08845	70.14063	2.89396
8 13/32	8.40625	.9246023	26.40908	55.50032	70.66504	2.89935
8 7/16	8.43750	.9262138	26.50725	55.91373	71.19141	2.90474
8 15/32	8.46875	.9278193	26.60543	56.32868	71.71973	2.91011
8 1/2	8.50000	.9294189	26.70360	56.74515	72.25000	2.91548
8 17/32	8.53125	.9300127	26.80178	57.16316	72.78223	2.91747
8 9/16	8.56250	.9326006	26.89995	57.58271	73.31641	2.92618
8 19/32	8.59375	.8341827	26.99813	58.00379	73.85254	2.93151
8 5/8	8.62500	.9357591	27.09630	58.42640	74.39063	2.93684
8 21/32	8.65625	.9373298	27.19448	58.85054	74.93066	2.94215
8 11/16	8.68750	.9388948	27.29265	59.27623	75.47266	2.94746
8 23/32	8.71875	.9404542	27.39083	59.70344	76.01660	2.95275
8 3/4	8.75000	.9420081	27.48900	60.13219	76.56250	2.95804

N^3	$\sqrt[3]{N}$	N^4	$\sqrt[4]{N}$	N^5	$\sqrt[5]{N}$	N
347.6143	1.91577	2444.163	1.62839	17185.52	1.47709	7 1/32
352.2698	1.91861	2487.906	1.63020	17570.84	1.47840	7 1/16
356.0667	1.92143	2532.232	1.63200	17963.02	1.47971	7 3/32
361.7051	1.92425	2577.149	1.63379	18362.19	1.48101	7 1/8
366.4852	1.92706	2622.660	1.63558	18768.41	1.48230	7 5/32
371.3074	1.92986	2668.772	1.63736	19181.80	1.48360	7 3/16
376.1716	1.93265	2715.489	1.63914	19602.44	1.48488	7 7/32
381.0781	1.93544	2762.816	1.64091	20030.42	1.48617	7 1/4
386.0271	1.93822	2810.760	1.64267	20465.85	1.48745	7 9/32
391.0188	1.94098	2859.325	1.64443	20908.81	1.48872	7 5/16
396.0533	1.94375	2908.516	1.64619	21359.41	1.48999	7 11/32
401.1309	1.94650	2958.341	1.64794	21817.77	1.49126	7 3/8
406.2516	1.94924	3008.801	1.64968	22283.93	1.49252	7 13/32
411.4158	1.95198	3059.905	1.65142	22758.04	1.49368	7 7/16
416.6235	1.95471	3111.657	1.65315	23240.19	1.49503	7 15/32
421.8750	1.95743	3164.063	1.65488	23730.47	1.40628	7 1/2
427.1705	1.96015	3217.128	1.65660	24229.00	1.49752	7 17/32
432.5100	1.96286	3270.857	1.65831	24735.86	1.49876	7 9/16
437.8939	1.96556	3325.257	1.66022	25251.17	1.50000	7 19/32
443.3223	1.96825	3380.333	1.66173	25775.04	1.50123	7 5/8
448.7953	1.97093	3436.089	1.66343	26307.56	1.50246	7 21/32
454.3133	1.97361	3492.533	1.66512	26848.85	1.50365	7 11/16
459.8762	1.97628	3549.669	1.66681	27399.01	1.50491	7 23/32
465.4844	1.97795	3607.504	1.66850	27958.16	1.50612	7 3/4
471.1380	1.98160	3666.042	1.67018	28526.39	1.50734	7 25/32
476.8372	1.98425	3725.291	1.67185	29103.84	1.50854	7 13/16
482.5883	1.98689	3785.350	1.67352	29691.13	1.50975	7 27/32
488.3731	1.98953	3845.938	1.67518	30286.76	1.51096	7 7/8
494.2101	1.99216	3907.349	1.67684	30892.48	1.51215	7 29/32
500.0935	1.99478	3969.493	1.67850	31507.85	1.51334	7 15/16
506.0234	1.99739	4032.375	1.68015	32132.99	1.51453	7 31/32
512.0000	2.00000	4096.000	1.68179	32768.00	1.51572	8
518.0235	2.00160	4160.376	1.68343	33413.02	1.51690	8 1/32
524.0940	2.00520	4225.508	1.68507	34068.16	1.51808	8 1/16
530.2118	2.00778	4291.402	1.68670	34733.54	1.51925	8 3/32
536.3770	2.01036	4358.063	1.68832	35409.26	1.52042	8 1/8
542.5897	2.01294	4425.497	1.68995	36095.46	1.52159	8 5/32
548.8504	2.01550	4493.713	1.69156	36792.28	1.52276	8 3/16
555.1589	2.01807	4562.712	1.69317	37499.79	1.52392	8 7/32
561.5156	2.02062	4632.504	1.69478	38218.16	1.52507	8 1/4
567.9207	2.02317	4703.093	1.69638	38947.49	1.52623	8 9/32
574.3743	2.02571	4774.487	1.69798	39687.92	1.52738	8 5/16
580.8765	2.02824	4846.688	1.69957	40439.55	1.52852	8 11/32
587.4278	2.03077	4919.708	1.70116	41202.56	1.52967	8 3/8
594.0280	2.03330	4993.548	1.70275	41977.01	1.53081	8 13/32
600.6775	2.03581	5068.217	1.70433	42763.08	1.53194	8 7/16
607.3765	2.03832	5143.720	1.70590	43560.89	1.53308	8 15/32
614.1250	2.04083	5220.625	1.70748	44375.13	1.53421	8 1/2
620.9234	2.04177	5297.253	1.70806	45192.19	1.53463	8 17/32
627.7718	2.04582	5375.296	1.71061	46025.97	1.53646	8 9/16
634.6703	2.04830	5454.198	1.71217	46872.01	1.53758	8 19/32
641.6192	2.05078	5533.966	1.71372	47730.46	1.53869	8 5/8
648.6185	2.05326	5614.604	1.71527	48601.42	1.53981	8 21/32
655.6687	2.05572	5696.122	1.71682	49485.06	1.54092	8 11/16
662.7697	2.05819	5778.524	1.71836	50381.51	1.54202	8 23/32
669.9219	2.06064	5861.816	1.71990	51290.89	1.54313	8 3/4

N	Decimal	Logarithm	Circumference	Area	N^2	$\sqrt{N}$
8 25/32	8.78125	.9435564	27.58718	60.56247	77.11035	2.96332
8 13/16	8.81250	.9450991	27.68535	60.99429	77.66016	2.96859
8 27/32	8.84375	.9466365	27.78353	61.42763	78.21191	2.97384
8 7/8	8.87500	.9481684	27.88170	61.86253	78.76563	2.97909
8 29/32	8.90625	.9496949	27.97988	62.29894	79.32129	2.98433
8 15/16	8.93750	.9512161	28.07805	62.73690	79.87891	2.98957
8 31/32	8.96875	.9527319	28.17623	63.17638	80.43848	2.99479
9	9.00000	.9542425	28.27440	63.61740	81.00000	3.00000
9 1/32	9.03125	.9557479	28.37258	64.05995	81.56347	3.00520
9 1/16	9.06250	.9572480	28.47075	64.50405	82.12891	3.01040
9 3/32	9.09375	.9587430	28.56893	64.94967	82.69629	3.01558
9 1/8	9.12500	.9602320	28.66710	65.39683	83.26563	3.02076
9 5/32	9.15625	.9617177	28.76528	65.84551	83.83691	3.02593
9 3/16	9.18750	.9631974	28.86345	66.29574	84.41016	3.03109
9 7/32	9.21875	.9646721	28.96163	66.74749	84.98535	3.03624
9 1/4	9.25000	.9661417	29.05980	67.20079	85.56250	3.04138
9 9/32	9.28125	.9676065	29.15798	67.65561	86.14160	3.04652
9 5/16	9.31250	.9690663	29.25615	68.11198	86.72266	3.05164
9 11/32	9.34375	.9705212	29.35433	68.56987	87.30566	3.05676
9 3/8	9.37500	.9719713	29.45250	69.02930	87.89063	3.06186
9 13/32	9.40625	.9734165	29.55068	69.49026	88.47754	3.06696
9 7/16	9.43750	.9748570	29.64885	69.95276	89.06641	3.07205
9 15/32	9.46875	.9762927	29.74703	70.41679	89.65723	3.07714
9 1/2	9.50000	.9777236	29.84520	70.88235	90.25000	3.08221
9 17/32	9.53125	.9791499	29.94338	71.34945	90.84473	3.08727
9 9/16	9.56250	.9805714	30.04155	71.81808	91.44141	3.09233
9 19/32	9.59375	.9819884	30.13973	72.28903	92.04004	3.09738
9 5/8	9.62500	.9834007	30.23790	72.75995	92.64063	3.10242
9 21/32	9.65625	.9848085	30.33608	73.23318	93.24316	3.10745
9 11/16	9.68750	.9862117	30.43425	73.70795	93.84766	3.11248
9 23/32	9.71875	.9876104	30.53243	74.18425	94.45410	3.11751
9 3/4	9.75000	.9890046	30.63060	74.66209	95.06250	3.12250
9 25/32	9.78125	.9903944	30.72878	75.14146	95.67285	3.12750
9 13/16	9.81250	.9917797	30.82695	75.62237	96.28516	3.13249
9 27/32	9.84375	.9931606	30.92513	76.10480	96.89941	3.13748
9 7/8	9.87500	.9945371	31.02330	76.58878	97.51563	3.14245
9 29/32	9.90625	.9959093	31.12148	77.07428	98.13379	3.14742
9 15/16	9.93750	.9972771	31.21965	77.56132	98.75391	3.15238
9 31/32	9.96875	.9986407	31.31783	78.04990	99.37598	3.15733
10	10.00000	1.0000000	31.41600	78.54000	100.00000	3.16228
10 1/32	10.03125	1.0013554	31.51418	79.03166	100.6260	3.16722
10 1/16	10.06250	1.0027059	31.61235	79.52481	101.2539	3.17214
10 3/32	10.09375	1.0040526	31.71053	80.01954	101.8838	3.17707
10 1/8	10.12500	1.0053950	31.80870	80.51575	102.5156	3.18198
10 5/32	10.15625	1.0066905	31.90688	81.01354	103.1494	3.18673
10 3/16	10.18750	1.0080676	32.00505	81.51290	103.7852	3.19179
10 7/32	10.21875	1.0093977	32.10323	82.01375	104.4229	3.19668
10 1/4	10.25000	1.0107239	32.20140	82.51609	105.0625	3.20156
10 9/32	10.28125	1.0120459	32.29958	83.02000	105.7041	3.20644
10 5/16	10.31250	1.0133640	32.39775	83.52548	106.3477	3.21131
10 11/32	10.34375	1.0146780	32.49593	84.03246	106.9932	3.21617
10 3/8	10.37500	1.0159881	32.59410	84.54093	107.6406	3.22105
10 13/32	10.40625	1.0172942	32.69228	85.05097	108.2900	3.22587
10 7/16	10.43750	1.0185965	32.79045	85.56258	108.9414	3.23071
10 15/32	10.46875	1.0198949	32.88863	86.07568	109.5947	3.23555
10 1/2	10.50000	1.0211893	32.98680	86.59035	110.2500	3.24037

N^3	$\sqrt[3]{N}$	N^4	$\sqrt[4]{N}$	N
677.1253	2.06309	5946.006	1.72143	$8\frac{25}{32}$
684.3802	2.06554	6031.101	1.72297	$8\frac{13}{16}$
691.6866	2.06798	6117.103	1.72448	$8\frac{27}{32}$
699.0450	2.07041	6204.025	1.72601	$8\frac{7}{8}$
706.4552	2.07284	6291.867	1.72752	$8\frac{29}{32}$
713.9178	2.07526	6380.640	1.72904	$8\frac{15}{16}$
721.4326	2.07767	6470.349	1.73054	$8\frac{31}{32}$
729.0000	2.08008	6561.000	1.73205	9
736.6201	2.08249	6652.600	1.73355	$9\frac{1}{32}$
744.2933	2.08489	6745.158	1.73505	$9\frac{1}{16}$
752.0194	2.08728	6838.676	1.73654	$9\frac{3}{32}$
759.7989	2.08967	6933.165	1.73803	$9\frac{1}{8}$
767.6317	2.09205	7028.628	1.73952	$9\frac{5}{32}$
775.5184	2.09443	7125.075	1.74100	$9\frac{3}{16}$
783.4587	2.09680	7222.510	1.74248	$9\frac{7}{32}$
791.4531	2.09917	7320.942	1.74396	$9\frac{1}{4}$
799.5017	2.10153	7420.375	1.74543	$9\frac{9}{32}$
807.6048	2.10389	7520.820	1.74690	$9\frac{5}{16}$
815.7623	2.10624	7622.278	1.74836	$9\frac{11}{32}$
823.9746	2.10858	7724.763	1.74982	$9\frac{3}{8}$
832.2419	2.11092	7828.275	1.75127	$9\frac{13}{32}$
840.5642	2.11326	7932.825	1.75273	$9\frac{7}{16}$
848.9419	2.11559	8038.419	1.75418	$9\frac{15}{32}$
857.3750	2.11791	8145.063	1.75562	$9\frac{1}{2}$
865.8638	2.12023	8252.765	1.75706	$9\frac{17}{32}$
874.4085	2.12255	8361.532	1.75850	$9\frac{9}{16}$
883.0091	2.12486	8471.369	1.75994	$9\frac{19}{32}$
891.6660	2.12716	8582.286	1.76137	$9\frac{5}{8}$
900.3793	2.12946	8694.287	1.76280	$9\frac{21}{32}$
909.1492	2.13175	8807.383	1.76422	$9\frac{11}{16}$
917.9758	2.13404	8921.577	1.76564	$9\frac{23}{32}$
926.8594	2.13633	9036.879	1.76706	$9\frac{3}{4}$
935.8001	2.13861	9153.294	1.76847	$9\frac{25}{32}$
944.7981	2.14089	9270.832	1.76988	$9\frac{13}{16}$
953.8536	2.14315	9389.496	1.77121	$9\frac{27}{32}$
962.9668	2.14542	9509.298	1.77270	$9\frac{7}{8}$
972.1379	2.14768	9630.241	1.77410	$9\frac{29}{32}$
981.3670	2.14994	9752.335	1.77549	$9\frac{15}{16}$
990.6543	2.15220	9875.585	1.77689	$9\frac{31}{32}$
1000.000	2.15444	10000.00	1.77828	10
1009.405	2.15668	10125.59	1.77967	$10\frac{1}{32}$
1018.867	2.15891	10252.35	1.78105	$10\frac{1}{16}$
1028.390	2.16115	10380.31	1.78243	$10\frac{3}{32}$
1037.971	2.16337	10509.45	1.78381	$10\frac{1}{8}$
1047.611	2.16553	10639.80	1.78514	$10\frac{5}{32}$
1057.312	2.16782	10771.37	1.78656	$10\frac{3}{16}$
1067.072	2.17003	10904.14	1.78793	$10\frac{7}{32}$
1076.891	2.17224	11038.13	1.78929	$10\frac{1}{4}$
1086.770	2.17445	11173.36	1.79065	$10\frac{9}{32}$
1096.711	2.17665	11309.83	1.79201	$10\frac{5}{16}$
1106.711	2.17884	11447.55	1.79337	$10\frac{11}{32}$
1116.771	2.18104	11586.50	1.79472	$10\frac{3}{8}$
1126.893	2.18322	11726.72	1.79607	$10\frac{13}{32}$
1137.076	2.18541	11868.23	1.79742	$10\frac{7}{16}$
1147.320	2.18759	12011.00	1.79877	$10\frac{15}{32}$
1157.625	2.18976	12155.06	1.80010	$10\frac{1}{2}$

N	Decimal	Logarithm	Circumference	Area	N^2	$\sqrt{N}$
$10\frac{17}{32}$	10.53125	1.0224800	33.08498	87.10651	110.9072	3.24519
$10\frac{9}{16}$	10.56250	1.0237668	33.18315	87.62425	111.5664	3.25000
$10\frac{19}{32}$	10.59375	1.0250498	33.28133	88.14348	112.2275	3.25480
$10\frac{5}{8}$	10.62500	1.0263289	33.37950	88.66428	112.8906	3.25960
$10\frac{21}{32}$	10.65625	1.0276045	33.47768	89.18665	113.5557	3.26439
$10\frac{11}{16}$	10.68750	1.0288761	33.57585	89.71051	114.2227	3.26917
$10\frac{23}{32}$	10.71875	1.0301442	33.67403	90.23586	114.8916	3.27395
$10\frac{3}{4}$	10.75000	1.0314085	33.77220	90.76279	115.5625	3.27872
$10\frac{25}{32}$	10.78125	1.0326691	33.87038	91.29128	116.2354	3.28348
$10\frac{13}{16}$	10.81250	1.0339261	33.96855	91.82127	116.9102	3.28824
$10\frac{27}{32}$	10.84375	1.0351795	34.06673	92.35275	117.5869	3.29299
$10\frac{7}{8}$	10.87500	1.0364293	34.16490	92.88580	118.2656	3.29773
$10\frac{29}{32}$	10.90625	1.0376755	34.26308	93.42042	118.9463	3.30246
$10\frac{15}{16}$	10.93750	1.0389181	34.36125	93.95654	119.6289	3.30719
$10\frac{31}{32}$	10.96875	1.0401571	34.45943	94.49422	120.3135	3.31191
11	11.00000	1.0413927	34.55760	95.03340	121.0000	3.31662
$11\frac{1}{32}$	11.03125	1.0426248	34.65578	95.57415	121.6885	3.32133
$11\frac{1}{16}$	11.06250	1.0438533	34.75395	96.11639	122.3789	3.32603
$11\frac{3}{32}$	11.09375	1.0450784	34.85213	96.66020	123.0713	3.33073
$11\frac{1}{8}$	11.12500	1.0463000	34.95030	97.20550	123.7656	3.33542
$11\frac{5}{32}$	11.15625	1.0475143	35.04848	97.75238	124.4619	3.34008
$11\frac{3}{16}$	11.18750	1.0487331	35.14665	98.30082	125.1602	3.34477
$11\frac{7}{32}$	11.21875	1.0499444	35.24483	98.85076	125.8604	3.34944
$11\frac{1}{4}$	11.25000	1.0511525	35.34300	99.40219	126.5625	3.35410
$11\frac{9}{32}$	11.28125	1.0523572	35.44118	99.95519	127.2666	3.35876
$11\frac{5}{16}$	11.31250	1.0535586	35.53935	100.50976	127.9727	3.36341
$11\frac{11}{32}$	11.34375	1.0547566	35.63753	101.06582	128.6807	3.36805
$11\frac{3}{8}$	11.37500	1.0559514	35.73570	101.62338	129.3906	3.37268
$11\frac{13}{32}$	11.40625	1.0571429	35.83388	102.18250	130.1025	3.37732
$11\frac{7}{16}$	11.43750	1.0583311	35.93205	102.74320	130.8164	3.38194
$11\frac{15}{32}$	11.46875	1.0595161	36.03023	103.30539	131.5322	3.38655
$11\frac{1}{2}$	11.50000	1.0606978	36.12840	103.86915	132.2500	3.39117
$11\frac{17}{32}$	11.53125	1.0618765	36.22658	104.43440	132.9697	3.39577
$11\frac{9}{16}$	11.56250	1.0630518	36.32475	105.00123	133.6914	3.40037
$11\frac{19}{32}$	11.59375	1.0641240	36.42293	105.56954	134.4150	3.40457
$11\frac{5}{8}$	11.62500	1.0653930	36.52110	106.13943	135.1406	3.40955
$11\frac{21}{32}$	11.65625	1.0665589	36.61928	106.71088	135.8682	3.41413
$11\frac{11}{16}$	11.68750	1.0677216	36.71745	107.28383	136.5977	3.41870
$11\frac{23}{32}$	11.71875	1.0688814	36.81563	107.85828	137.3291	3.42327
$11\frac{3}{4}$	11.75000	1.0700379	36.91380	108.43429	138.0625	3.42783
$11\frac{25}{32}$	11.78125	1.0711915	37.01198	109.01187	138.7979	3.43238
$11\frac{13}{16}$	11.81250	1.0723418	37.11015	109.59095	139.5352	3.43693
$11\frac{27}{32}$	11.84375	1.0734893	37.20833	110.17151	140.2744	3.44148
$11\frac{7}{8}$	11.87500	1.0746336	37.30650	110.75365	141.0156	3.44601
$11\frac{29}{32}$	11.90625	1.0757750	37.40468	111.33736	141.7588	3.45054
$11\frac{15}{16}$	11.93750	1.0769134	37.50285	111.92256	142.5039	3.45507
$11\frac{31}{32}$	11.96875	1.0780488	37.60103	112.50934	143.2510	3.45959
12	12.00000	1.0791812	37.69920	113.09760	144.0000	3.46410

N^3	$\sqrt[3]{N}$	N^4	$\sqrt[4]{N}$	N
1167.992	2.19193	12300.41	1.80144	$10\frac{17}{32}$
1178.420	2.19410	12447.06	1.80278	$10\frac{9}{16}$
1188.910	2.19626	12595.01	1.80411	$10\frac{19}{32}$
1199.463	2.19842	12744.29	1.80544	$10\frac{5}{8}$
1210.078	2.20057	12894.90	1.80676	$10\frac{21}{32}$
1220.760	2.20272	13046.83	1.80808	$10\frac{11}{16}$
1231.494	2.20486	13200.08	1.80941	$10\frac{23}{32}$
1242.297	2.20700	13354.69	1.81072	$10\frac{3}{4}$
1253.163	2.20914	13510.67	1.81204	$10\frac{25}{32}$
1264.092	2.21127	13668.00	1.81335	$10\frac{13}{16}$
1275.083	2.21340	13826.68	1.81466	$10\frac{27}{32}$
1286.138	2.21552	13986.75	1.81596	$10\frac{7}{8}$
1297.258	2.21764	14148.22	1.81727	$10\frac{29}{32}$
1308.441	2.21976	14311.07	1.81857	$10\frac{15}{16}$
1319.689	2.22187	14475.34	1.81987	$10\frac{31}{32}$
1331.000	2.22398	14641.00	1.82116	11
1342.376	2.22608	14808.09	1.82245	$11\frac{1}{32}$
1353.817	2.22818	14976.60	1.82374	$11\frac{1}{16}$
1365.322	2.23028	15146.55	1.82503	$11\frac{3}{32}$
1376.892	2.23237	15317.92	1.82631	$11\frac{1}{8}$
1388.528	2.23445	15490.77	1.82759	$11\frac{5}{32}$
1400.230	2.23655	15665.08	1.82887	$11\frac{3}{16}$
1411.996	2.23863	15840.84	1.83015	$11\frac{7}{32}$
1423.828	2.24070	16018.07	1.83142	$11\frac{1}{4}$
1435.726	2.24278	16196.79	1.83269	$11\frac{9}{32}$
1447.691	2.24484	16377.01	1.83396	$11\frac{5}{16}$
1459.722	2.24691	16558.72	1.83522	$11\frac{11}{32}$
1471.818	2.24897	16741.93	1.83649	$11\frac{3}{8}$
1483.982	2.25103	16926.66	1.83775	$11\frac{13}{32}$
1496.213	2.25308	17112.93	1.83900	$11\frac{7}{16}$
1508.510	2.25513	17300.72	1.84026	$11\frac{15}{32}$
1520.875	2.25718	17490.06	1.84152	$11\frac{1}{2}$
1533.307	2.25922	17680.94	1.84276	$11\frac{17}{32}$
1545.807	2.26126	17873.39	1.84401	$11\frac{9}{16}$
1558.374	2.26312	18067.39	1.84515	$11\frac{19}{32}$
1571.010	2.26533	18262.98	1.84650	$11\frac{5}{8}$
1583.714	2.26736	18460.17	1.84773	$11\frac{21}{32}$
1596.486	2.26938	18658.93	1.84897	$11\frac{11}{16}$
1609.325	2.27140	18859.28	1.85021	$11\frac{23}{32}$
1622.234	2.27342	19061.25	1.85144	$11\frac{3}{4}$
1635.213	2.27543	19264.86	1.85267	$11\frac{25}{32}$
1648.260	2.27744	19470.07	1.85390	$11\frac{13}{16}$
1661.375	2.27945	19676.91	1.85512	$11\frac{27}{32}$
1674.560	2.28145	19885.40	1.85634	$11\frac{7}{8}$
1687.816	2.28345	20095.56	1.85756	$11\frac{29}{32}$
1701.140	2.28545	20307.36	1.85878	$11\frac{15}{16}$
1714.535	2.28744	20520.85	1.86000	$11\frac{31}{32}$
1728.000	2.28943	20736.00	1.86121	12

INDEX

E

F

J

L

M

R